ASTR323 HW1 Timothy Allen 66522411

1. (a)

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

For the Wein approximation we take $\frac{h\nu}{kT} >> 1$ so

$$e^{\frac{h\nu}{kT}} >> 1$$
$$\frac{1}{e^{\frac{h\nu}{kT}} - 1} \simeq e^{-\frac{h\nu}{kT}}$$
$$B_\nu(T) \simeq \frac{2h\nu^3}{c^2}e^{-\frac{h\nu}{kT}}$$

(b) The series expansion for $e^{\frac{h\nu}{kT}}$

$$e^{\frac{h\nu}{kT}} = 1 + \frac{h\nu}{kT} + (\frac{h\nu}{kT})^2\frac{1}{2} + \cdots$$

With $\frac{h\nu}{kT} << 1$

$$e^{\frac{h\nu}{kT}} \simeq 1 + \frac{h\nu}{kT}$$

Substituting into Planck's law

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{1 + \frac{h\nu}{kT} - 1}$$
$$B_\nu(T) = \frac{2\nu^2 kT}{c^2}$$

2.

$$T_{ff} \approx \sqrt{\frac{R^3}{GM}}$$

With $R_{sol} = 6.957 * 10^{10}cm$ and $GM_{sol} = 1.32712442 * 10^{26}cm^3s^{-2}$ (Pols. Table 2)

$$
\begin{aligned}
T_{ff} &\approx \sqrt{\frac{(6.957 * 10^{10})^3}{1.32712442 * 10^{26}}} \\
&\approx 1592.85845363s \\
&\approx 26.6476408939\ minutes \\
&\approx 30\ minutes(1sf)
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
\rho(r) &= \rho_c(1 - (\frac{r}{R})^2) \\
\rho = \frac{\mathrm{d}m}{\mathrm{d}r\,\mathrm{d}s} &= \frac{\mathrm{d}m}{\mathrm{d}v} \\
\mathrm{d}m &= \rho\,\mathrm{d}v \\
m(r) &= \int_0^r 4\pi r'^2 \rho(r')\,\mathrm{d}r' \\
&= 4\pi \int_0^r r'^2 \rho_c(1 - (\frac{r'}{R})^2)\,\mathrm{d}r' \\
&= 4\pi\rho_c \int_0^r r'^2 - \frac{r'^4}{R^2}\,\mathrm{d}r' \\
&= 4\pi\rho_c \left(\frac{r'^3}{3} - \frac{r'^5}{5R^2}\right|_0^r \\
m(r) &= 4\pi\rho_c \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right) \\
m(R) &= 4\pi\rho_c \left(\frac{R^3}{3} - \frac{R^5}{5R^2}\right) \\
&= 4\pi\rho_c R^3 \left(\frac{1}{3} - \frac{1}{5}\right) \\
&= \frac{8}{15}\pi\rho_c R^3
\end{aligned}
$$

(b) With $r = xR$

$$
\begin{aligned}
m(x) &= 4\pi\rho_c \left(\frac{(xR)^3}{3} - \frac{(xR)^5}{5R^2}\right) \\
&= 4\pi\rho_c R^3 \left(\frac{x^3}{3} - \frac{x^5}{5}\right)
\end{aligned}
$$

For $m(x) = 1/2\ m(R)$

$$
\begin{aligned}
\frac{x^3}{3} - \frac{x^5}{5} &= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) \\
5x^3 - 3x^5 &= 1
\end{aligned}
$$

For $0 \leq x \leq 1$ this has the solution
$x \approx 0.643139$

(c) Average density can be found as total mass / total volume

$$\bar{\rho} = \frac{\frac{8}{15}\pi\rho_c R^3}{\frac{4}{3}\pi R^3}$$
$$\bar{\rho} = 0.4\rho_c$$

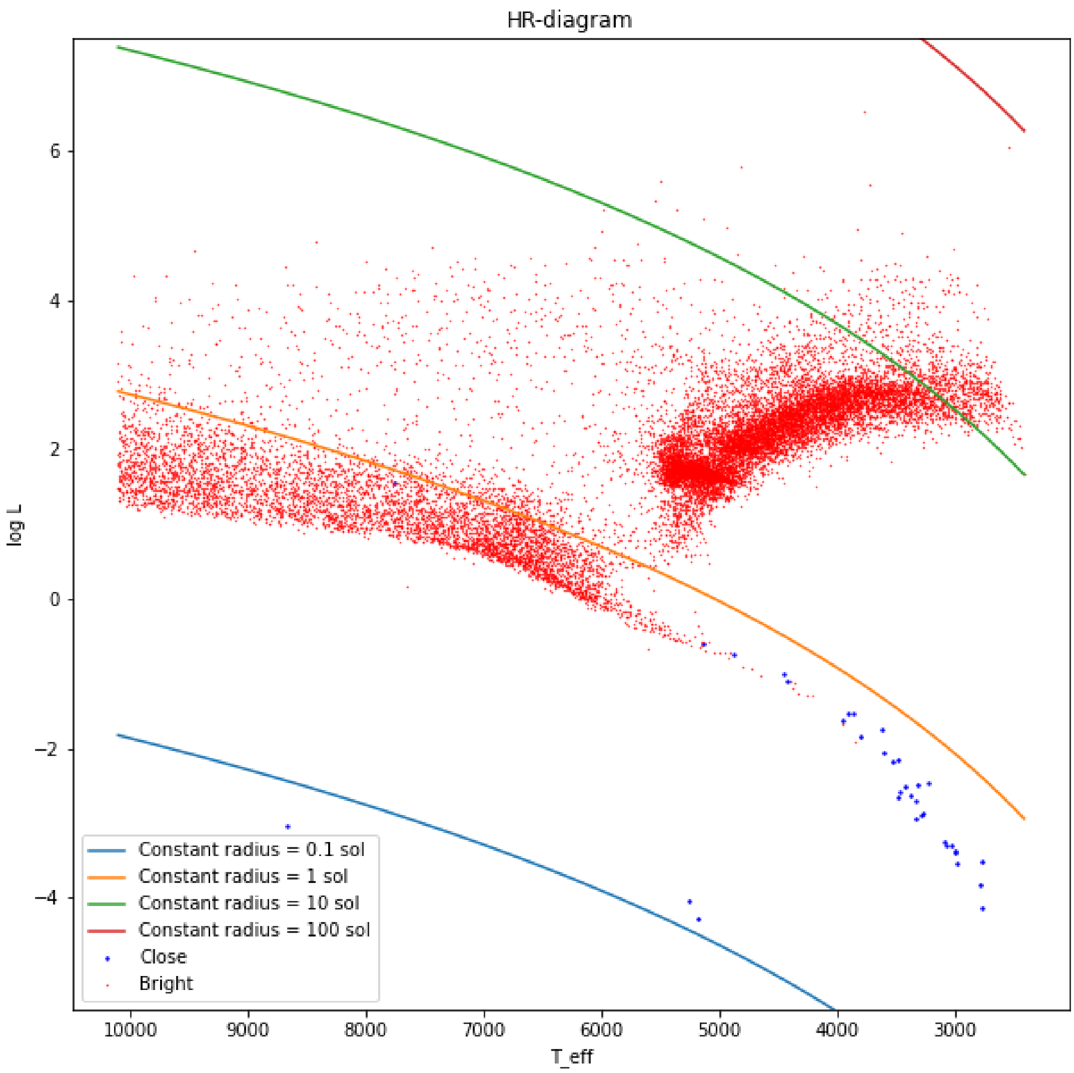

Figure 1: HR diagram for close vs bright stars in Gaia DR3 Part 1

4. (a)

(b) i. The faintest main-sequence stars are just under one solar radius and the brightest are just over 1 solar radius.

ii. The faintest white dwarfs stars are just over 0.1 solar radius and the brightest are just under 0.1 solar radius.

iii. The faintest red giants stars are around 1 solar radius and the brightest are just under 100 solar radius.