PS 4

"Sources Consulted: None"

Problem 1 a) $a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0$

when there is a even number of 1's in the bits it will give us 0 and a odd number of 1's will give a 1

b) $\overline{a_0}$

Negating the bit will give us the opposite number

c) $a_4 + a_3 + a_2 + a_1 + a_0$

By adding each bit, we can evaluate whether the number is 1 or 0.

d) $(a_4 \oplus a_3 \oplus a_2 \oplus a_1 \oplus a_0) \oplus (a_4 + a_3 + a_2 + a_1 + a_0) \cdot \overline{a_0}$

Problem 2 a) $(x+\bar{y})(\bar{x}+\bar{z})(y+z)(\bar{x}+\bar{v})$

satisfiable

when x, y, z, v is in order

F, F, T, F or F, F, T, T or T, T, F, F

x	y	z	v	
F	F	F	F	F
F	F	F	T	F
F	F	T	F	T
F	F	T	T	T
F	T	F	F	F
F	T	F	T	F
F	T	T	F	F
F	T	T	T	F
T	F	F	F	F
T	F	F	T	F
T	F	T	F	F
T	F	T	T	F
T	T	F	F	T
T	T	F	T	F
T	T	T	F	F
T	T	T	T	F

b) $\overline{\bar{x}\bar{y} + (x+y+z)}$ not satisfiable

$= \overline{\bar{x}\bar{y}} \cdot \overline{(x+y+z)}$

$= xy \cdot (\bar{x} \cdot \bar{y} \cdot \bar{z})$

$= x\bar{x}y \cdot xy\bar{y} \cdot xy\bar{z}$

$= 0 \cdot 0 \cdot xy\bar{z}$

$= 0$

x	y	z	
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	F

"Sources Consulted: None"

Problem 3 a) exactlyOnce$(x1, x2, x3)$

$$(x1 \land \neg(x2 \land x3)) \lor (x2 \land \neg(x1 \land x3)) \lor (x3 \land \neg(x1 \land x2))$$

$$(x1 \cdot \overline{x2} \cdot \overline{x3}) + (x2 \cdot \overline{x1} \cdot \overline{x3}) + (x3 \cdot \overline{x1} \cdot \overline{x2})$$

b) differentTimeSlots$(x1, x2, x3, y1, y2, y3)$

$$(x1 \oplus y1) + (x2 \oplus y2) + (x3 \oplus y3)$$

c) is it valid$(a1, a2, a3, b1, b2, b3, c1, c2, c3, d1, d2, d3, e1, e2, e3, f1, f2, f3)$

$$(\neg(a1 \land b1)\ \neg(a2 \land b2)\ \neg(a3 \land b3))(\neg(a1 \land c1)\neg(a2 \land c2)\neg(a3 \land c3))$$
$$(\neg(a1 \land d1)\neg(a2 \land d2)\neg(a3 \land d3))(\neg(b1 \land e1)\neg(b2 \land e2)\neg(b3 \land e3))$$
$$(\neg(c1 \land e1)\neg(c2 \land e2)\neg(c3 \land e3))(\neg(c1 \land d1)\neg(c2 \land d2)\neg(c3 \land d3))$$
$$(\neg(d1 \land f1)\neg(d2 \land f2)\neg(d3 \land f3))(\neg(B1 \land F1)\neg(B2 \land F2)\neg(B3 \land F3))$$

Problem 4 a)
$$\begin{array}{c} A \land M \\ \hline M \end{array}$$
simplification

b)
$$\begin{array}{c} d \lor P \\ \neg d \\ \hline P \end{array}$$
disjunctive syllogism

c)
$$\begin{array}{c} L \\ L \to W \\ \hline W \end{array}$$
modus ponens

d)
$$\begin{array}{c} S \\ \hline S \lor b \end{array}$$
addition

e)
$$\begin{array}{c} e \to a \\ a \\ \hline e \end{array}$$
Invalid when $e = f$ and $a = T$

f)
$$\begin{array}{c} h \to e \\ e \to m \\ \hline h \to m \end{array}$$
hypothetical syllogism

Problem 5 a.1) $(c \wedge h) \to j$ "Sources Consulted: friend"

$$\frac{\neg j}{-c}$$

The argument is not valid. When $c=T$, $h=j=F$, the hypothesis are both true and the conclusion $\neg c$ is false

a.2) $(c \wedge h) \to j$

$\neg j$

h

$\therefore$ — $\neg c$

$(c \wedge h) \to j$	hypothesis
$\neg j$	hypothesis
$\neg(c \wedge h)$	Modus tollens 1,2
$\neg c$	Simplication 3

The argument is valid

b) $(r \wedge \neg s) \vee (q \wedge \neg s)$

$\neg s \to ((p \wedge r) \to u)$

$u \to (s \wedge \neg t)$

$\therefore p \to q$

1	$(r \wedge \neg s) \vee (q \wedge \neg s)$	Hypothesis
2	$\neg s \wedge (r \vee q)$	Distributive law 1
3	$(r \vee q)$	Simplification 2
4	$\neg s$	Simplification 2
5	$\neg s \to ((p \wedge r) \to u)$	Hypothesis
6	$((p \wedge r) \to u)$	Modus ponens 4,5
7	$u \to (s \wedge \neg t)$	Hypothesis
8	$u \to (F \wedge \neg t)$	Complement law 4
9	$u \to F$	Domination law 8
10	$\neg u$	Conditional Identity 9
11	$(p \wedge r) \to F$	Complement law 6,10
12	$\neg(p \wedge r)$	Conditional Identity 11
13	$\neg p \vee \neg r$	De Morgan's law 12
14	$p \to \neg r$	Conditional Identity 13
15	$\neg r \to q$	Conditional Identity 3
16	$p \to q$	Hypothetical syllogism 14,15

c) $\neg r \to \neg s$

$p \to u$

$\neg t \to \neg r$

$u \to s$

$t \to q$

$\therefore p \to q$

1	$\neg r \to -s$	hypothesis
2	$\neg t \to -r$	hypothesis
3	$\neg t \to -s$	Hypothetical Syllogism 1,2
4	$p \to u$	hypothesis
5	$u \to s$	hypothesis
6	$p \to s$	Hypothetical Syllogism 4,5
7	$s \to t$	Contrapositive rule 3
8	$p \to t$	Hypothetical Syllogism 6,7
9	$t \to q$	Hypothesis
10	$p \to q$	Hypothetical Syllogism 8,9

Problem 6 "Sources Consulted: friend"

a) $\{x! \mid x \in \mathbb{N}\} \setminus \mathbb{Z}^+$
$\{0\}$, $\{x! \mid x \in \mathbb{N}\}$ can be 0 when $\mathbb{Z}^+$ can't.

b) $\{2j+1 \mid j \in \mathbb{Z}\} \cap \{nm \mid n,m \in \mathbb{N} \land n \neq m\}$
$\{\mathbb{Z}^-\}$, $\{nm \mid n,m \in \mathbb{N} \land n \neq m\}$ can't be a negative integer.

c) $\{-8, 16, -24, 32, -40, \ldots\}$
$\{-(-1)^{i+1} \cdot 8i \mid i \in \mathbb{Z}^+\}$ Alternating positive/negative multiples of 8.

d) $\{10; 1{,}000; 100{,}000; 10{,}000{,}000; 1{,}000{,}000{,}000; \ldots\}$
$\{10^{2i+1} \mid i \in \mathbb{N}\}$

e) $\{0\} \subseteq \{0\}$ versus $\{\emptyset\} \subseteq \{0\}$
$\{0\} \subseteq \{0\}$ is true since same integers can be in both subset. However subset of empty set can't be in subset of $\{0\}$.

f) $\{\emptyset\} \in \{\emptyset\}$ versus $\emptyset \in \{\emptyset\}$
An empty set can be in a set of a empty set. However the empty set should be itself.