Math 1003

Section 3.8

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Outline

Models of Population Growth

- One model for population growth assumes that the population grows at a rate proportional to its size.
- In other words, if a certain number of bacteria produce a certain number of offspring in a certain time, then ten times that many bacteria produce ten times that many offspring in the same time.
- This is plausible when the population has unlimited food and environment and no restrictions on its size.
- Name the variables:

  $t =$ *time*

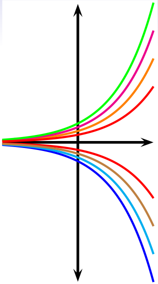
  $P =$ *the number of individuals in the population*

- The rate of growth is $\mathrm{d}P/\mathrm{d}t$.
- Then "rate of growth proportional to population size" means

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$$

where k is the proportionality constant.

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$$



- This is a differential equation.
- Exponential functions satisfy this condition.
- Let $P(t) = Ce^{kt}$ (C is a constant). Then
$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(Ce^{kt}) = Cke^{kt} = kCe^{kt} = kP(t)$$
- Therefore any function of the form $P(t) = Ce^{kt}$ satisfies the equation. We will see later that there is no other solution.
- Letting C vary over the real numbers gives a family of solutions.
- Since populations are non-negative, only solutions with $C > 0$ are relevant.