A First Course in

LINEAR ALGEBRA

Lecture Notes

for Math 1503

6.2: Complex Numbers; Polar Form

A First Course in Linear Algebra

Lecture Slides

These lecture slides were originally developed by Karen Seyffarth of the University of Calgary. Edits, additions, and revisions have been made to these notes by the editorial team at Lyryx Learning to accompany their text A First Course in Linear Algebra based on K. Kuttler's original text.

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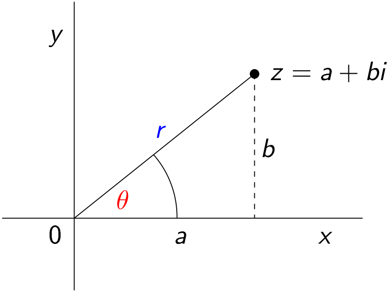
Complex Numbers in Polar Form

Suppose $z = a + bi$, and let $r = |z| = \sqrt{a^2 + b^2}$. Then r is the distance from z to the origin. Denote by θ the angle that the line through 0 and z makes with the positive x-axis (measured clockwise).



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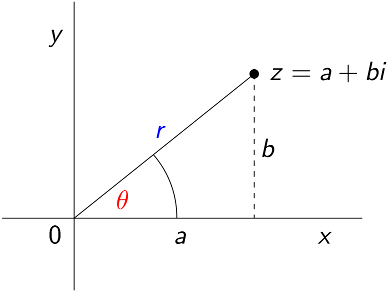

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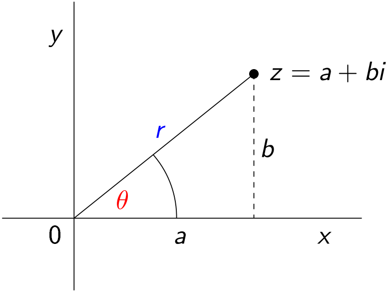

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θ is called an argument of z, and is denoted $\arg z$.

The Polar Form

Definition (Polar Form of a Complex Number)

Let z be a complex number with $|z| = r$ and $\arg z = \theta$. Then

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is called a polar form of z.

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Let z be a complex number with $|z| = r$. The principal argument of z is the unique angle $\theta = \arg z$ (measured in radians) such that

$$-\pi < \theta \leq \pi.$$

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Since sine and cosine have periodicity 2π, we may add (or subtract) multiples of 2π to any argument.

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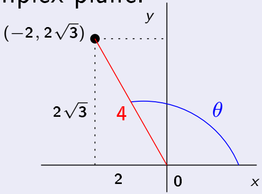
There are two approaches to finding an argument, θ. One is to graph $-2 + 2\sqrt{3}$ in the complex plane.



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By graphing the point $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$, we again determine that $\theta = \frac{2\pi}{3}$, and thus z can be written in polar form as $z = 4e^{i(2\pi/3)}$.

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Theorem

If $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$ are complex numbers, then

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If θ is any angle and n is a positive integer, then $\left(e^{i\theta}\right)^n = e^{in\theta}$. Equivalently,

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As an immediate consequence of De Moivre's Formula, we have that for any real number $r > 0$ and any positive integer n,

$$\begin{aligned} \left(re^{i\theta}\right)^n &= r^n e^{in\theta} \\ \left(r\left(\cos\theta + i\sin\theta\right)\right)^n &= r^n\left(\cos n\theta + i\sin n\theta\right) \end{aligned}$$

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&= 64e^{-\pi i} \\
&= 64e^{\pi i} \\
&= 64(\cos\pi + i\sin\pi) \\
&= -64.
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