Problem 1

"Sources Consulted: None"

a) zyBook 1.4.2b) $\neg(P \leftrightarrow q)$ and $\neg P \leftrightarrow q$

A: True

P	q	¬P	P↔q	¬P↔q	¬(P↔q)
F	F	T	T	F	F
F	T	T	F	T	T
T	F	F	F	T	T
T	T	F	T	F	F

zyBook 1.4.5b) $\neg j \rightarrow (l \vee \neg r)$

$(r \wedge \neg l) \rightarrow j$

A: logically equivalent

j	l	r	¬j	¬l	¬r	l∨¬r	r∧¬l	¬j→(l∨¬r)	(r∧¬l)→j
F	F	F	T	T	T	T	F	T	T
F	F	T	T	T	F	F	F	F	F
F	T	F	T	F	T	T	F	T	T
F	T	T	T	F	F	T	F	T	T
T	F	F	F	T	T	T	F	T	T
T	F	T	F	T	F	F	T	T	T
T	T	F	F	F	T	T	F	T	T
T	T	T	F	F	F	T	F	T	T

c) $j \rightarrow \neg l$

$\neg j \rightarrow l$

A: not logically equivalent

j	l	¬j	¬l	j→¬l	¬j→l
F	F	T	T	T	F
F	T	T	F	T	T
T	F	F	T	T	T
T	T	F	F	F	T

zyBook 1.4.6 b) $P \vee e$

$\neg(P \vee e)$

$\neg P \wedge \neg e$

A: The applicant does not have written permission from his parents and is not at least 18 years old.

b) zyBook 1.5.4 b) $P \rightarrow q$ is not logically equivalent to $\neg P \rightarrow \neg q$

If P=T and q=F, then $P \rightarrow q$ is false and $\neg P \rightarrow \neg q$ is True.

Therefore $P \rightarrow q$ is not logically equivalent to $\neg P \rightarrow \neg q$

d) $q \rightarrow P$ is logically equivalent to $\neg P \rightarrow \neg q$

If P=T and q=F, then $q \rightarrow P$ is true and $\neg P \rightarrow \neg q$ is True.

Therefore $q \rightarrow P$ is logically equivalent to $\neg P \rightarrow \neg q$.

zyBook 1.5.5a) P = x is a rational number

q = y is a rational number

r = x−y is a rational number

- $(P \wedge \neg q) \rightarrow \neg r$
- $(P \wedge r) \rightarrow q$

A: They are logically equivalent

P	q	r	¬q	¬r	P∧¬q	P∧r	(P∧¬q)→¬r	(P∧r)→q
F	F	F	T	T	F	F	T	T
F	F	T	T	F	F	F	T	T
F	T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	T	T
T	F	F	T	T	T	F	T	T
T	F	T	T	F	T	T	F	F
T	T	F	F	T	F	F	T	T
T	T	T	F	F	F	T	T	T

Problem 2.

"Sources Consulted: None"

a) $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$(P \rightarrow Q) \wedge (Q \rightarrow P) \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ Conditional Identities

$(\neg P \vee Q) \wedge (\neg Q \vee P) \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ Conditional Identities

$((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge P) \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ Distributive Law

$((\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)) \vee ((\neg P \wedge P) \vee (Q \wedge P)) \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ Distributive Law

$(\neg P \wedge \neg Q) \vee (Q \wedge P) \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ Complement Law

$(\neg Q \wedge \neg P) \vee (Q \wedge P) \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$ Commutative Law

$\top \equiv \top$

b) $((P \rightarrow Q) \vee (P \rightarrow R)) \rightarrow (P \rightarrow (Q \vee R))$ is a tautology.

$\neg((P \rightarrow Q) \vee (P \rightarrow R)) \vee (P \rightarrow (Q \vee R))$ Conditional Identities

$\neg((\neg P \vee Q) \vee (\neg P \vee R)) \vee (\neg P \vee (Q \vee R))$ Conditional Identities

$\neg(\neg P \vee (Q \vee R)) \vee (\neg P \vee (Q \vee R))$ Distributive Law

$\neg(\neg P \vee (Q \vee R)) \vee (\neg P \vee (Q \vee R))$ Complement Law

c) $\neg((P \rightarrow Q) \vee (Q \rightarrow R))$ is a contradiction.

$\neg((\neg P \vee Q) \vee (\neg Q \vee R))$ Conditional Identities

$\neg((Q \vee \neg Q) \vee (R \vee \neg P))$ Distributive laws

$\neg \top \vee (R \vee \neg P) \equiv \top$

d) M = "I am motivated", S = "I study", P = "I pass"

1. If I am motivated, I study $M \rightarrow S$
2. If I study, I pass $S \rightarrow P$
3. It is not true that if I am motivated, I pass $\neg(M \rightarrow P)$

$(M \rightarrow S) \wedge (S \rightarrow P) \wedge \neg(M \rightarrow P) \equiv \top$

$(\neg M \vee S) \wedge (\neg S \vee P) \wedge \neg(M \rightarrow P)$

$((\neg M) \vee (S \wedge \neg S) \text{ or } P) \wedge \neg(M \rightarrow P)$ associative laws

$((\neg M) \vee \text{False}) \vee P) \wedge \neg(M \rightarrow P)$ complement laws

$((\neg M) \vee P) \wedge \neg(M \rightarrow P)$ identity laws

$(M \rightarrow P)$ and not $(M \rightarrow P)$ conditional identities

False complement laws

Problem 3

"Sources Consulted: a friend"

a) Alice: $\neg(r_2 \wedge r_3)$, Bob: $\neg(r_1 \wedge r_3)$ (when Chloe is wearing red)

$\neg(r_2 \wedge r_3) \wedge \neg(r_1 \wedge r_3)$

$= (\neg r_2 \vee \neg r_3) \wedge (\neg r_1 \vee r_3)$

$= \neg r_3 \vee (\neg r_2 \wedge \neg r_1)$

$r_3 \rightarrow (\neg r_2 \wedge \neg r_1)$

r_3 is false. Chloe wearing blue

b) Alice and Bob would not know the colors of the hats

options: $\neg r_1 \wedge \neg r_2$ / $\neg r_1 \wedge r_2$ / $r_1 \wedge \neg r_2$

Problem 4

A = F_1 = Living forever by choosing box 1.

B = F_2 = Living forever by choosing box 2.

C = M_1 = Living miserably by choosing box 1.

D = M_2 = Living miserably by choosing box 2.

Box1 $= F_1 \wedge M_2$

Box2 $= \overline{(F_1 \wedge M_2)} \vee (F_2 \wedge M_1)$

a) $(F_1 \wedge M_2) \oplus (\overline{(F_1 \wedge M_2)} \vee (F_2 \wedge M_1))$

b) $(F_1 M_2) \oplus ((\overline{F_1 M_2}) + (F_2 M_1))$

c) $(F_1 M_2) \oplus ((\overline{F_1 M_2}) + (\overline{F_2 M_1}))$

$= ((\overline{F_1 M_2})((\overline{F_1 M_2}) + (\overline{F_2 M_1}))) + ((\overline{\overline{F_1 M_2}})((F_1 M_2) + (F_2 M_1)))$

$= (\overline{F_1 M_2})(F_1 M_2) + (\overline{F_1 M_2})(\overline{F_2 M_1}) + (F_1 M_2) \cdot (\overline{F_1 M_2}) \cdot (\overline{F_2 M_1})$

$= (\overline{F_1 M_2})(\overline{F_2 M_1}) + (F_1 M_2 \cdot \overline{F_1 M_2}) \cdot (\overline{F_2 M_1})$

$= (\overline{F_1 M_2})(\overline{F_2 M_1})$

d) Since $F_2 \cdot M_1 \cdot (\overline{F_1 M_2})$ is True, the function is True when F_2 is true

e) Box1 $= F_1 \wedge M_2$, and there is a not over $F_1 \wedge M_2$ which mean Box 1 is a lie

f) Yes, since these two boxs can't be true or false at the same time, they are mutually exclusive