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The effects of homophily and recognition bias on the gender pay gap and retention

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Abstract

- The gender pay gap is an important issue facing the workforce today. Moral and practical reasons are discussed as to why differences based solely on gender are detrimental. Models are needed to see the impacts of factors based on gender to establish the problem to help it to be resolved.

- A model was made utilising differential equations to predict salary changes over time with a term to represent a difference in increase between women and men taken as recognition of performance. The model also includes equations for the number of individuals from a cohort with a term for homophily to account for differences between men and women, amplifying the effects of initial conditions.

  Real-world data is applied to the model for determining some parameters to better evaluate the gender gaps for pay and number of individuals.
- This report finds a very significant impact of the recognition of performance, $p_f = 0.5$, on salary (women receive around half the recognition men do) and non-zero homophily effects, $\lambda = 10, \mu_h = 0.001$, (one-tenth of all other attrition effects).
- This indicates a significant remaining bias against women in the workforce which is a pressing issue morally and practically.

  Expansion of the model should be done to include other factors known to affect salary such as retirement, job preference, location, and family-based interruptions. Different datasets should also be applied to the model to create a more complete picture of these effects. To better fit available data the model could be adjusted to examine a certain age of individual with growth included instead of attempting to follow one cohort.

Introduction

The gender pay gap represents an unfair imbalance in the treatment of women in the workplace as stated by (Business Ethics 2020). This has moral implications as well as lowering the overall performance of the workforce. According to (NZ 2023) and (Business Ethics

2020) public opinion along with legislation such as the Equal Pay Act 1972 has shown over the past 50 years that discriminating against groups, in this case women, to provide them with unequal pay is immoral.

Furthermore, unfair treatment of women in the workforce like reduced pay and promotion opportunities logically leads to increased attrition, that is individuals leaving the workforce. This is logically pre-empted by demotivation leading to worse performance. Alienating half of the potential workforce like this should lead to overall less output from the workforce.

This leads to the idea of homophily, a bias towards similar people as oneself. Since historically men dominated the workforce, homophily represents a difficulty in correcting the ratio of men and women. Homophily is one of the factors used in this model to model the differences in pay.

The other factor of difference that will be considered in this model of the gender pay gap is recognition of performance. If the performance of women is not properly recognized then they will likely receive a lower increase in salary over time.

Fair treatment of men and women in the workforce is moral and will improve the overall output of the workforce.

The model presented in this report is simple and clear, making it accessible and allowing for additions to expand it without requiring a huge amount of data to perform a regression fit. Although assumptions about what exactly the parameters refer to may be unclear they should contain all the information required to model the salaries.

The addition of real-world data as inputs to help define some of the parameters improves the quality and utility of the model.

Similar models as used by (Olsen and Walby 2004) and (Barnard 2008) accept a wider range of small inputs when possible such as age, experience, education, motivation, job preference, location and family-based interruptions. Significant within these are the factors used in this report, homophily and direct discrimination (modelled in this report as recognition of performance). Instead of modelling differential equations, salary is modelled using the Blinder–Oaxaca decomposition, which gives the natural log of wages from a combination of explanatory variables and coefficients along with an error term to qualify the fit.

This report will present a model of change in salary and number of individuals in the workforce. Factors affecting men and women equally will be modelled with combined terms(R, μ) and factors affecting men and women differently will be modelled with homophily and performance recognition $(\mu_h h(x), p)$.

This report finds a very significant impact of recognition of performance on salary (around half) and non-zero homophily effects (one-tenth of all other attrition effects).

Model

The simple gender pay gap model described in class takes a change in salary (S) is given by $\frac{\mathrm{d}S}{\mathrm{d}t} = pRS$ where p is a ratio of the difference in salary changes between men and women (taken as recognition of performance), R represents a holistic change due to factors equally impacting men and women (eg. inflation) and F and M subscripts can be used to differentiate female and male terms.

Then taking one year of individuals entering the workforce a change in the number of individuals working (N) is given by $\frac{\mathrm{d}N}{\mathrm{d}t} = -\mu N$ where μ represents attrition. This attrition term can be expanded to include a homophily function $h(x) = \dfrac{1 - exp(-\lambda x)}{1 + exp(-\lambda x)}$ where $x = 1 - \dfrac{N_F}{N_M}$ and multiplying this by its own attrition term μ_h.

Putting all this together gives the system

$$\begin{aligned}
\frac{\mathrm{d}S_F}{\mathrm{d}t} &= pRS_F \\
\frac{\mathrm{d}S_M}{\mathrm{d}t} &= RS_M \\
\frac{\mathrm{d}N_F}{\mathrm{d}t} &= -\mu N_F - \mu_h h(x) N_F \\
\frac{\mathrm{d}N_M}{\mathrm{d}t} &= -\mu N_M
\end{aligned}$$

Using data from USA (Labor Statistics 2023) provides an average weekly earnings from 25-34 of \$1,121 for 16,520,000 men and \$966 for 13,524,000 women. The 55-64 year age

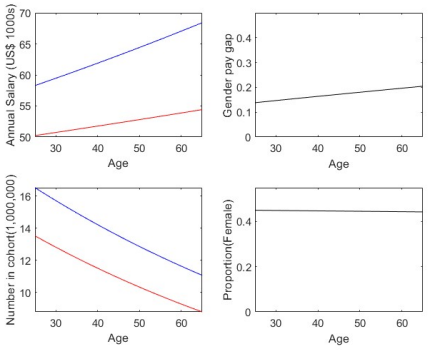

Figure 1: Model outputs for data initial conditions, $p_f = 0.5, \lambda = 10, \mu = 0.01, \mu_h = 0.001$, R = 0.004

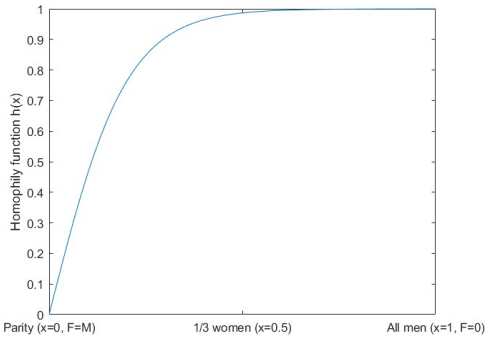

Figure 2: Homophily function with $\lambda = 10$

group has an average of $1,341 for 10,995,000 men and $1,043 for 8,995,000 women. These values allow for more realistic setting of initial conditions and the parameters μ and R.

Results

Evaluating the model with real-world data provided gives values of p and various possible values for μ_h and λ by manually adjusting the parameters.

Figure 1 shows graphs of salary, number of individuals, pay gap and proportion of women in relation to age for values matching the data from (Labor Statistics 2023). A very significant value $p_f = 0.5$ is observed showing very low recognition of performance for women with this model. Smaller yet still significant values of $\lambda = 10, \mu_h = 0.001$ are found for the homophily. This is less significant than other attrition effects but notable non-zero. Figure 2 shows the homophily curve used, the data is also matched for lower values of $\lambda = 10$ and slightly increased values of μ_h.

For this model three of the differential equations can be solved exactly using the separation of variables however this was omitted as it is not particularly insightful or relevant to the actions of the model. The final equation is an inexact differential equation and so can be solved exactly but is outside the scope of this report.

There are no proper fixed points in this system as it evolves with exponentials over time however, the trivial N=0 or S=0 conditions lead to no changes in salary or number as ex-

pected(a nullcline). Nullclines exist for the systems of the difference between female and male salary with the same initial conditions and p=1. For the number of men and women, a nullcline is at equal initial conditions or $\lambda = 0$ or $\mu_h = 0$.

Discussion

Key findings in the report were a recognition of the performance of women of half that of men, along with small homophily effects. This was a much more extreme reduction in recognition of women than other studies such as (Olsen and Walby 2004) found, 0.5 vs 0.089. This is likely due more than anything else to a lack of confounding factors taken into account in this reports' model such as education, family care and firm size, additional terms could be added to correct this. A different dataset was also used which may mean there has been some change over time or due to location.

The homophily effect matches much better with (Olsen and Walby 2004) finding 1% per 10% more men. Unfortunately, there are many assumptions in the model that will have a significant effect on this result.

The model has no way to deal with attrition and salary changes due to retirement. Although this is generally at standard ages due to benefits so the model is cut off, some individuals choose to retire before this age affecting the higher ages of the model. Additionally, salary is observed to peak not around 60 but around 40(Chen et al. 2023). The model has no way to deal with any other reasons for this decrease in wages other than retirement, although it could simply be adjusted to do so by adding terms to the relevant equations.

Since available data does not follow a group the parameter R corresponds more to age or experience impacts than inflation, this should not impact results but is notably not how the model was designed.

A more difficult-to-address issue with the model is its assumption that relations to changes will be linear. If impacts are best modelled by other functions they could misconstrue the data.

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