wooclap 7

PDE

6.2.1 in cursus

wooclap

Wat hoort samen? Je mag ervan uitgaan dat T gekend is op de rand ($t = 0$ / $x = x_{min}$ / $x = x_{max}$ van het (x, t)-gebied van verdeeld is in n deelintervallen in de x-richting en m deelintervallen in de t-richting

$\varphi(x, t)$

$\varphi_{i,j} = \varphi(x_i, t_j)$

09:36 76

richting en m deelintervallen in de t-richting

You did not answer

$\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial x^2}$, RL evalueren op volgend tijdstip

(no answer)

C. het oplossen van m stelsels van dimensie $(n-1)\times(n-1)$

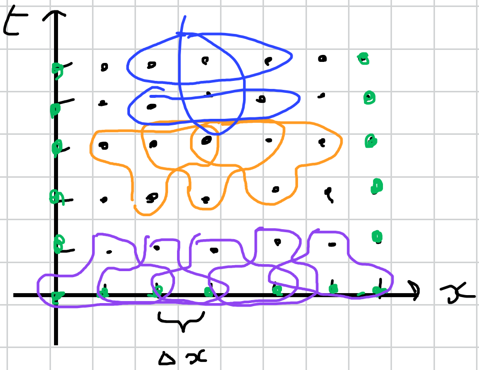
(1) m stelsels van dimensie $(n-1)\times(n-1)$

$\frac{\partial^2 T}{\partial t^2} = 2\frac{\partial^2 T}{\partial x^2}$

(no answer)

B. het oplossen van 1 stelsel van dimensie $n\,m \times n\,m$

1 stelsel van dimensie $nm \times nm$



deze punten moet je gegeven krijgen

$\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial x^2}$, RL evalueren op vorig tijdstip

(2)

(no answer)

A. er moeten geen stelsels worden opgelost

(1) $$\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial x^2}$$

1 onbekende in deze vergelijkingen

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = 2\,\frac{T_{i+1,j+1} - 2T_{i,j+1} + T_{i-1,j+1}}{(\Delta x)^2}$$

② $$\frac{\partial^2 T}{\partial t^2} = 2\frac{\partial^2 T}{\partial x^2}$$

deze kleur is grafisch mogelijk om op te lossen

$$\frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta t)^2} = 2\,\frac{T_{i+1,j+1} - 2\,T_{i,j+1} + T_{i-1,j+1}}{(\Delta x)^2}$$

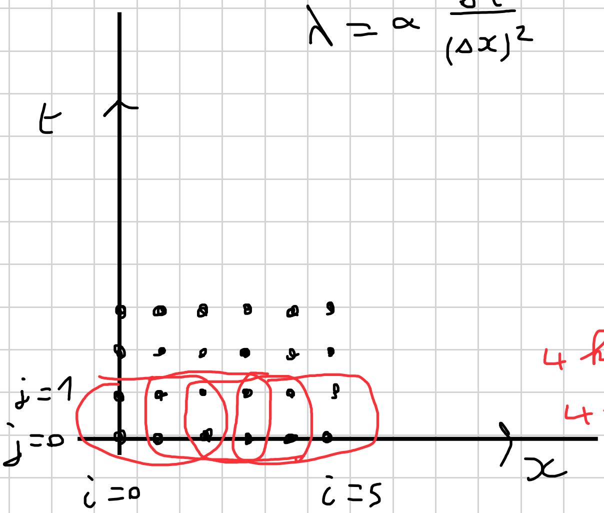
oef 4 cursus p55

$$\frac{\partial \varphi}{\partial t} = \alpha \frac{\partial^2 \varphi}{\partial x^2} \qquad x \in [0,L] \qquad \varphi(x,t)$$

gekend: $\varphi(x,0)$, $\varphi(0,t)$, $\varphi(L,t)$

$$\frac{\varphi_{i,j+1} - \varphi_{i,j}}{\Delta t} = \alpha\,\frac{\left(\varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i+1,j}\right) + \varphi_{i+1,j+1} - 2\varphi_{i,j+1} + \varphi_{i-1,j+1})}{2(\Delta x)^2}$$

$$\lambda = \alpha\frac{\Delta t}{(\Delta x)^2}$$



4 knopen
4 vgl 4 onbekenden

$$-\varphi_{i,j+1} + \varphi_{i,j} + \frac{\lambda}{2}\left(\varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i-1,j}\right) + \frac{\lambda}{2}\left(\varphi_{i+1,j+1} - 2\varphi_{i,j+1} + \varphi_{i-1,j+1}\right) = 0$$

$$\Downarrow$$

$$\frac{\lambda}{2}\varphi_{i+1,j+1} + (-1-\lambda)\varphi_{i,j+1} + \frac{\lambda}{2}\varphi_{i-1,i+1}$$

$$= -\frac{\lambda}{2}\varphi_{i+1,j} + (-1+\lambda)\varphi_{i,j} - \frac{\lambda}{2}\varphi_{i-1,j}$$

$$\Big(\quad\Big)\Big(\;\Big) = \underbrace{\Big(\quad\Big)\Big(\;\Big)}_{\text{bekend}}$$

↳ nu in matrixvorm gieten

gaat over de elementen de rij erboven

posities op x-as

correctie voorwaarden

$$\underbrace{\begin{pmatrix} -1-\lambda & \frac{\lambda}{2} & 0 & 0 \\ \frac{\lambda}{2} & -1-\lambda & \frac{\lambda}{2} & 0 \\ 0 & \frac{\lambda}{2} & -1-\lambda & \frac{\lambda}{2} \\ 0 & 0 & \frac{\lambda}{2} & -1-\lambda \end{pmatrix}}_{\text{getallen}} \underbrace{\begin{pmatrix} \varphi_{1,1} \\ \varphi_{2,1} \\ \varphi_{3,1} \\ \varphi_{4,1} \end{pmatrix}}_{\text{onbekenden}} = \underbrace{\begin{pmatrix} -1+\lambda & -\frac{\lambda}{2} & 0 & 0 \\ -\frac{\lambda}{2} & -1+\lambda & -\frac{\lambda}{2} & 0 \\ 0 & -\frac{\lambda}{2} & -1+\lambda & -\frac{\lambda}{2} \\ 0 & 0 & -\frac{\lambda}{2} & -1+\lambda \end{pmatrix}}_{\text{getallen}} \underbrace{\begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \\ \varphi_{3,0} \\ \varphi_{4,0} \end{pmatrix}}_{\text{onbekenden}} + \underbrace{\begin{pmatrix} -\frac{\lambda}{2}\left(\varphi_{0,1} + \varphi_{0,0}\right) \\ 0 \\ 0 \\ -\frac{\lambda}{2}\left(\varphi_{5,1} + \varphi_{5,0}\right) \end{pmatrix}}_{\text{getallen}}$$

Lineaire stelsels

p 65 2

$$A v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad L \underbrace{U x}_{V} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 7 & 8 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad A = LU$$

je mag kiezen als L of U 1'tjes in de diagonaal hebt

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{7} & 1 & 0 \\ \frac{4}{7} & \frac{1}{2} & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 7 & 8 & 0 \\ 0 & 6/7 & 3 \\ 0 & 0 & \frac{9}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1/7 & 1 & 0 \\ 4/7 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \iff \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 13/7 \\ \frac{3}{2} \end{pmatrix}$$

$$\begin{pmatrix} 7 & 8 & 0 \\ 0 & 6/7 & 3 \\ 0 & 0 & 9/2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 13/7 \\ 3/2 \end{pmatrix} \iff \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1/3 \end{pmatrix}$$