Source Consulted: None

P1 a) $\left(\sum_{i=1}^{n} i\right)^2 = \sum_{i=1}^{n} i^3$

Base case: for $n=1$, right side and left side of the equation are 1, therefore $\sum_{i=1}^{1} i^3 = 1 = \frac{1^2(1+1)^2}{4}$

Inductive step: Suppose that for positive integer k, $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$ then we should show that

$$\sum_{i=3}^{k+1} i^3 = \left(\frac{k(k+1)}{2} + (k+1)\right)^2 = \left(\frac{(k+2)(k+1)}{2}\right)^2 = \left(\frac{(n+1)n}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$$

starting with the left side of the equation to be proven:

$$\sum_{i=1}^{k+1} i^3 = \sum_{i=1}^{k} i^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3 = (k+1)^2\left(\frac{k^2}{4} + (k+1)\right) = (k+1)^2\frac{k^2+4k+4}{4} = \frac{n^2(n+1)^2}{4}$$

By separating the last term, by the inductive hypothesis, therefore $\sum_{i=1}^{k+1} i^3 = \frac{(k+1)^2(k+2)^2}{4}$ and $\forall n \in \mathbb{Z}^+$

b) $\sum_{i=1}^{n} 2^i(i+1) = n \cdot 2^{n+1}$

Base case: for $n=1$, right side and left side of the equation are 1, therefore

$$\sum_{i=1}^{1} 2^i(i+1) = 4 = 1 \cdot 2^2 = 4$$

Inductive step: Suppose that for positive integer k, $\sum_{i=1}^{k} 2^i(i+1) = k \cdot 2^{k+1}$, then we should show that

$$\sum_{i=1}^{k+1} 2^i(i+1) = (k+1) \cdot 2^{k+2}$$

starting with the left side of the equation to be proven:

$$\sum_{i=1}^{k+1} 2^i(i+1) = \sum_{i=1}^{k} 2^i(i+1) + 2^{k+1}(k+2) = k \cdot 2^{k+1} + 2^{k+1}(k+2) = 2^{k+1}(k+1) \cdot 2 = (k+1) \cdot 2^{k+2}$$

By separating the last term, by the inductive hypothesis, therefore $\sum_{i=1}^{k+1} 2^{k+1}(k+2) = (k+1) \cdot 2^{k+2}$ and $\forall n \in \mathbb{Z}^+$.

Source Consulted: None

P2. Base case: For $n=2$, 1 wire is required. For $n=3$, 2 wire is required.
For $n=3$, 3 wire is required.

For any two computers to be connected by a unique route, $n-1$ wire is needed. Since the network should be always connected, first and last $(k+1)$ needs 1 connection, while other ha two connection. Because each connection is counted twice, the equation for the number of wire required for k computer is $\frac{2+2(k-2)}{2}$, and $\frac{2+2(k-2)}{2}=k-1$.

By proof of contradiction since the statement that such network does not need $n-1$ wires is wrong, the proof is correct.

Source Consulted: friend

P3 a) $f_0=0, f_1=1$ and $f_k=f_{k-1}+f_{k-2}$ so $k\geq 2$

$f_0+f_1+\cdots+f_n=f_{n+2}-1$

Since $f_k=f_{k-1}+f_{k-2}$, we can say that $f_2=f_1+f_0=1$

when $P(0)$, $f_0=f_2-1$, because $f_2=f_1+f_0=1$. $P(0)$ is true.

when $P(1)$, $f_0+f_1=f_3-1$, because $f_3=f_2+f_1=2$. $P(1)$ is true.

Since $P(0)$ and $P(1)$ is true, lets assume $P(n)$ is true.

Adding f_{n+1} to both side, we get $f_0+f_1+\cdots+f_n+f_{n+1}=f_{n+1}+f_{n+2}-1=f_{n+3}-1=f_{(n+1)+2}-1$

Since $f_{n+3}=f_{n+1}+f_{n+2}$, $P(n+1)$ is True. Therefore $P(n)$ is True for all $n\geq 0$.

b) $f_0=0, f_1=1$ and $f_k=f_{k-1}+f_{k-2}$ so $k\geq 2$

$n\geq 0, f_n\leq 2^{n-1}$

When $P(0)$, $f_0=0\leq 2^{-1}=\frac{1}{2}$. $P(0)$ is true.

when $P(1)$, $f_1=1\leq 2^{1-1}=1$. $P(1)$ is true.

Assume that $P(k-1)$ and $P(k-2)$ is true

$f_k=f_{k-1}+f_{k-2}\leq 2^{k-2}+2^{k-3}=f_k\leq 2^{k-2}\cdot(1+2^{-1})=f_k\leq 2^{k-2}\cdot 1.5<2^{k-2}\cdot 2$

Using the equation we get that $P(k)$ is true.

Since $P(k)$ is true and $n\geq 0$, $f_n\leq 2^{n-1}$ is true, it is proved by strong induction.

Source Consulted: None

P4 Base case: For $n=1$, we can say Yida picks $c=1$ cookies from a jar and Nika picks $c=1$ cookies from a different jar.

Inductive Step: let $n \leq k$ true for some k

Proof by cases:

1. For $n=k+1$, if Yida picks 1 cookie from the jar leaving k cookies, Nika will also leave k cookies since they are both choosing the same number of cookie. The equation becomes $n=k+1-1=k$, which is True.
2. For $n=k+1$, if Yida picks several cookies calling 's', Nika will also pick s cookies which makes the equation $n=(k+1-s)$. Since $1<s$, $n(k+1)+s<k$ is true.

Therefore, the strategy is True for any number of cookies n by strong induction.

Source Consulted: friend

P5 Base Case: $n=0$. If $n=0$, we can start at any point, because there is no booths.

Inductive Case: Assume any road with k toll booths and k reward booths. And have road D with $k+1$ toll booths and $k+1$ reward booths. Since there is at least one reward booth and one toll booth $(k+1 \geq 1)$, call the r_{k+1} and t_{k+1}. Consider a road D', which is the same road as D, but without r_{k+1} and t_{k+1}. There is a starting point for D', by the inductive hypothesis.

Proof by cases:

case 1: Lets assume the starting point for D' is between r_{k+1} and t_{k+1}.
Starting point before r_{k+1}, we can go along the road as we earn \$1 at r_{k+1} and lose \$1 at t_{k+1}. We can travel along the road like and road D', which we can by the inductive hypothesis

case 2: Lets assume the starting point for D' is not between r_{k+1} and t_{k+1}.
Starting by passing the same booths as D', we can do by the inductive hypothesis. When passing r_{k+1}, the money will be same on D and D'. As we go around the road, since we are passing r_{k+1} first, we would always get \$0 as we go through t_{k+1}. Since we passed all of them, the money we have will be same as D'. Since the booth we passed are same on both D and D', we are able to go by the inductive hypothesis.