Source Consulted: None

Problem 1.

10.1.3 (c) $3\times10\times10\times10\times3$
$= 3^2\times10^3$
$= 9{,}000$ A: 9,000 ways

2 days with only vegeterian food (3) and 3 days with all kind of food (10). If we all multiply them, we get 9,000 ways.

10.2.4 (a) $(7\times3\times6)+(3\times7\times2)$
$= 126+42$
$= 168$ A: 168 ways

For any coders, if we start with the junior, we get $7\cdot3\cdot6 = 126$. Starting with the senior, we get $3\cdot7\cdot2 = 42$. If we add them, we get 168 ways.

10.3.2 (c) Define the function $f: P_n \to B^n$ such that if $x \in P_n$, then $f(x)$ is obtained by dropping the last n bits of x. P_7 is a palindrome and contains 7-bit binary string, where each bit can be either 0 or 1. There are 2 ways when bit 1 and bit 7 is equal, bit 2 and bit 6 is equal, bit 3 and bit 5 is equal, and bit 4. Therefore, the cardinality of P_7 is equal to the total number of 16 ways. To create a bijection function between P_n and B_n, $16 = 2^n$, $2^4 = 2^n$. Therefore the value $n=4$.

10.4.2 (b) $(10\times9\times8\times7)+(10\times9\times8\times7)$
$= 5040+5040$
$= 10080$ A: 10080 different numbers

Since the last 4 digits are different, one number can't be used after the one before. Because it is 4 digits, it can be shown as $10\cdot9\cdot8\cdot7$. Also, there are two cases: starting with 824 or 825. Therefore $2(10\cdot9\cdot8\cdot7) = 10080$ different numbers.

10.5.5 (a) A: $C(35,10)\times C(35,10)$

Since the choir director is selecting a 10 subset from the 35 boys and 10 subset from the 35 girls, the number of choices is $C(35,10)\times C(35,10)$.

10.5.6 (b) A: $C(37,2)$

Since 3 of the computers in the network have a copy of a particular file, we only have to choose from 37 computers of which 2 fails. Therefore, $C(37,2)$.

Source Consulted: None

10.6.4(b) $\frac{20!}{4!\times4!\times4!\times4!\times4!}$

$= \frac{20!}{(4!)^5}$

A: $\frac{20!}{(4!)^5}$ ways

Since there are 20 comicbook that can be divided evenly so that 4 books go to each kid which is 5 kids, we can divide $20!/(4!)^5$.

10.7.3 (c) $(1\times11\times10\times9\times8)\times3$

$= 23{,}760$ ways

A: 23,760 ways.

Since there are only 3 players who play center, if 1 player plays center, the remain players will fill the spot as $1\times11\times10.9\times8$. Since there are 3 options we can multiply 3. Therefore we get 23,760 ways.

10.8.2 (b) $C(52,5)-(C(13,5)\times4^5)$

$=2{,}598{,}960-1{,}317{,}888$

$=1{,}281{,}072$

A: 1,281,072 ways

To find at least two cards with the same rank, we have to subtract the total number of 5 cards using 52 cards from total number of combination with no card with same rank. The total number of 5 cards using 52 cards will be $C(52,5)$. The total number of combination with no card with same rank can be found by the 5 cards with 13 rank combination $C(13,5)$ multiplied by 4^5, which is the possible ways that are 4 suits in a deck. Therefore, the number of 5 cards with at least two cards with same rank is 1,281,072.

10.8.4 (a) $10!-(9!\times2)$

$=3{,}628{,}800-725{,}760$

$=2{,}903{,}040$

To find when the president is not next to the vice president is subtracting to total number of making lines from the number when president and vice president are next to each other. The total number of making line is 10! and the number when president and vice president can be shown as 9!, because we can assume president and vice president are one person. Since they can switch sides, we need to multiply 2. Therefore, the number when the president is not next to the vice president is 2,903,040.

Source Consulted: None

Problem 2

10.9.2(a) A: 621 people → True
620 people → False

Assuming that there are 31 days in a month, if we divide 621 by 31, we get 20.032. Since there is a remainder, there are at least 21 who are born on the same day of the month. However, if there is only 620 people, there is no remainder if we divide by 31. Therefore, it is false if there are only 620 people.

10.9.3(b) $20 \times 12 - 11$
$= 229$
A: 229 people.

Since there are 12 month in a year, and 20 people must be selected, there should be 240 people. However, the question says at least 20 who are born in the same month, which means only 1 out 12 month needs 20 people. Therefore subtracting 240 from 11, we get 229 people

10.9.4(a) Among set $\{1,2 \ldots 13,14\}$, the sets we can make that sums up to 15 is $\{1,14\}, \{2,13\}, \{3,12\}, \{4,11\}, \{5,10\}, \{6,9\}$, and $\{7,8\}$. If we choose 8 numbers from the set, at least two of the selected numbers must sum to 15. Therefore the statement is True.

11.1.3(c) A: 5^{20}

Since there are no restriction on 5 books that can be given to 20 children, we get 5^{20} ways.

11.3.1(g) $(C(9,2) \times 2^7 + C(9,3) \times 2^6)) - C(9,2) \times C(7,3)$
$= 4,608 + 5,376 - 1,260$
$= 8,724$
A: 8,724

To find the string that has exactly 2a's or exactly 3b's, we have to add the number of the strings that have exactly 2a's and the strings that have exactly 3b's, and subtract the string that has exactly 2a's or exactly 3b's. 2a's total is $C(9,2) \times 2^7$ and 3b's total is $C(9,3) \times 2^6$. Lastly 2a's and 3b's are $C(9,2) \times C(7,3)$. Therefore the string that has exactly 2a's or exactly 3b's is 8,724.

Source Consulted: None

Problem 3

11.2.8 (a) A: $\frac{25!}{9! \times 2! \times 5! \times 7! \times 2!}$

The coefficient of the term $v^9w^2x^5y^7z^2$ is $\frac{25!}{9! \times 2! \times 5! \times 7! \times 2!}$.

11.2.8 (b) A: $C(29, 25)$

The equation for the number of coefficient is $\binom{n+m-1}{n}$. When n is the power of the equation and m is the number of variable we get $C(29, 25)$.