Quantitative management modeling
Assignment 3 solution

In the previous assignment we Consider the problem

The Weigela Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

1. Solve the problem using lpsolve, or any other equivalent library in R.

   See oakintoy_3.rmd

2. Identify the shadow prices, dual solution, and reduced costs

   Shadow prices:
   0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

   Dual solution:
   0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

   Reduced cost:
   0 0 -24 -40 0 0 -360 -120 0

3. Further, identify the sensitivity of the above prices and costs. That is, specify the range of shadow prices and reduced cost within which the optimal solution will not change.

```
> cbind(get.sensitivity.rhs(lprec)$duals[1:11],
get.sensitivity.rhs(lprec)$dualsfrom[1:11], get.sensitivity.rhs(lprec)$dualstill[1:11])
      price         lower        upper
 [1,]  0.00 -1.000000e+30 1.000000e+30
 [2,]  0.00 -1.000000e+30 1.000000e+30
 [3,]  0.00 -1.000000e+30 1.000000e+30
 [4,] 12.00  1.122222e+04 1.388889e+04
 [5,] 20.00  1.150000e+04 1.250000e+04
 [6,] 60.00  4.800000e+03 5.181818e+03
 [7,]  0.00 -1.000000e+30 1.000000e+30
 [8,]  0.00 -1.000000e+30 1.000000e+30
 [9,]  0.00 -1.000000e+30 1.000000e+30
[10,] -0.08 -2.500000e+04 2.500000e+04
[11,]  0.56 -1.250000e+04 1.250000e+04

> cbind(get.sensitivity.rhs(lprec)$duals[12:20],
get.sensitivity.rhs(lprec)$dualsfrom[12:20], get.sensitivity.rhs(lprec)$dualstill[12:20])
     cost         lower        upper
 [1,]    0 -1.000000e+30 1.000000e+30
 [2,]    0 -1.000000e+30 1.000000e+30
 [3,]  -24 -2.222222e+02 1.111111e+02
 [4,]  -40 -1.000000e+02 1.000000e+02
 [5,]    0 -1.000000e+30 1.000000e+30
 [6,]    0 -1.000000e+30 1.000000e+30
 [7,] -360 -2.000000e+01 2.500000e+01
 [8,] -120 -4.444444e+01 6.666667e+01
 [9,]    0 -1.000000e+30 1.000000e+30
```

4. Formulate the dual of the above problem and solve it. Does the solution agree with what you observed for the primal problem?

Objective Function:

Maximize $Z = 420 L_1 + 360 M_1 + 300 S_1 + 420 L_2 + 360 M_2 + 300 S_2 + 420 L_3 + 360 M_3 + 300 S_3)$

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x1	L1	+ M1	+ S1							≤ 750
x2				L2	+ M2	+ S2				≤ 900
x3							L3	+ M3	+ S3	≤ 450
x4	20 L1	+ 15 M1	+ 12 S1							≤ 13000
x5				20 L2	+ 15 M2	+ 12 S2				≤ 12000
x6							20 L3	+ 15 M3	+12 S3	≤ 5000
x7	L1			+ L2			+ L3			≤ 900

X8	M1	+ M2		+ M3	≤ 1200
X9	S1	+ S2		+ S3	≤ 750
X10	900 L1 +900 M1 + 900 S1 -750 L2 + 450M2 +750S2				= 0
X11	450 L1 +450 M1 + 450 S1		- 750L3 - 750M3 - 750S3		= 0

L1, L2, L3, M1, M2, M3, S1, S2, S3 ≥ 0

- ## The dual problem

N: the dual objective variable to minimize cost

Objective Function:

Min.

N = 750 x1 + 900 x2 + 450 x3 + 13000 x4 + 12000 x5 + 5000 x6 + 900 x7 + 1200 x8 + 750 x9 + 0 x10 + 0 x11)

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x1 + 20 x4 + x7 +900 x10 + 450 x11 ≥ 420
x1 + 15 x4 + d8 +900 x10 + 450 x11 ≥ 360
x1 + 12 g4 + a9 +900 x10 + 450 x11 ≥ 300
x2 + 20 x5 + g7 -750 x10 ≥ 420
x2 + 15 x5 + d8 -750 x10 ≥ 360
x2 + 12 x5 + d9 -750 x10 ≥ 300
x3 + 20 x6 + g7 -750 x11 ≥ 420
x3+ 15 x6 + d8 -750 x11 ≥ 360
x3 + 12 x6 + a9 -750 x11 ≥ 300

x1, x2, x3, x4, x5, x6, x7, x8, x9 ≥ 0
x10, x11 = unrestricted

See oakintoy-dual_3.rmd for solving the dual LP problem.

The solution of the dual is the same as the shadow price in the primal problem. The optimal objective value is the same as that of the primal problem.