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1. (a) Finding general equation for the time spent on main-sequence:

$$\log \tau_{MS} = 7.719 - 0.655 \log\left(\frac{M}{M_\odot}\right)$$
$$\tau_{MS} = 10^{7.719-0.655\log\left(\frac{M}{M_\odot}\right)}$$
$$\tau_{MS} = 10^{7.719}\left(\frac{M}{M_\odot}\right)^{-0.655} \qquad \text{for M} > 0$$

Finding general equation for mass depending on time (adjusted equation)

$$\log -\dot{M} = -12.76 + 1.3 \log \frac{L}{L_\odot}$$
$$\log \frac{L}{L_\odot} = 0.781 + 2.760 \log\left(\frac{M}{M_\odot}\right)$$

Substituting and solving by seperation of variables

$$\log -\dot{M} = -12.76 + 1.3(0.781 + 2.76 \log\left(\frac{M}{M_\odot}\right))$$
$$-\dot{M} = 10^{-12.76+1.3(0.781+2.76\log\left(\frac{M}{M_\odot}\right))}$$
$$\dot{M} = -10^{-12.76+1.3*0.781} * \left(\frac{M}{M_\odot}\right)^{1.3*2.76}$$
$$\left(\frac{M}{M_\odot}\right)^{-1.3*2.76} \mathrm{d}(\frac{M}{M_\odot}) = -10^{-12.76+1.3*0.781}\,\mathrm{d}t \quad \dot{M} \text{ rewritten as} \frac{\mathrm{d}\frac{M}{M_\odot}}{\mathrm{d}t}$$
$$\frac{1}{1-1.3*2.76}\left(\frac{M}{M_\odot}\right)^{1-1.3*2.76} = -10^{-12.76+1.3*0.781}t + C$$
$$\left(\frac{M}{M_\odot}\right) = (-(1-1.3*2.76)*10^{-12.76+1.3*0.781}t + C)^{\frac{1}{1-1.3*2.76}}$$
$$\left(\frac{M}{M_\odot}\right) = (4.6586949296*10^{-12}t + C)^{-0.386398763524}$$

Finding C from initial masses, substituting into get main-sequence lifetime and solving for final mass and mass fraction

M_{ini}	C	τ_{MS}	$\frac{M}{M_\odot}$	Mass fraction	Mass frac(2sf)
25	0.000241063388856	6358421.21288	23.9051360127	0.956205440508	0.96
40	0.0000714280415271	4673591.02558	36.0920150558	0.902300376395	0.90
60	0.0000250117779252	3583529.06284	49.2434502749	0.820724171248	0.82
85	0.0000101546233696	2852532.51612	61.5199754451	0.723764417001	0.72
120	0.00000415986211232	2275811.48006	73.5587513663	0.612989594719	0.61

(b) The products of nuclear burning appear at the surface when the mass of the star is 83% of its initial mass(the envelope has been lost).

$$\begin{aligned} 0.83 * 85 &= (4.6586949296 * 10^{-12} t + 0.0000101546233696)^{-0.386398763524} \\ t &= 1.3507 * 10^{6} (years) \\ t &= 1.4 * 10^{6} (years)(2sf) \end{aligned}$$

2. (a) Assuming the radiation is isotropic the force due to radiation pressure is

$$\begin{aligned} P &= \frac{L}{4\pi r^2 c} \\ F &= \frac{L}{4\pi r^2 c} A \end{aligned}$$

Equating to force due to change in momentum of wind

$$\begin{aligned} F &= \dot{M} v \\ \frac{L}{4\pi r^2 c} A &= \dot{M} v \\ \dot{M} &= \frac{L}{vc} \end{aligned}$$

(b) Rate of addition of kinetic energy to the wind

$$\begin{aligned} \frac{1}{2} \dot{M} v^2 &= \frac{1}{2} \frac{L}{vc} v^2 \\ &= L \frac{v}{2c} \end{aligned}$$

Since 2c is much larger than v, even with this maximum mass loss rate, the rate of addition of kinetic energy to the wind is only a small fraction of the luminosity.

3. From the article "Populating the periodic table: Nucleosynthesis of the elements" by Jennifer Johnson, there are main groups of nucleosynthesis: Big Bang nucleosynthesis, stellar nucleosynthesis (proton-proton chain, CNO cycle, the triple-alpha process or helium burning, and burning up to iron), supernovae, the r-process, the s-process, and type Ia supernovae.

Big Bang nucleosynthesis was the original formation of elements following the Big Bang. For a short time after the universe cooled to 1 billion K, neutrons fused with protons, forming deuterium. The products of these reactions then fused with each other, protons, and neutrons, mostly creating the first hydrogen and helium, with a little lithium.

Regarding stellar nucleosynthesis, the proton-proton chain dominates for lower-mass stars. It is a series of chains beginning with the fusion of two protons into deuterium due to quantum tunnelling. This requires one proton to decay into a neutron and produces a positron and electron neutrino. Later branches in the chain can produce hydrogen up to beryllium, with amounts dependent on temperature. Importantly, this is a way of converting protons into stable neutrons, which is required for the formation of heavier nuclei. The triple-alpha process describes fusion reactions involving three helium nuclei. It generally occurs in stars after they have run out of hydrogen in their core, producing beryllium, carbon, and oxygen. A sufficiently massive star will create carbon then begin the CNO cycle, which will quickly become dominant. Various fusions of carbon, nitrogen, oxygen, and protons lead to the release of a lot of energy and adjust the composition of the star, also creating fluorine and neon. The largest stars will continue past this point with carbon, neon, oxygen, and even silicon burning, producing the elements up to iron.

Once silicon burning is complete, a supernova will spread the elements produced in a star, causing additional nucleosynthesis and exciting the surrounding gas to produce cosmic rays. These cosmic rays can fission heavier elements to create a significant amount of lithium, beryllium, and boron.

The r-process occurs in the rare situation of a binary merger involving two neutron stars or a neutron star and a black hole. The r-process describes the rapid fusion of neutrons with elements (neutron captures) due to the release of neutrons from the rupturing of the neutron star's core. This process produces most elements, especially extremely heavy elements beyond an atomic mass of 250; however, these elements will decay below plutonium due to their very short half-lives.

The s-process is a much slower creation of elements through neutron captures, which occurs in stars too small to burn oxygen. Since it is slower, elements that are formed and are initially unstable often have the time to decay and become stable before they can experience another neutron capture. As a result, the s-process won't reach thorium and uranium, leading to the creation of "almost-stable" nuclei and different neutron-to-proton ratios compared to the r-process.

Type Ia supernovae occur after a white dwarf is ignited, causing carbon and oxygen burning after somehow collapsing. This process will then spread elements up to iron throughout the universe.