Problem Set 5

Source Consulted: None

Problem 1. a) $A-(B\cap A) = A\cap\overline{(B\cap A)}$ set subtraction law

$= A\cap(\overline{B}\cup\overline{A})$ de morgan's law

$=(A\cap\overline{B})\cup(A\cap\overline{A})$ distribution law

$=(A\cap\overline{B})\cup\emptyset$ complement law

$= A\cap\overline{B}$ identity law

$=A-B$ set subtraction law

b) $(A-B)-A=(A\cap\overline{B})-A$ set subtraction law

$=(A\cap\overline{B})\cap\overline{A}$ set subtraction law

$=(A\cap\overline{A})\cap(\overline{B}\cap\overline{A})$ distribution law

$=\emptyset\cap(\overline{B}\cap\overline{A})$ complement law

$=\emptyset$ domination law

Problem 2 a) $A\cap C=B\cap C$ is not true

ex) when $A=\{1,2,3\}$, $B=\{2,3,4\}$, and $C=\{2,3\}$

$A\cap C=\{2,3\}$ and $B\cap C=\{2,3\}$ which means $A\cap C=B\cap C$.

However $A\neq B$ so we cannot conclude $A=B$.

b) $A\cup C=B\cup C$ is not true

ex) when $A=\{1,2,3\}$, $B=\{4,5,6\}$, and $C=\{1,2,3,4,5,6\}$

$A\cup C=\{1,2,3,4,5,6\}$ and $B\cup C=\{1,2,3,4,5,6\}$ which means $A\cup C=B\cup C$

However $A\neq B$ so we cannot conclude $A=B$

c) $A\cup C=B\cup C$ and $A\cap C=B\cap C$

$$\begin{array}{l} A\cup C=B\cup C \\ \underline{A\cap C=B\cap C} \\ \therefore A=B \end{array}$$

$A=A\cap(A\cup B)$

$A=A\cap B$

Source Consulted: friend

Problem 3 $A \neq B$

a) $\neg(A \subseteq B \land B \subseteq A)$

$\equiv \neg(\forall x \in U(x \in A \to x \in B) \land \forall x \in U(x \in B \to x \in A))$

b) $\neg(\forall x \in A(x \in B) \land \forall x \in B(x \in A))$

c) $(\exists x \in U \neg(x \in A \to x \in B)) \lor (\exists x \in U \neg(x \in B \to x \in A))$

d) $(\exists x \in A \neg(x \in B)) \lor (\exists x \in B \neg(x \in A))$

Problem 4 a) $\exists d \in D \exists r \in d \exists a \in r \forall s \in d \forall b \in s\ (a \neq b \to \neg F(a,b))$

b) $\forall d \in D \exists r \in d \exists s \in r \forall a \in r\ (a \neq s \to F(a,s)) \land$
$\neg \exists b \in r(b \neq s \to F(b,s)) \land b \neq a)$

Problem 5 Source Consulted: friend

c) Test case 1.

A = ['happiness', 'is', 'not', 'a', 'gift'], B = ['happiness', 'is', 'a', 'gift']

Similarity1 (A, B) = 0.8

A = [1,1,1,1,1], B = [1,1,0,1,1]

Similarity2 (A, B) = 0.894

Test case 2

A = ['happiness', 'is', 'not', 'a', 'gift'], B = ['happiness', 'is', 'a', 'gift']

Similarity1 (A, B) = 0.8

A = [1,1,2,1,1], B = [2,1,1,0,1]

Similarity2 (A, B) = 0.802

Test case 3

A = ['happiness', 'is', 'not', 'a', 'gift'], B = ['happiness', 'is', 'a', 'gift']

Similarity1 (A, B) = 0.8

A = [1,1,2,1,1], B = [3,1,1,0,1]

Similarity2 (A, B) = 0.714

Test case 4

A = ['happiness', 'is', 'not', 'a', 'gift'], B = ['happiness', 'is', 'a', 'gift']

Similarity1 (A, B) = 0.8

A = [1,1,2,1,1], B = [4,1,1,0,1]

Similarity2 (A, B) = 0.649

Similarity 1 does not change because the repetition does not take account of the words. However, Similarity 2 changes when there is a diffence in the frequency of words