PS 6

"Sources Consulted: a friend"

1. b) divisors(b) ∩ divisors(a−b) ⊆ divisors(a) ∩ divisors(b)

1. Suppose C is an arbitrary element of divisors(b) ∧ divisors(a−b)	Universal Instantiation
2. $C \in$ divisors(b) ∧ $C \in$ divisors(a−b)	given from, 1
3. $C \in$ divisors(b)	Simplification, 2
4. $C \in$ divisors(a−b)	Simplification, 2
5. divisors(b) $= \{C \in \mathbb{Z} : C \mid b\}$	definition of divisors(b), 3
6. divisors(a−b) $= \{C \in \mathbb{Z} : C \mid (a-b)\}$	definition of divisors(a−b), 3
7. $\exists q \in \mathbb{Z} (b = q \cdot c)$	definition of |, 4
8. $\exists q \in \mathbb{Z} ((a-b) = q \cdot c)$	definition of |, 5
9. Suppose k_1 is a particular element in $\mathbb{Z}$ where $b = k_1 \cdot c$	Existential Instantiation
10. Suppose k_2 is a particular element in $\mathbb{Z}$ where $(a-b) = k_2 \cdot c$	Existential Instantiation
11. $b = k_1 \cdot c$	algebra, 8
12. $a = b + (k_2 \cdot c)$	algebra, 9
13. $a = (k_1 \cdot c) + (k_2 \cdot c)$	algebra, 10, 11
14. $a = (k_1 + k_2) C$	algebra, 12
15. $(k_1 + k_2)$ is a particular element in $\mathbb{Z}$	algebra, 8, 9
16. $\exists q \in \mathbb{Z} (a = q \cdot c)$	Existential Generalization, 13, 14
17. $c \mid a$	definition of |, 15
18. $c \in$ divisors(a)	definition of divisors(a), 16
19. $C \in$ divisors(a) ∧ divisors(b)	Conjunction, 3, 18
$\forall x (x \in (\text{divisors}(b) \cap \text{divisors}(a-b)) \rightarrow x \in (\text{divisors}(a) \cap \text{divisors}(b)))$	Universal Generalization 1, 19

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2. b) $P(A \cap B) \subseteq P(A) \cap P(B)$

$P(A \cap B) \equiv \forall C(C \in P(A) \land C \in P(B))$

1.	Suppose C is an arbitrary element of $P(A \land B)$	Universal Instantiation
2.	$C \subseteq (A \land B)$	Def of powerset, 1
3.	$C \subseteq A$	Simplification 2
4.	$C \subseteq B$	Simplification 2
5.	$C \in P(A)$	Def of powerset 3
6.	$C \in P(B)$	Def of powerset 4
7.	$\forall C(C \in P(A) \land C \in P(B))$	Universal Generalization 2,5,6
	$\therefore P(A) \cap P(B)$	Def of $\cap$, 7

3. a) $F = \{(x,y) \mid x-y \in \mathbb{Z}\}$ R is an equivalence relation "Sources Consulted: a friend"

$xRy = x-y \in \mathbb{Z}$

When R is reflective: $\forall x \in \mathbb{Z}(xRx)$

1. Suppose c is an arbitrary integer number	Universal Instantiation
2. $c-c=0$	a fact from algebra
3. $c-c \in \mathbb{Z}$	algebra, 2
4. $\forall x \in \mathbb{Z}(x-x \in \mathbb{Z})$	Universal Generalization
$\therefore \forall x \in \mathbb{Z}(xRx)$	

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when R is symmetric: $\forall x \forall y \in \mathbb{Z}(xRy \to yRx)$

1. Suppose c,d are arbitrary integer number	Universal Instantiation
2. $c-d \in \mathbb{Z}$	def of R, 1
3. $\exists q \in \mathbb{Z}(c-d=q)$	def of int, 2
4. let k be a particular element in $\mathbb{Z}$ where $c-d=k$	Existential Instantiation, 3
5. $-(c-d)=-k$	algebra, 4
6. $d-c=-k$	algebra, 5
7. $\exists q \in \mathbb{Z}(d-c=q)$	Existential Generalization, 4, 6
8. $d-c \in \mathbb{Z}$	def of int, 7
9. dRc	def of R, 8
$\therefore \forall x \forall y \in \mathbb{Z}(xRy \to yRx)$	Universal Generalization

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when R is transitive: $\forall x \forall y \forall z \in \mathbb{Z}(xRy \land yRz \to xRz)$

1. Suppose a,b,c are abitrary integers	Universal Instantiation
2. $a-b \in \mathbb{Z}$	def of R, 1
3. $b-c \in \mathbb{Z}$	def of R, 1
4. $\exists q \in \mathbb{Z}(a-b=q)$	def of int, 2
5. $\exists q \in \mathbb{Z}(b-c=q)$	def of int, 3
6. Let k_1 be a particular integer s.t. $a-b=k_1$	Existential Instantiation, 4
7. Let k_2 be a particular integer s.t. $b-c=k_2$	Existential Instantiation, 5
8. $(a-b)-(b-c)=k_1-k_2$	Algebra, 6, 7
9. $a-c=k_1-k_2$	Algebra 8
10. $\exists q \in \mathbb{Z}(a-c=q)$	Existential Generalization, 9
11. $a-c \in \mathbb{Z}$	def of int, 10
12. aRc	def of R, 11
$\therefore \forall x \forall y \forall z(xRy \land yRz \to xRz)$	Universal Generalization 1, 12

b) when y is a negative integer, $x-y$ can't be a set of natural numbers. This is why the set R is not an equivalence relation.

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4.

a)

1. suppose (a,b), (c,d), (e,f) are arbitrary integer number	Universal Instantiation
2. $a \le c$ and $b \le d$	Hypothesis
3. $c \le e$ and $d \le f$	def of R, 1, 2
4. $a \le c \le e$	algebra, 2, 3
5. $b \le d \le f$	algebra, 2, 3
6. $(a,b)R(e,f)$	def of R, 1, 4, 5
$\therefore \forall (a,b),(c,d),(e,f) \in \mathbb{Z}^2 \left((a,b)R(c,d) \land (c,e)R(d,f) \Rightarrow (a,b)R(e,f)\right)$	Universal Generalization, 1, 5, 6

b) Yes. If $a \le c$ and $c \le e$, then $a \le e$. Also, if $b \le d$ and $d \le f$, then $b \le f$. Since a, c, e and b, d, f shows a transitivity, R is transitive