Math 1003

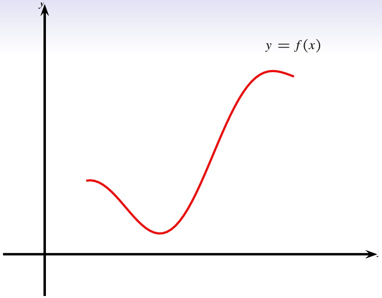
Functions and Models

Dr. Tim Alderson

University of New Brunswick Saint John

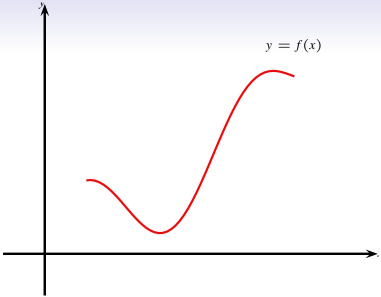
Chapter 1

Outline



Definition (Function)

A function f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.



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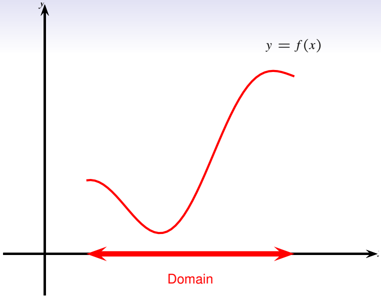

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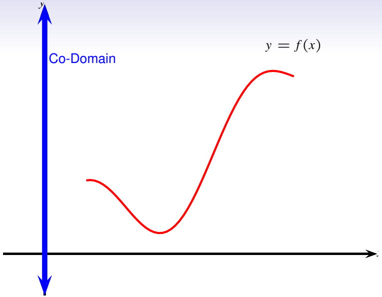
Definition (Domain)

The set D in the definition of f is called the domain of f.



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Definition (Co-domain)

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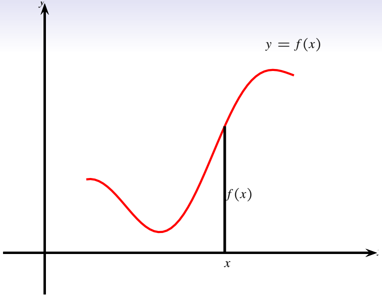

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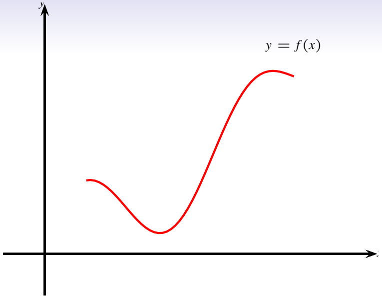
Definition (Value of f at x)

The number $f(x)$ is called *the value of f at x* and is read "f of x".



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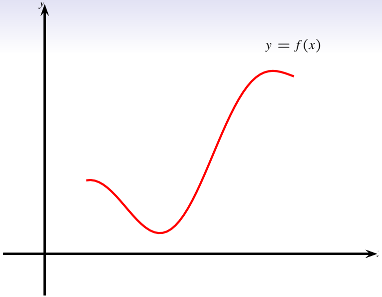

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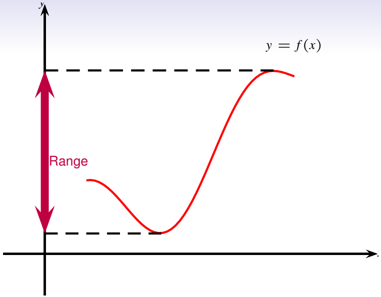
- In the expression $f(x)$, x is referred to as the *argument* of f.



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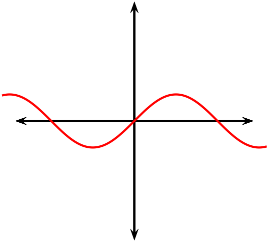
Definition (Range)

The set of all possible values taken by $f(x)$ as the element x runs over elements of D is called the range of f.

The Vertical Line Test

Question

Given a curve in the plane, is it the graph of a function or not?



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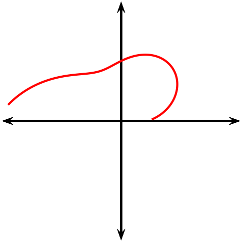

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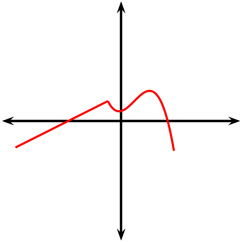

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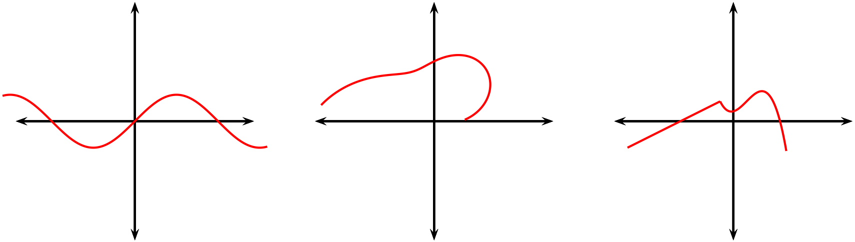
Given a curve in the plane, is it the graph of a function or not?

The answer is as follows.

Proposition (The Vertical Line Test)

A curve in the plane is the graph of a function if and only if no vertical line intersects it more than once.



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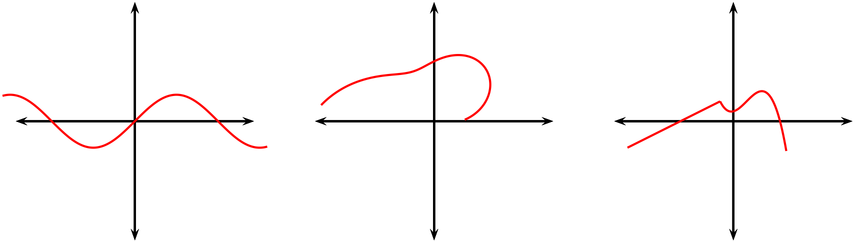

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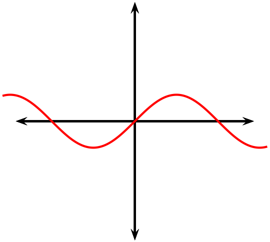
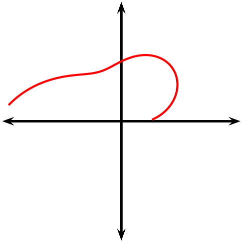


Function



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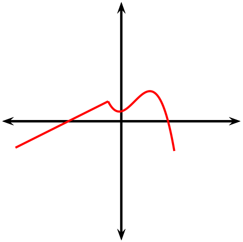

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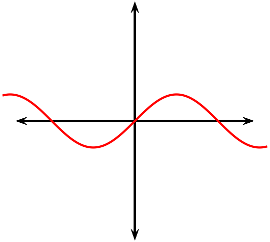

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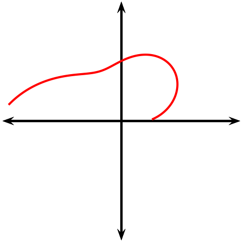

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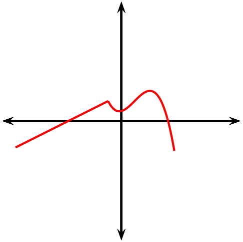

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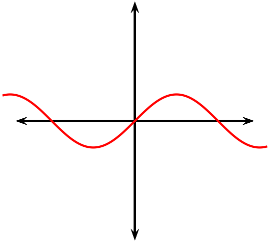

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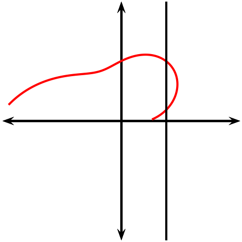
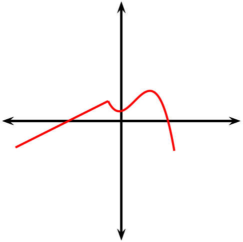


Not a function



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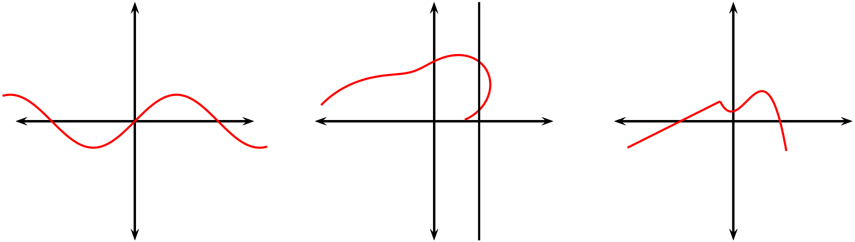

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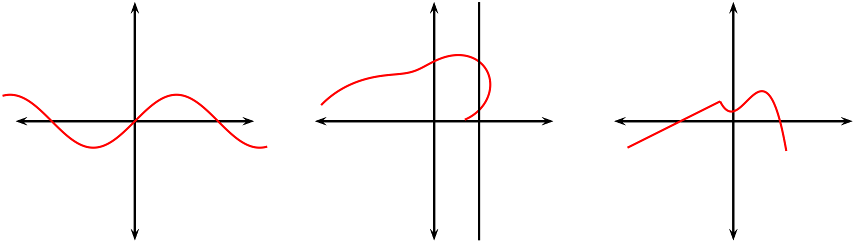

Function | Not a function | Function

Piecewise Defined Functions

Definition (Piecewise Defined Function)

A piecewise defined function is a function that is defined by different algebraic formulas on different subsets of its domain.

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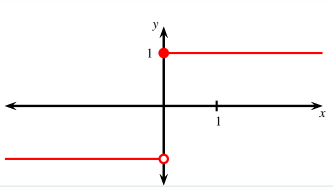
Example



$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ -1 & \text{if} \quad x < 0 \end{cases}$$

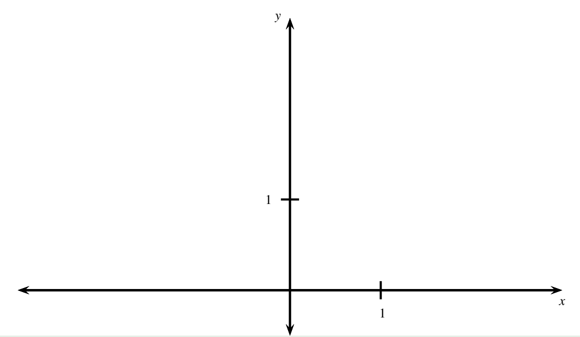
The filled red circle means $(0, 1)$ is on the curve.
The open circle means $(0, -1)$ is not on the curve.

Example

The absolute value $|x|$ of a number a is defined to be

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0. \end{cases}$$

Sketch a graph of the function $f(x) = |x|$.



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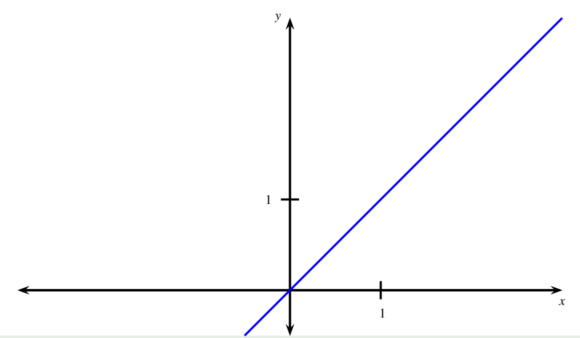

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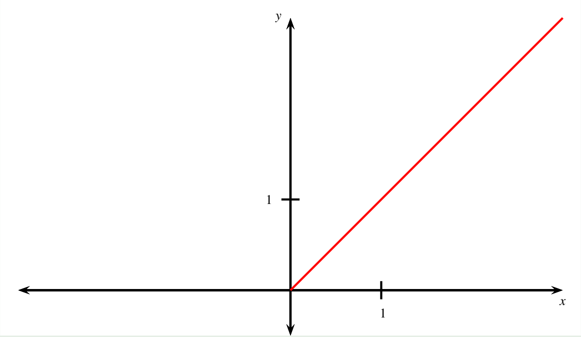

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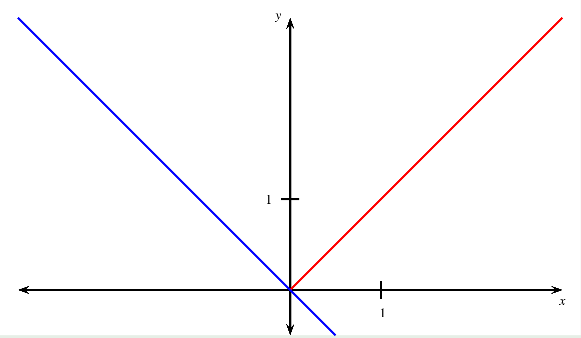

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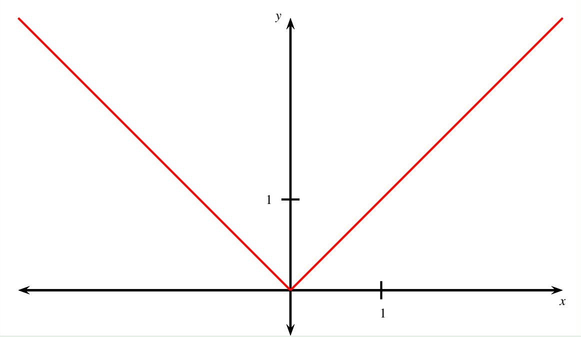

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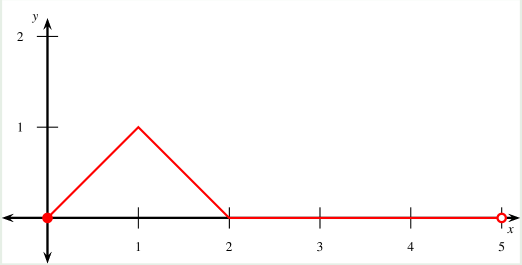

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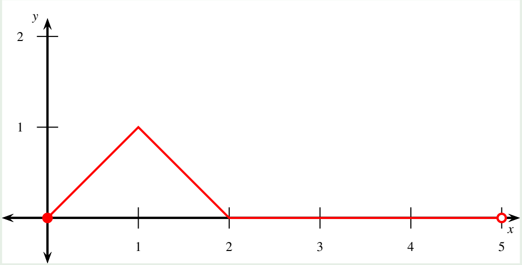

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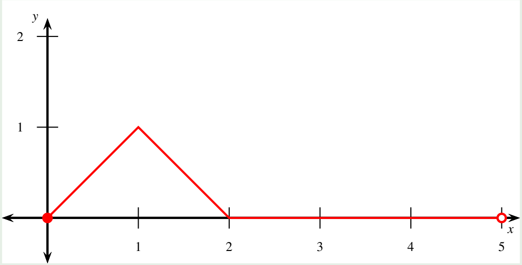

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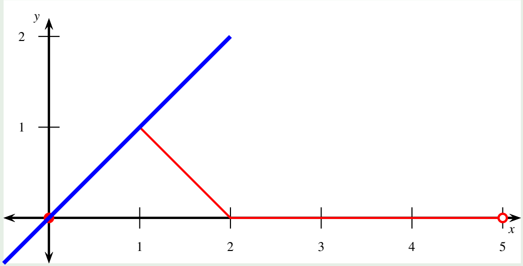

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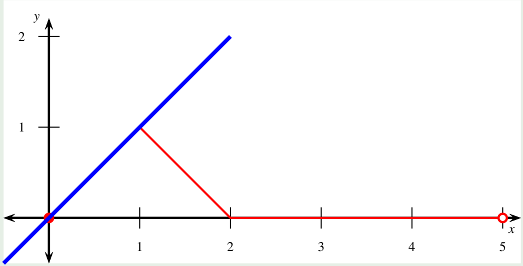

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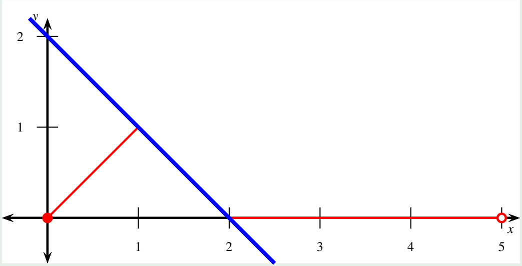

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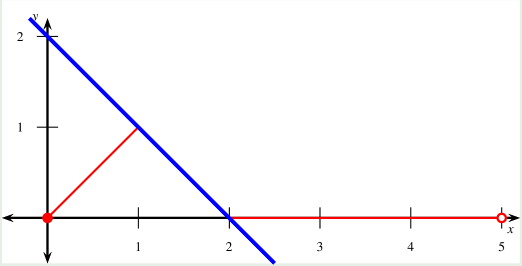

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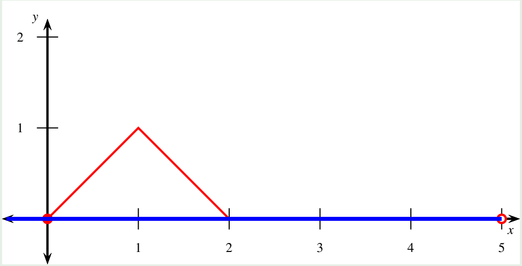

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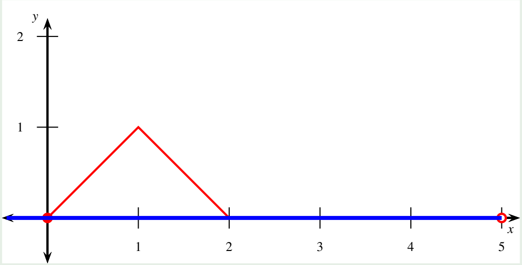

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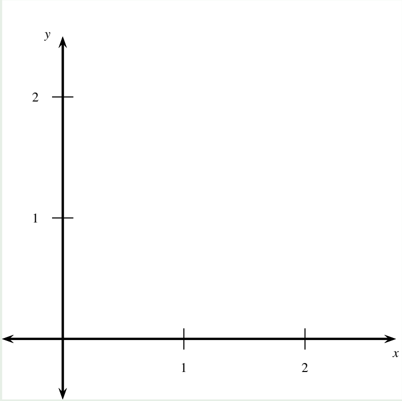
Sketch the function $f(x) = |2x - 3|$.



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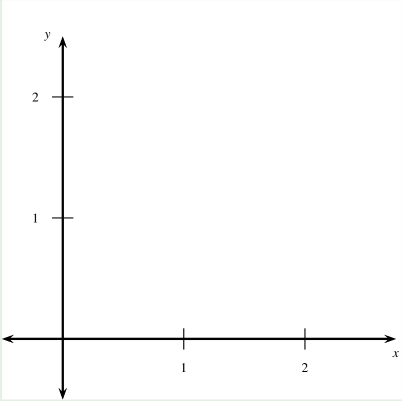

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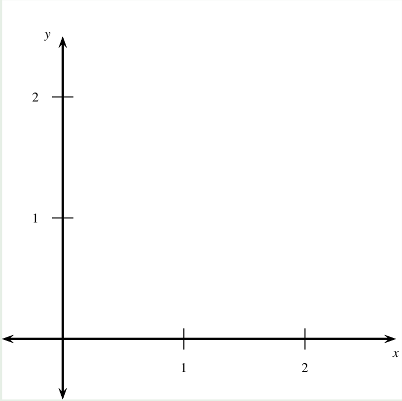

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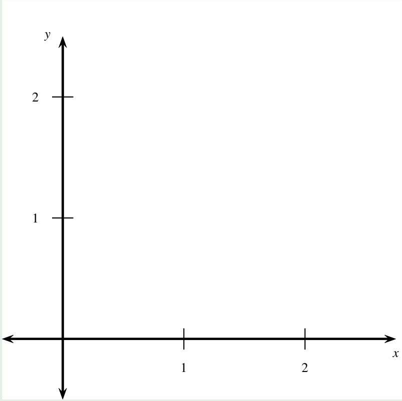

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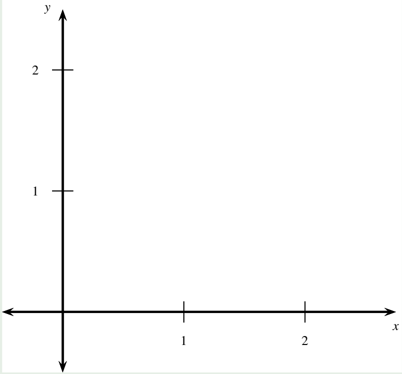

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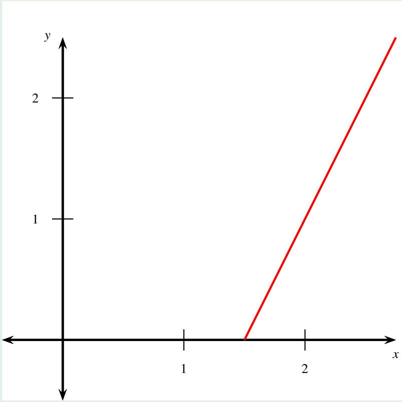

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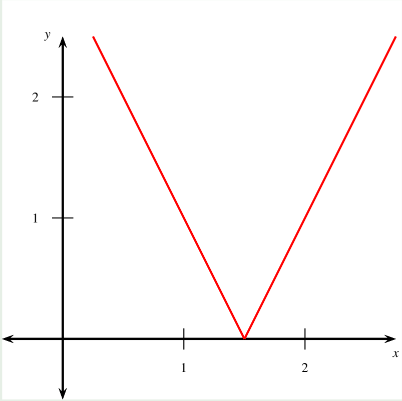

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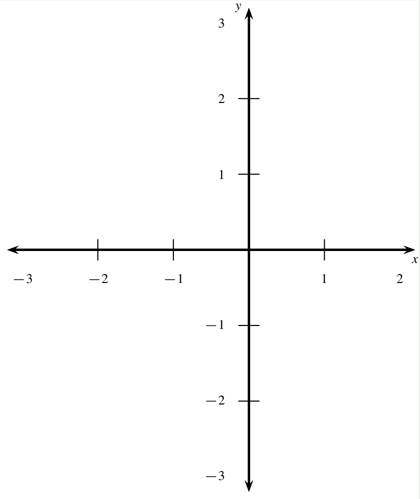
Example

Sketch the function $f(x) = \dfrac{|4x+2|}{2x+1}$.



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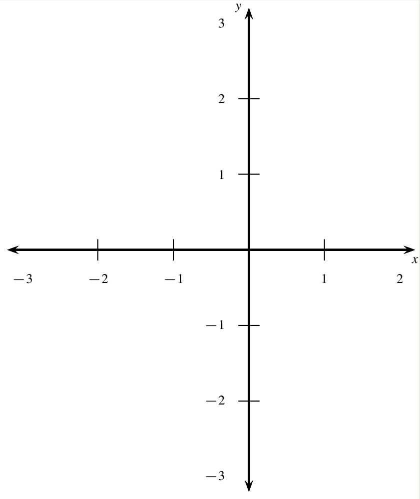

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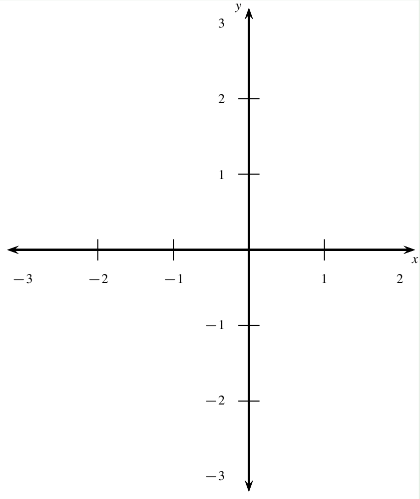

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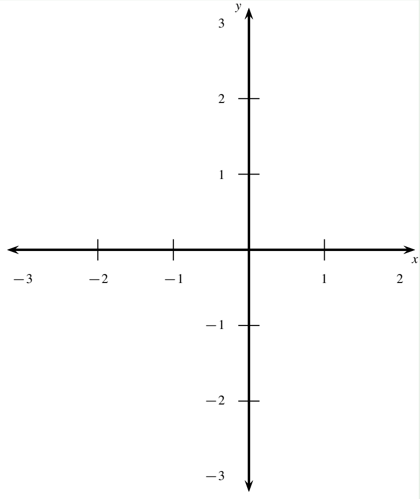

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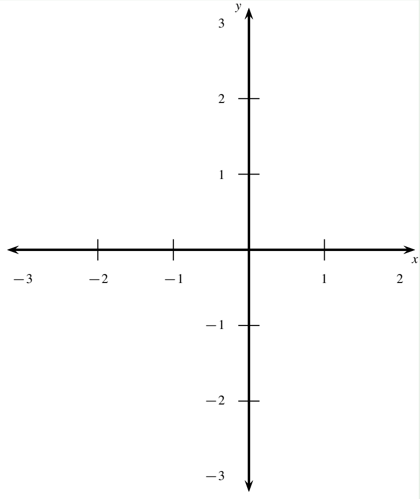

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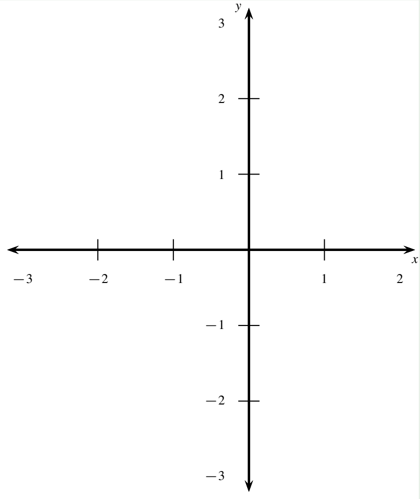

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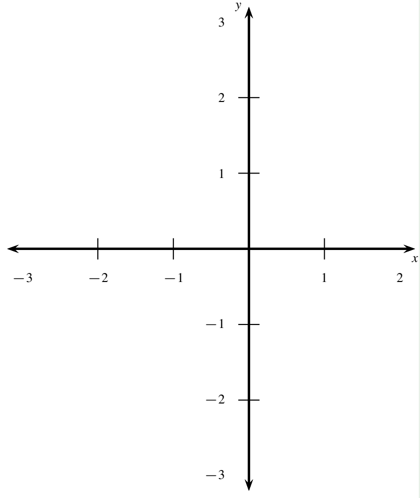

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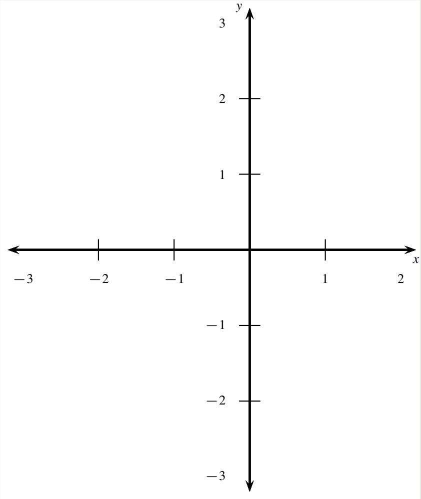

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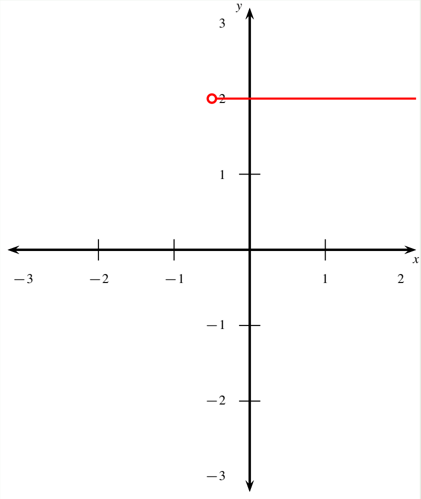

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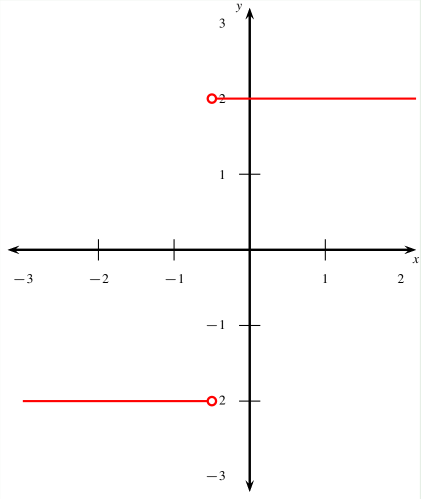

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Determine whether each of the following functions is even, odd, or neither even nor odd.

$$f(x) = x^5 + x \qquad g(x) = 1 - x^4 \qquad h(x) = 2x - 1$$

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Therefore h is neither even nor odd.

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A function f is called increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
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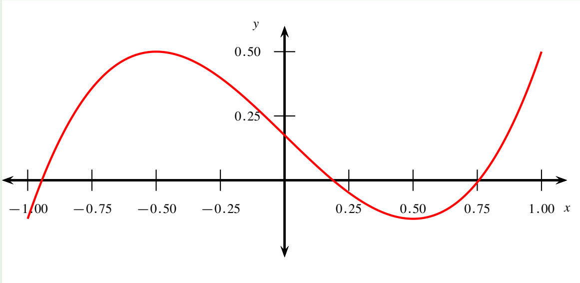

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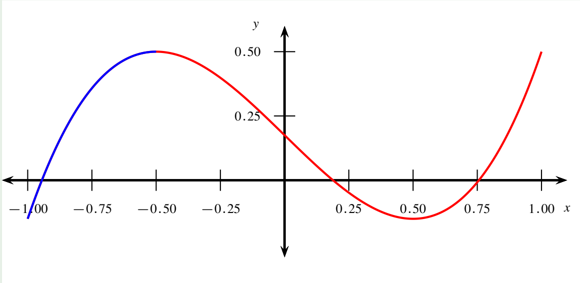

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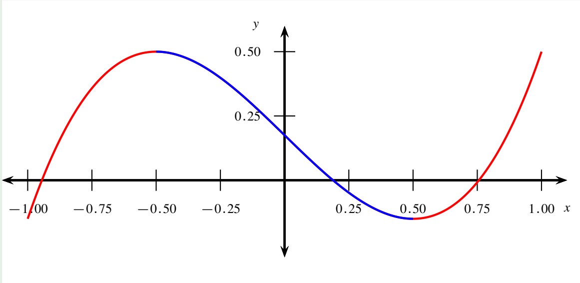

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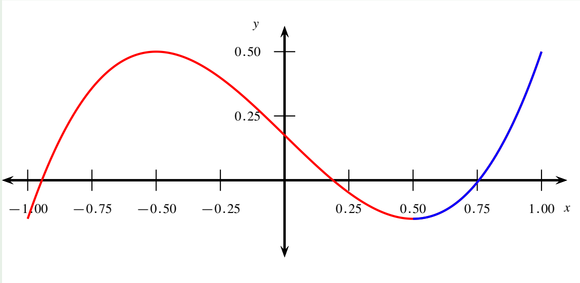

- f is increasing on $[-1, -\frac{1}{2}]$.
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- Can't take $\log x$ if $x \leq 0$.

Example (Two Functions and Their Domains)

Find the domains of the following two functions:

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$$\begin{aligned} x^2 - x - 6 &\neq 0 \\ (x-3)(x+2) &\neq 0 \\ x &\neq 3 \text{ or } -2 \end{aligned}$$

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Domain is all real numbers except 3 and -2; that is, $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.