Electric Guitar Strings as Magnetically-Damped Oscillating Systems

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I. ABSTRACT

The electric guitar is the perfect marriage between electromagnetic-physics and music. While both can be highly analytical and mathematically-driven, guitarists often lean hard in the opposite direction. This leads to many ideas in the guitar world to be endorsed or discounted seemingly arbitrarily. Such as the case with the debate over single pickup guitars. Many see them as limiting and pointless, while others view them as more precise instruments that offer a sound with less interference from unused pickups.

This project endeavors to computationally simulate a guitar string with multiple different configurations of pickups in order to settle the debate. Using a discretized wave equation and Euler's method, the position of a string over time is solved for with multiple different pickup configurations. It was found that there is a distinct interference from extra pickups, yet it is unclear if this is analogous or relevant to real-world scenarios.

II. INTRODUCTION

A modern electric guitar uses ferromagnetic Nickel strings whose vibration is converted into an electrical signal via a "pickup". The basic idea of a guitar pickup is a magnet inside a coil of wire that generates current corresponding to a magnetic string perturbing its field.

Multiple variations of these pickups exist, featuring different magnet sizes and strengths as well as coil winding patterns. Some pickups utilize specific patterns and materials with the direct intention of altering sound signatures by manipulating the magnetic field of its magnets.While Hum-buckers, Soapbar-pickups, p90's, and many more offer different coil counts and sizes, each advertising different sounds, our main interest will be the simple single-coil. Single-coil pickups usually feature 6 aluminum-nickel-cobalt slugs surrounded by a single coil of wires, and are commonly referred to alnico single-coils or simply single-coils [1].

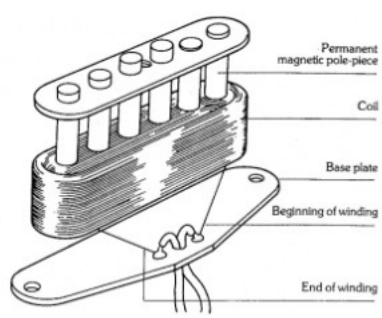

FIG. 1. Diagram of a Seymour-Duncan brand single-coil pickup [1]

The vast majority of electric guitars come equipped with multiple pickups that can be toggled on and off. With the assumption that turning a pickup off will allow different tones to be explored without affecting the overall system. Many musicians opt for multi-pickup configurations in the name of versatility. However, there is a small leaning in the guitar community that tend toward guitars with only one pickup, believing the guitars to produce superior and less-altered sounds than their multi-pickup counterparts.

The aim of this project is to determine via computational simulation if this idea bears any substance. Specifically, does unused pickup count influence the oscillating guitar strung?

To answer this question, a guitar string can be modeled via as a damped oscillating wave evolving over time. The methodology of doing so in the programming language Python is detailed below for three separate systems: Zero pickups, One pickup, and Three pickups.

FIG. 2. Two Stratocaster-style electric guitars. The left guitar uses only a single pickup in the neck position whereas the right guitar features the traditional 3 single coil pickups. [2]

III. METHODOLOGY

The basic structure of the modelled guitar string is a series of coupled masses along the x direction that can move side-to-side in the y direction. 101 masses were used in this approximation; similar systems found accurate and real-world analogous results with fewer than 50 masses [3].

The propagation of a wave throughout the system is modelled by the wave equation, equation 1.

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} - \gamma\frac{\partial y}{\partial t} - \epsilon\frac{\partial y}{\partial t} - l^2\frac{\partial^4 y}{\partial x^4} = 0 \tag{1}$$

In this equation, c is the speed of the wave(m/s), γ is the damping constant(s/m), and l is the characteristic stiffness of the string. The final coefficient, ϵ, represents the magnetic damping force. The magnetic damping force in this contexts refers to the product of the magnitudes of the magnetic charge of the pickup, each incident point of the string, and the permeability of the medium(air).

The length of the string, L, was 0.647 meters. This corresponds to a 25.5 inch scale length; the modern standard for electric guitars. The speed of the wave, c, was set with equation 2.

$$f = c/2L \tag{2}$$

In equation 2, f represents the frequency we tuned our string too, 83Hz. This frequency correspond to the low E or 6th string on a guitar. The air damping constant γ was set equal to l, the characteristic stiffness, both 5^{-5} [4].

The initial ϵ constant was set as a product of the two magnetic fields from the string and pickup, both of which were 1 mT [1]. The localized constant was then properly scaled for each mass along the string. This scaling was taken with respect to the center of each pickup as an approximation.

The wave equation was then rewritten as a set of first-order differential equations and discretized. The discretized set was iterated through using Euler's method for 2.5 seconds with 500,000 steps. The python implementation branches from a modified template created by Luke Polson as part of an educational video in which he plays acoustic guitar audio created by the digital system [4].

The inclusion of the pickup parameters(count, positions, field strength) were retrofitted to the original program and wave equation. The python package, NUMBA, was used to accelerate the calculations. The function took initial position of the string as well as positions of the pickups. The epsilon constant was different for each mass, effectively localized and scaled at each position by the inverse-square of the difference in distance.

This constant was also used as a toggle; set to zero to effectively discount it from the system when analyzing different configurations. When the pickups were "engaged", they were placed at the 80th, 85th, and 95th positions similar to a modern guitar configuration.

The output of the function was a 3D array of the string position over time. A sample output after some time t below, figure 3

The position of a single mass was selected from the string and it's oscillation was used for Fourier decomposition/analysis. In the interest of computation time, every 100th time-step was selected for use in the analysis. This discontinuity was later corrected for when calculating the resulting frequency.

IV. RESULTS AND DISCUSSION

Performing the simulation for the three cases yielded the following wave forms, figure 4. From these wave forms, we can see the clear distortions present in the systems that included pickups. The single pickup system features a

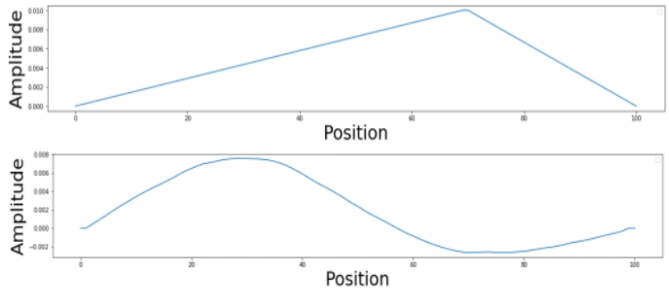

FIG. 3. TOP: Initial position of string at $t = 0$. BOTTOM: string position at some later time t

secondary oscillation that is clearly visible. The three pickup system also features an additional perturbation, yet its pattern appears much noisier. While we cannot know if these differences are audibly significant, they are clearly a departure from the baseline configuration featuring no pickups.

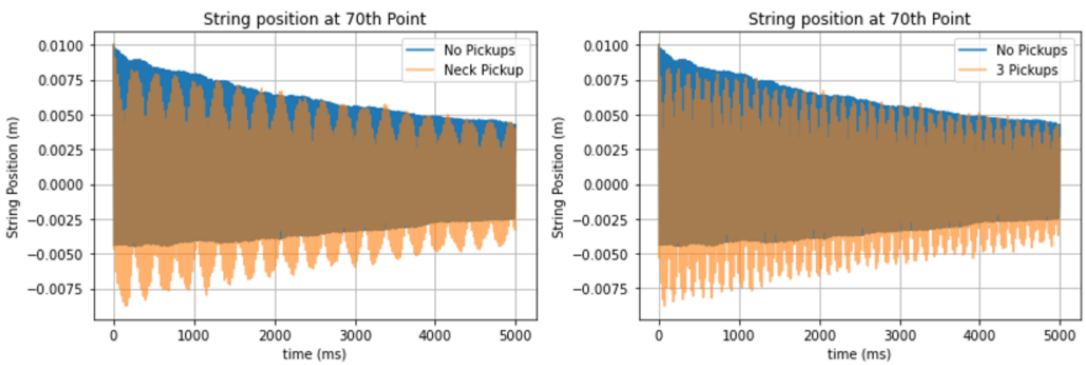

FIG. 4. Position of the 70th mass on the string over 2.5 seconds. In blue is a system with no pickups. The left figure uses only one pickup, referred to as the neck pickup due to its proximity to the guitar neck. The right figure results from the three pickup configuration

The wave forms were then passed into a function that calculated the Fourier transform and produces figure 5. As you can can see, the differences between pickup configurations were very minuscule in terms of the resulting frequency.

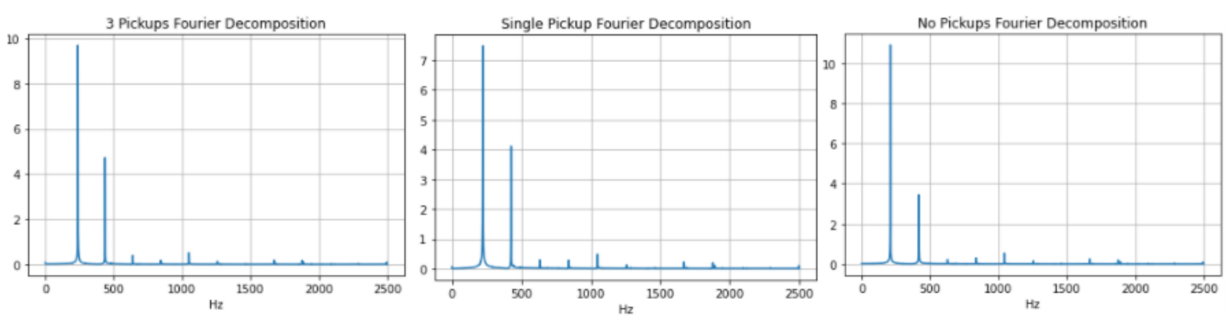

FIG. 5. The Fourier transform of the wave resulting from the each pickup configuration as labelled. The Hz label refers strictly to the positional data and can be scaled by $\frac{5}{14}$ to account for the data selection/pre-processing

From the decomposed waveform data, we can extract the exact frequencies in Hz that which the mass on the string was oscillating, figure 6 This is done by taking the index of the list that corresponds to the peak in the Fourier transform. That index number is then appropriately scaled back to a rel-world representation of Hz. In this case, it was scaled by a factor of 5/14.

Pickup Count	Hz
0.0	74.642857
1.0	79.642857
3.0	85.357143

FIG. 6. Table of pickup counts and resulting system frequency

As we can see from figure 6, there seems to be a positive relation between pickup count and frequency of the system. This follows intuition as the pickup exerts a force on the string that increases the tension, resulting in a higher frequency or note. It is unclear whether this would have an effect in a real world scenario as before any instrument playing were to occur, the relevant strings would be adjusted into the desired tuning.

V. CONCLUSIONS AND PERSPECTIVES

From the results, it was determined that magnetic pickups do tangible affect the sound of a plucked string, and that there is a positive relation between pickup count and frequency. The inclusion of a single pickup raised the frequency by 5Hz, and adding two more pickups added 10.71Hz from the baseline of zero pickups. The "noisiness" of the waveform also scaled with the number of pickups.

While it remains unclear whether the distortion would be apparent strictly from the sound of the string, we must also consider that the signal produced by a guitar pickup may incur interference from a number of sources. Each different pickup configuration necessitates a different series of potentiometers and switches to traverse until passed through to an amplifier or effects pedal. Each step in the circuit could introduce different perturbations that this model cannot account for. Even light bulbs in the vicinity of guitars can cause audible interference that may dwarf the effects of pickup configurations. From personal experience, even orienting my guitar so its pickups are exposed to my monitor produces significant and perceptible interference.

This model also only examine the low E string on a guitar in standard E-A-D-G-B-e tuning, commonly referred to as the 6th string. The 6th string will measure a net force toward the center of the pickup at each amplitude whereas the middle strings will experience more oscillation in the direction of the net force. This geometric difference may lead to different effects for the outer-most guitar string relative to the inner strings. Strings at different tunings that feature different tensions may also alter the effects of the pickups as magnetic dampers; alongside scale -length(l).

On a separate note, the computational methods worked surprisingly well. The NUMBA package drastically pared down the run times of each simulation and aided in the development and estimation of the model and its parameters. Though, different iterative methods than the Euler method I used, such as Velocity-Verlet, Euler-Cromer, or RK4, could improve the accuracy of the experiment. Other configurations and types of pickups could also be tested. A common single pickup system consists of a Humbucker located near the edge or bridge section of the guitar. A Humbucker is essentially two single-coil pickups wired on opposite polarities, which may yield interesting results when compared to a true single-coil.

There also exists Piezo-electric pickups that directly translate the motion of the guitar string into an electric signal via piezo electricity [5]. Such a pickup would not experience any magnetic damping, but may fall victim to increased frictional damping in it's stead. Such a system would be very interesting to examine within this framework.

This model also only accounted for 1D motion of each mass on the string. Future investigations could account for 2D motion. Though a 3D model can be created, it would likely not provide any novel information as there is little motion along the x axis that would act as the third spatial coordinate.

VI. APPENDICES

Here below is the opening statement used to define the function for solving the discretized wave equation. The function itself is rather large and unwieldy to include in LATeX format, though the function can be found and explored in the project notebook attached.

```
def compute_d(d, times, length, dt, dx, l,
    gamma,e1,e2,e3,pickup_1_pos,pickup_2_pos,pickup_3_pos,height)
```

This function takes in many self explanatory parameters mentioned previously. Though some naming conventions follow: d is a array of the initial string positions, *times* is the number of steps to iterate through, *length* is the number of masses throughout the string-length, dt and dx are the respective step sizes in each category, the e arguments are the local epsilon values that are further adjusted within the function as it loops through d. This function structure will return an array of the same size as d which models the string position over time given it's arguments. The function itself was heavily based on the work of Luke Polson in his educational video in which he created guitar audio in python and exported the wave function as a .wav file to be played [4]. The video itself is very interesting and I highly recommend the reader view it.

The following loop is what was used to select the position of a single mass over time for a specific pickup configuration. In the program, $strum_custom$ is set with the usual arguments, except $e2$ and $e3$ are set to zero. This effectively isolates the single pickup as the only active magnetic influence on the string. It's basic principle was used to plot the other two configuration's waveforms.

```
strum_custom = compute_d(sol, Nt, Nx, dt, dx, l,
    gamma,e1,0,0,pickup_1_pos,pickup_2_pos,pickup_3_pos, height)
single1 = []
for i in range(5000):
    single1.append(strum_custom[i*100][70])
```

Below is the *dft* function used for the Fourier decompositions. This function originates from the Day-11 Pre-Class assignment. It takes an array as an argument and returns the transform.

```
def dft(y):
    N = len(y)
    c = np.zeros(N//2 + 1, complex)
    for k in range(N//2 + 1):
        for n in range(N):
            c[k] += y[n]*np.exp(-2j*np.pi*k*n/N)
    return c
```

[1] C. D. Corporation, The Anatomy Of Single Coil Pickups, (2020).
[2] TDPI Forum User: Sharkfin, My Electric Guitars (2012).
[3] J. Feinberg and B. Yang, Natural-frequency splitting of a guitar string caused by a non-uniform magnetic field, The Journal of the Acoustical Society of America **144**, EL460 (2018), https://doi.org/10.1121/1.5080465.
[4] Luke Polson, I Generated Guitar Audio in python using NUMBA (2021).
[5] CS Guitars, (almost) EVERYTHING About Pickups — Too Afraid To Ask (2021).