Math 1003

Section 3.9

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Outline

Related Rates

- Suppose we are pumping a balloon with air.
- The balloon's volume is increasing.
- The balloon's radius is increasing.
- The rates of increase of these quantities are related to one another.
- It is easier to measure the rate of increase of volume.

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- The balloon's radius is increasing.
- The rates of increase of these quantities are related to one another.
- It is easier to measure the rate of increase of volume.
- In a related rates problem, we compute the rate of change of one quantity in terms of the rate of change of another (which may be more easily measured).

Related Rates: Guideline

- **Draw a picture, introduce notation:** If possible, draw a schematic picture with all the relevant information, introduce variables/labels.
- **Identify:** Identify quantities whose rates of change are either given or are required.
- **Find an equation:** Find an equation that relates only the quantities with rates that are given or required.
- **Differentiate the equation with respect to t (time):** This will often involve implicit differentiation.
- **Evaluate the appropriate equation at the desired values:** The known/given values should allow you solve for the required rate.

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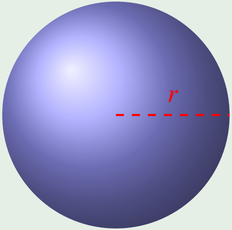

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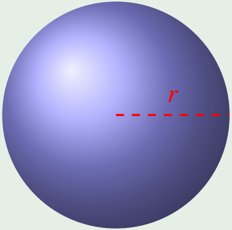

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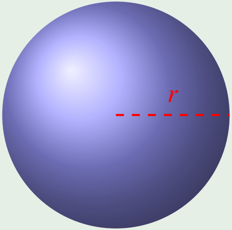

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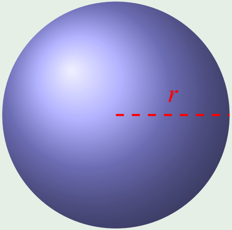

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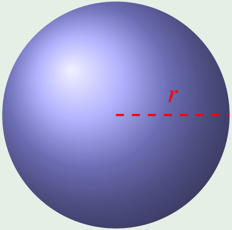

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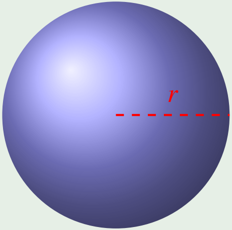

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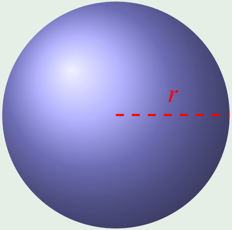

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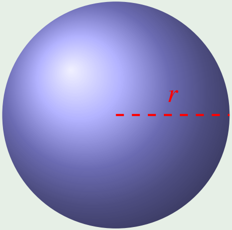

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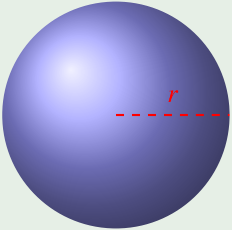

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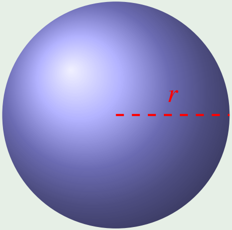

- **Find an equation** relating the quantities with given/required rates.

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- Now: **Differentiate** (implicitly) this equation with respect to time.

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Therefore the radius is increasing at a rate of $\frac{1}{25\pi}$ cm/s when $r = 25$ cm.

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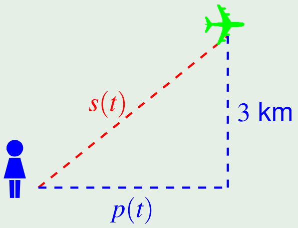

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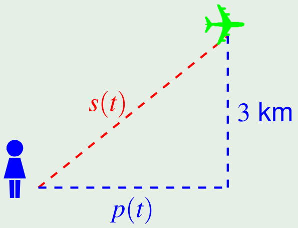

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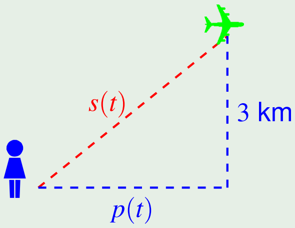

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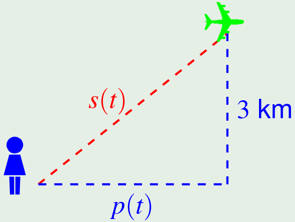

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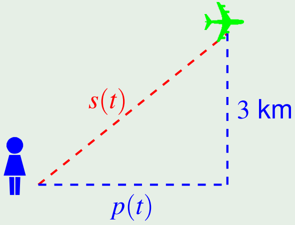

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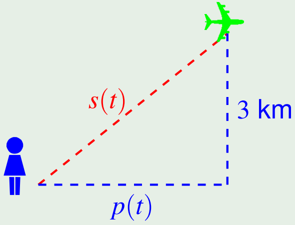

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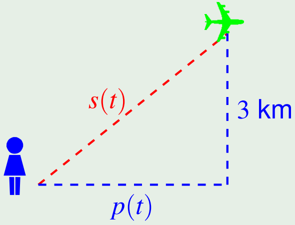

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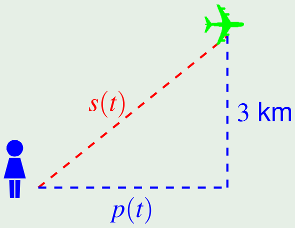

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Required: Find $\frac{\mathrm{d}s}{\mathrm{d}t} = s'(t)$ when $p = 4$.

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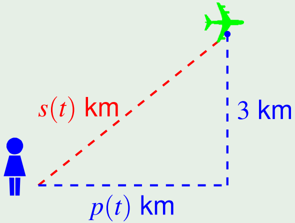

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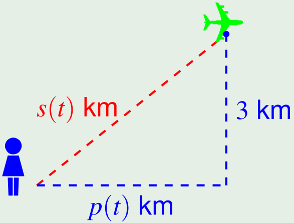

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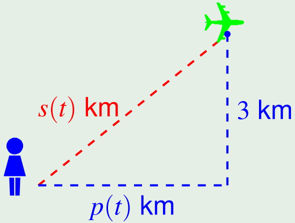

Given: $p'(t) = 500$ km/h
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Find an equation relating quantities with given/required rates: (ty Pythagoras)

$$p^2 + 3^2 = s^2$$

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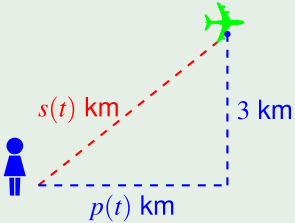

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Differentiate (implicitly with respect to t):

$$2p \cdot p'(t) + 0 = 2s \cdot s'(t)$$

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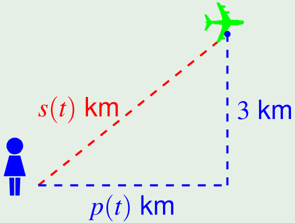

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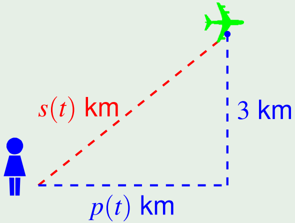

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Now **evaluate** when $p(t) = 4$ and $p'(t) = 500$.
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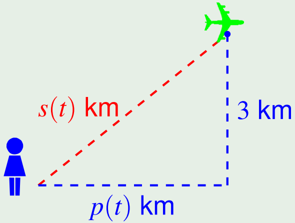

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$$2(4)(500) = 2(5)s'(t),$$

thus $s'(t) = 400$ km/h.

Example

Water is poured into a conical container at the rate of 10 cm^3/sec. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm. How fast is the water level rising when the water is 4 cm deep?

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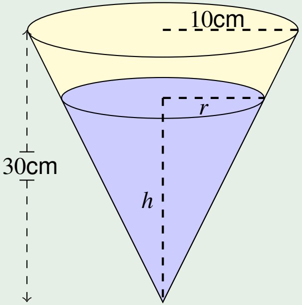

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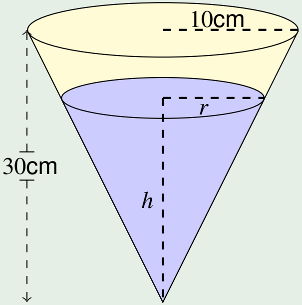

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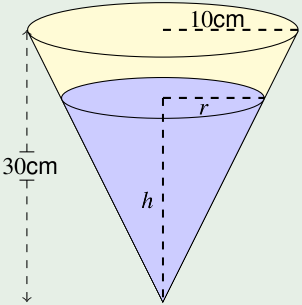

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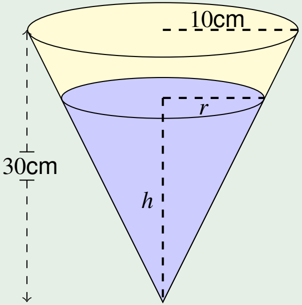

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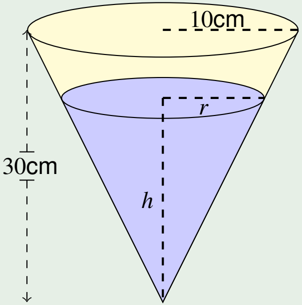

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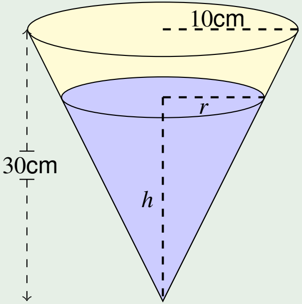

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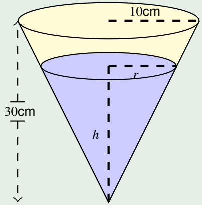

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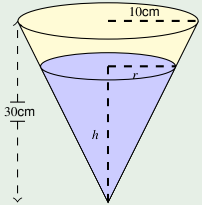

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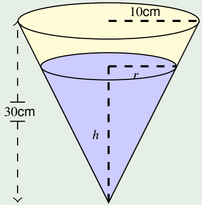

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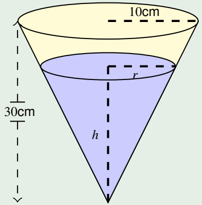

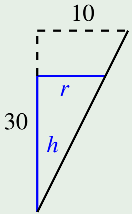

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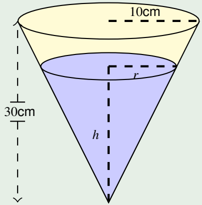

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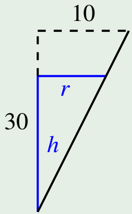

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This gives us

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 \cdot h = \frac{1}{27}\pi h^3$$

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$$h'(t) = \frac{9}{\pi h^2} \cdot V'(t) = \frac{9}{\pi \cdot 4^2} \cdot 10 = \frac{90}{16\pi} \text{ cm/s}$$

thus $s'(t) = 400$ km/h.

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Therefore the top of the ladder is falling at a rate of $3/4$ ft/s.