ASTR323 HW3 Timothy Allen 66522411

1. (a) i.

$$
\begin{aligned}
d &= \frac{1}{p} \\
&= \frac{1}{0.379} \\
d &= 2.63852242744 \\
d &= 2.64 \ pc \ (3sf)
\end{aligned}
$$

ii. With parsecs and light years defined in terms of kilometres as 9 460 730 472 580.8 and $30.857 \cdot 10^{12}$ respectively by the IAU [1].

$$
\begin{aligned}
2.63852242744(pc) &= \frac{9460730472580.8}{30.857 \cdot 10^{12}} d(ly) \\
d(ly) &= 8.60577169802 \ ly \\
d(ly) &= 8.61 \ ly \ (3sf)
\end{aligned}
$$

iii. With parsecs defined as $\frac{648000}{\pi}$ by the IAU [2].

$$
\begin{aligned}
2.63852242744(pc) &= \frac{648000}{\pi} d(au) \\
d(au) &= 544234.317276 \ au \\
d(au) &= 5.44 * 10^5 \ au \ (3sf)
\end{aligned}
$$

(b) With $m_\lambda - M_\lambda$ as the distance modulus.

$$
\begin{aligned}
m_\lambda - M_\lambda &= 5log_{10}(d) - 5 \\
&= 5 \cdot log_{10}(2.63852242744) - 5 \\
&= -2.89319604984 \\
m_\lambda - M_\lambda &= -2.89 \ mag \ (3sf)
\end{aligned}
$$

(c) Neglecting interstellar absorption the absolute visual magnitude is

$$
\begin{aligned}
M_V &= 2.89319604984 + m_V \\
&= 2.89319604984 - 1.44 \\
&= 1.45319604984
\end{aligned}
$$

To find the absolute bolometric magnitude the bolometric correction is used. Since Sirius is a binary system with spectral type A0mA1Va [3] a bolometric correction of -0.38 can be estimated [4].

$$\begin{aligned} M_{bol} &= B.C + M_V \\ &= -0.38 + 1.45319604984 \\ &= 1.07319604984 \\ &= 1.1 \; mag \; (2sf) \end{aligned}$$

The absolute bolometric magnitude of the Sun is approximately 4.74 [2]. Comparing these magnitudes in the classical way

$$\begin{aligned} M_{bol,*} - M_{bol,\odot} &= 1.1 - 4.74 \\ &= -3.64 \\ &= -3.6 \; mag \; (2sf) \\ &= -2.5 log_{10}\left(\frac{L_*}{L_\odot}\right) \\ L_* &= L_\odot * 10^{-0.4*-3.6} \\ &= L_\odot 27.5422870334 \\ L_* &= L_\odot 28 \; (2sf) \end{aligned}$$

So Sirius's luminosity is around 28 times the Sun's.

2. (a) From the given light curve of Mira its visual magnitude appears to vary between 3 and 9. Converting to luminosity

$$\begin{aligned} \frac{L_{max}}{L_{min}} &= 10^{-0.4*-6} \\ &= 251.188643151 \\ \frac{L_{max}}{L_{min}} &= 3 * 10^2 \; (2sf) \end{aligned}$$

(b) Around half the pulsation cycle should be visible to the naked eye as the magnitude will be below 6.

(c) L $\propto R^2$ so

$$\begin{aligned} \left(\frac{R_{max}}{R_{min}}\right)^2 &= 251.188643151 \\ \frac{R_{max}}{R_{min}} &= 15.8489319246 \\ \frac{R_{max}}{R_{min}} &= 20 \; (1sf) \end{aligned}$$

(d) L $\propto T^4$ so

$$\left(\frac{T_{max}}{T_{min}}\right)^4 = 251.188643151$$
$$\frac{T_{max}}{T_{min}} = 3.98107170554$$
$$\frac{T_{max}}{T_{min}} = 4\ (1sf)$$

3. The main sequence lifetime of a star with the assumption that one the main sequence $M \propto L^{\frac{1}{4}}$ can be simplified to $t_{MS} = \left(\frac{L}{L_\odot}\right)^{-\frac{3}{4}} t_{\odot,MS}$ where properties of the sun are known and main-sequence luminosity shall be determined from spectral type. For K0 and M0 stars spectral types correspond to effective temperatures of 5000 and 3500 Kelvin respectively and radii while on the main-sequence of 0.85 solar radii and 0.60 solar radii respectively [5]. Since luminosity is proportional to R^2T^4

$$\begin{aligned}
t_{MS} &= \left(\frac{L}{L_\odot}\right)^{-\frac{3}{4}} t_{\odot,MS} \\
&= \left(\frac{R^2T^4}{R_\odot^2T_\odot^4}\right)^{-\frac{3}{4}} t_{\odot,MS} \\
&= \left((\frac{R}{R_\odot})^2\frac{T^4}{5800^4}\right)^{-\frac{3}{4}} 1.1 * 10^{10} \\
t_{MS}(K0) &= \left((0.85)^2\frac{5000^4}{5800^4}\right)^{-\frac{3}{4}} 1.1 * 10^{10} \\
&= 2.1909792487 * 10^{10} \\
t_{MS}(K0) &= 2.2 * 10^{10}\ yr\ (2sf) t_{MS}(M0) \qquad = \left((0.6)^2\frac{3500^4}{5800^4}\right)^{-\frac{3}{4}} 1.1 * 10^{10} \\
&= 1.0770742848 * 10^{11} \\
t_{MS}(M0) &= 1.1 * 10^{11}\ yr\ (2sf)
\end{aligned}$$

With recent estimates of the age of the universe around $13.8 * 10^9$ years [6] these K0 and M0 stars will live for around two and ten times the current age of the universe respectively.

4. First finding the distance modulus for these two stars. The star closest to the line-of-best-fit has a log P of 1.38 ± 0.01 and a m_V of 26.4 ± 0.1. Using the Cepheid Period-Luminosity relation [7] this gives an absolute magnitude of -5 ± 0.1. Combining this with the mean visual extinction gives $m_V - M_V - A_V = 31.4 \pm 0.4$. Following the same process for the second closest star, log P is 1.62 ± 0.01, m_V is 25.5 ± 0.1 giving an absolute magnitude of -5.5 ± 0.1 and $m_V - M_V - A_V = 31.0 \pm 0.4$.

Then distance modulus calcualtions

$$
\begin{aligned}
d &= 10^{0.2(m_V - M_V - A_V + 5)} \\
d_1 &= 10^{0.2(31.4+5)} \\
&= 19054607.1796 \\
d_1 &= 19 \pm 3 Mpc \\
d_2 &= 10^{0.2(31.0+5)} \\
&= 15848931.9246 \\
d_2 &= 16 \pm 3 Mpc
\end{aligned}
$$

This is in good agreement with the value of 17.1 ± 1.8 Mpc obtained by Wendy Freedman and her colleagues. Stricter error propogation and direct access to the data instead of reading off a graph should further improve this agreement.

References

[1] IAU, *International astronomical union*, 2013.

[2] E. E. Mamajek, G. Torres, A. Prsa, et al., *Iau 2015 resolution b2 on recommended zero points for the absolute and apparent bolometric magnitude scales*, 2015.

[3] R. O. Gray, C. J. Corbally, R. F. Garrison, M. T. McFadden, and P. E. Robinson, "Contributions to the Nearby Stars (NStars) Project: Spectroscopy of Stars Earlier than M0 within 40 Parsecs: The Northern Sample. I.", Astronomical Journal **126**, 2048–2059 (2003).

[4] Princeton, *Astronomical terms and constants*, 2007.

[5] A. N. Cox, *Allen's astrophysical quantities* (Springer New York, 2001).

[6] and N. Aghanim, Y. Akrami, M. Ashdown, et al., "Planck 2018 results vi. cosmological parameters", Astronomy & Astrophysics **641**, A6 (2020).

[7] J. Storm, W. Gieren, P. Fouqué, et al., "Calibrating the cepheid period-luminosity relation from the infrared surface brightness technique", Astronomy & Astrophysics **534**, A94 (2011).