Source Consulted: None

PS. 8

P1 1.b) The proof did not show the step where we plug in the expression kw for x and
3.2 jy for z in the expression xz to get $xz=(kw)(jy)$. And then we can say that since k and j are integers, kj is also an integer such that $xz=kj \cdot wy$. We cannot just make up an integer m not explaining it.

e) The proof cannot use a same integer j for n and m. They should use
3.3 a different like k and j.

2. m) False. For example, if $x=4$, $y=1$, and $z=3$, then x divides $y+z=4$, but 4 does not
4.4 divide 1 or 3, so x does not divide y or z.

n) Since x divides $y+z$ and y, $y+z=kx$ and $y=jx$ for some integer k
4.4 and j. Since k, j, and are all integers, kx and jx are both integers. If we add z on both side for $y=jx$, we get $y+z=jx+z$. Since $y+z=kx=jx+z$, we can say that $z=kx-jx$. Since k and j are both integers, we can say that $k-j$ is an integer which means that x divides z.

3. b) We assume that for a real number x and y that x is not irrational and y is not irrational
5.3 and prove that $x+y$ is rational.

x and y are real numbers and every real number is either rational or irrational. Therefore since x and y are not irrational and x and y are real, x and y must be rational. By the definition of a rational number, $x=\frac{a}{b}$ and $y=\frac{c}{d}$, where a,b,c,d are integers and $b\neq 0$ and $d\neq 0$. Therefore $x+y=\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$. We can conclude that $x+y=\frac{ad+bc}{bd}$ is rational because $ad+bc$ is an integer and bd is a non-zero integer. Therefore $x+y$ is rational

b) We assume for real numbers x and y, it is not the case that $x>10$ or $y>10$ and prove
5.4 that $x+y\leq 20$.

Since it is not true that $x>10$ and $y>10$, by De Morgan's law, the inequalities $x>10$ and $y>10$ are false. Therefore, it must be true that $x\leq 10$ and $y\leq 10$. Since both x and y are lower or equal to 10, we can add y to both sides of the inequality $x\leq 10$ to get that $x+y\leq 10+y$. We can also add both side of the inequality $y\leq 10$ by 10 to get that $y+10\leq 20$. Putting the inequalities together gives: $x+y\leq y+10\leq 20$, and therefore $x+y\leq 20$.

Source Consulted: None

P1 3d) Using proof by contrapositive, assume $x \geq 0$, where x is a real number. Since $x \geq 0$, then
5.5
$x^3 \geq 0$ and $2x \geq 0$. Since x^3 and $2x$ is all greater than 0, their sum is also greater than 0. $x^3+2x \geq 0$. Therefore the theorm is true.

e) Using proof by contrapositive, assume n and m are integers such that m and n are
5.5
even, then n^2+m^2 is even. Since n is an even integer, then $n=2k$ for some integer k. Since m is an even integer, then $m=2j$ for some integer j. Pluggin in $2k$ for n and $2j$ for m into the expression n^2+m^2 gives $n^2+m^2=(2k)^2+(2j)^2=4k^2+4j^2=2(2k^2+2j^2)$. Since k and j are integers, $(2k^2+2j^2)$ is also an integer. Since $n^2+m^2=2(2k^2+2j^2)$ is equal to the definition of even numbers, we can conclude that n^2+m^2 is even.

Source Consulted: None

P2 1a) Proof by contrapositive. We shall assume that 3 does not divide n, then 3 does not divide n^2
6.3
Since $3 \nmid n$, $n=3k+1$ for some integer k and $n=3k+2$ for some integer k. Let $n=3k+1$. If we square both side, we get $n^2=(3k+1)^2=9k^2+6k+1=3(3k^2+2k)+1$, where we assume $3k^2+2k$ is an integer. Therefore, $3\nmid n^2$, because n^2 can't be both $3s$ for some integer s and $3t+1$ for some integer t. Let $n=3k+2$. If we square both side, we get $n^2=(3k+2)^2=9k^2+12k+4$ $=3(3k^2+4k+1)+1$, and assume $3k^2+4k+1$ is an integer. Same as before, $3\nmid n^2$ because n^2 can't be both $3y$ for some integer y and $3z+1$ for some integer z. We can conclude that if $3\nmid n$, then $3\nmid n^2$.

1b) Proof by contradiction. Suppose that $\sqrt{3}$ is rational. Therefore $\sqrt{3}$ can be expressed as the ratio of two
6.3
integers $\frac{n}{d}$, where $d\neq 0$. Squaring both side of the equation $\sqrt{3}=\frac{n}{d}$ gives $3=\frac{n^2}{d^2}$. Then multiplying both sides of the equations by d^2 gives $3d^2=n^2$. 3 dividing d^2 means 3 divides n, where we get $n=3s$ for some integer s. $n^2=9s^2$, and we can assume that $d^2=3r^2$. Therefore, d^2 is multiple of 3 and also d is multiple of 3. This means n and d have a common factor of 3, however this contradicts the idea that $\sqrt{3}$ is a rational number.

2.c) $(x+y+z)/3$ $\geq$ x or y or z
6.6
Proof by contradiction. Assume that there exists three real numbe x, y, and z such that at least one of x, y and z is greater or equal to the average of three real number $\frac{x+y+z}{3}$. Let $a=\frac{x+y+z}{3}$ be their average value. Suppose x, y and z are greater than a. If $a+a+a < x+y+z$ $= a < \frac{x+y+z}{3} = a$. We get the contradiction. Therefore the assumption is false.

h) For all integers x and y, $x^2-4y \neq 2$.
6.6
Proof by contradiction. Assume that $x^2-4y=2$. By simple algebra, $x^2=4y+2=2(2y+1)$. Let $y+1$ be an integer which makes $2(y+1)$ an even number. Since $2(y+1)$ is even, x must be even because odd times itself is always odd. If we let $x=2z$, where $2z$ is an integer. $(2z)^2=2(2y+1)=4z^2=2(2y+1)$. Dividing 2 will make the equation $2z^2=2y+1$, which is also $x^2=2y+1$. This equation shows that x^2 is an odd number. However we said that x is even, which contractics the fact that x^2 is odd. Therefore $x^2-4y=2$ is false.

Source Consulted: None

P3 $b_1 = a \cdot q_1 + r_1$, $b_2 = a \cdot q_2 + r_2$
$0 \leq r_1 \leq a$ & $0 \leq r_2 \leq a$

Proof by cases:

Case 1: $r_1 - r_2 \geq 0$. If we subtract b_1 and b_2, $b_1 - b_2 = a \cdot (q_1 - q_2) + r_1 - r_2$. In this equation, we can say that $b_1 - b_2$ is the dividen, a is the divisors, $q_1 - q_2$ is the quotient, and $r_1 - r_2$ is the remainder. Therefore $b_1 - b_2$ has a remainder $r_1 - r_2$ when divided by a.

Case 2: $r_1 - r_2 < 0$. According to the division algorithm, the remainder can't be less than 0. So if we subtract 1 from the quotients and add a to the remainder, we get the equation $b_1 - b_2 = a \cdot (q_1 - q_2 - 1) + r_1 - r_2 + a$, which is equal to $b_1 - b_2 = a \cdot (q_1 - q_2) + r_1 - r_2$. In this case $r_1 - r_2 + a$ is the remainder and it is not less than 0, so it is proved that $b_1 - b_2$ has a remainder $r_1 - r_2 + a$ when divided by a.

The above two cases prove that $b_1 - b_2$ has a remainder $r_1 - r_2$ or $r_1 - r_2 + a$ when divided by a.

Source Consulted: friend

P4. Proof that $T=U$ i.e. $T \subseteq U$ & $U \subseteq T$

$T = \{d : d|x \text{ and } d|y\}$

$U = \{d : d|y \text{ and } d|(x\%y)\}$

Suppose $d \in T$ and $d \in U$. Applying the divisional algorithm, such that $x = qy + r$, $r = x\%y$. By the definition of divisors, if $d|x$ and $d|qy$, $d|(x-qy)$, which is equivalent to $d|r$. So, $d|y$ and $d|x\%y$, which is $d \in U$ and $T \subseteq U$. Using the same method, since $x\%y = r$, by divional algorithm and definition of divisors $y|x-r$. For arbitrary element q, $qy = x-r$, which is equivalent to $x = qy+r$. Because of the definition of divisors, $d|y$ then $d|qy$. But $d|x\%y$ since $x\%y = r$. So $d|qy+r$, therefore $d|x$ and $d|y$ $d \in T$ thus $U \subseteq T$. In conclusion $T=U$.

Source Consulted = friend

P5. Let a and $b \neq 0$ be two integers. Then, there exist two integers c and $d \neq 0$ such that $\frac{a}{b} = \frac{c}{d}$ and $\gcd(c,d) = 1$

Let $g = \gcd(a,b)$. This shows that $g \mid a$ and $g \mid b$. Using the division algorithm, we can write the equation $a = g \cdot c$ and $b = g \cdot d$. Since $\frac{a}{b} = \frac{gc}{g \cdot d} = \frac{c}{d}$, we proofed that $\frac{a}{b} = \frac{c}{d}$

Using proof of contradiction, suppose $\gcd(c,d) \neq 1$. ?