

Neurodynamics - Fall 2017

BENG 260 / BGGN 260 / PHYS 279

Homework 3: Due October 27

Computational Lab

In this assignment we will study the single neuron stability dynamics in two variables: membrane potential V , and accommodation (K^+ slow persistent activation gating) variable w . For the neuron, we will consider two models: the [Morris-Lecar \(ML\) model](#) and a reduced HH model (substituting w for n like in homework 2). The ML model is replicated here:

$$C_m \frac{dV}{dt} = -I_{Ca} - I_K - I_L + I_{ext} \quad (1)$$

$$I_{Ca} = g_{Ca} m_\infty(V) (V - E_{Ca}) \quad (2)$$

$$I_K = g_K w (V - E_K) \quad (3)$$

$$I_L = g_L (V - E_L) \quad (4)$$

$$\frac{dw}{dt} = \frac{w_\infty(V) - w}{\tau_w(V)} \quad (5)$$

with voltage dependence (V in units mv) of the kinetics of gating variables:

$$m_\infty(V) = (1 + \tanh((V + 1)/15))/2 \quad (6)$$

$$w_\infty(V) = (1 + \tanh(V/30))/2 \quad (7)$$

$$\tau_w(V) = 5 / \cosh(V/60) \quad [\text{units } ms] \quad (8)$$

and with electrical parameters:

$$\begin{aligned} C_m &= 1 \mu F/cm^2 \\ E_{Ca} &= 100 mV; \quad g_{Ca} = 1.1 mS/cm^2 \\ E_K &= -70 mV; \quad g_K = 2.0 mS/cm^2 \\ E_L &= -50 mV; \quad g_L = 0.5 mS/cm^2 \end{aligned} \quad (9)$$

Note: Similar to previous assignments, the external current I_{ext} is varied as a parameter which governs the stability of the dynamics.

1. Stability Analysis [40 points].

Determine the stability of the Morris-Lecar model when injected with current for three cases: $I_{ext} = 5 \mu A/cm^2$, $24.3 \mu A/cm^2$ and $30 \mu A/cm^2$. The following questions evaluated at all three injected currents will guide you through the process.

- Plot the null-clines and stationary point(s) in V - w space.
- Compute the eigenvectors and eigenvalues of the Jacobian around the stationary point(s) and determine the stability.

- (c) Verify the stability by plotting the simulated trajectory of V and w in the V - w plane starting from several initial conditions in the region of the stationary point(s).
- (d) If the value of external current is over $200\mu A/cm^2$, what is the expected stability and trajectory?

Homework Problems

2. *Extended Stability Analysis* [40 points].

Homework 2 explored the properties of the full and reduced Hodgkin-Huxley (HH) models. To compare the stability of the HH model to the ML model, we need to reduce the dimensionality of HH to 2 variables, V and n . To do this, we will let $h = 0.8 - n$, and approximate the m gating dynamics with its equilibrium value:

$$m_{\infty}(V) = \alpha_m(V)/(\alpha_m(V) + \beta_m(V))$$

Perform the stability analysis from Problem 1 using the reduced HH model and compare your results with the ML model. Are the reduced HH model results what you would expect for the full HH model? If not, what would account for the differences?

3. *Dimensionality Reduction* [Reading - 20 points].

In this question, you will explore dimensionality reduction which can be used to improve your stability analysis (systems framework), as a statistical framework, and as a tool that you can use for your own research or for the class project.

- (a) Given a set of data, Principle Component Analysis (PCA) finds the linear lower-dimensional representation of the data such that the variance of the reconstructed data is preserved. What does that mean and what is its significance?
- (b) Geometrically describe what PCA does.
- (c) Conceptually describe Independent Component Analysis (ICA).
- (d) What is the difference between PCA and ICA?
- (e) What scenarios would you use PCA and when would you use ICA?
- (f) Can PCA or ICA be used for non-linear dimension reduction?

Submission Guidelines

Solutions without work or explanations where applicable will receive no credit. Submit a single **.zip** file containing solutions, plots, and Matlab/Python code to both computational lab and homework problems by 3:00pm of due date on TritonEd.

The submission file should follow the naming scheme **LastFirst_A12345678_HW3.zip**.