Neurodynamics Homework 3

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1 Stability Analysis

1.1 a

We computed the null-clines and stationary points for $\frac{dV}{dt}$ for I_{ext} =5, 24.3, and 30 $\mu V/cm^2$ and $\frac{dw}{dt}$ in V-w space (Figure 1).

1.2 b

The real values of the eigenvalues around the stationary points are:

I_{ext}	λ
5	-0.396
24.3	-0.091
30	0.270

The values for 5 and 24.3 $\mu V/cm^2$ are negative and thus stable while the values for 30 $\mu V/cm^2$ are positive and thus not stable.

1.3 c

To further assess the stability of these points, we plotted the trajectories of initial values near these stationary points. For the first two values, I_{ext} = 5 and 24.3 $\mu V/cm^2$, we see that if we start near the stationary point we quickly converge to an equilibrium (Figures 2, 3). For the third value of I_{ext} = 30 $\mu V/cm^2$,we see a non-converging behavior in which values spiral out (Figure 4). These confirm our stability findings from the eigenvalues.

1.4 d

When we inject much higher amounts of current, $I_{ext}200\mu V/cm^2$, we find the system is stationary with stable real eigenvalues of [-3.46, -0.317] at I_{ext} = $220\mu V/cm^2$ (Figure 5,6).

2 Extended Stability Analysis

Using our reduced HH model, with linear terms for both m and h, we performed a stability analysis. With an external stimulation I_{ext} this model has a stationarity at -40 mV (Figure 7) that is stable with real eigenvalues of [-0.500, -0.209] and converging fields (Figure 8).

We additionally looked at the stability of the HH model using the same values of I_{ext} as before. Here we found some surprising behavior whereby none of the higher current injections demonstrated equilibrium points with $\frac{dn}{dt}$ (Figure 9).

3 Dimensionality Reduction

3.1 a

Principal Components Analysis (PCA) is a technique which represents a data set as a series of Principal Components (PCs) sorted in the order of how much variance they explain in a data set. This can be used to reduce the dimensionality of a data set by removing the less informative PCs while still maintaining the majority of the variance in the reconstructed data. This means that any data that has a non-zero covariance matrix can be reduced to much lower dimensional representation, say 1/2, without losing 1/2 of the relevant information needed to recreate the original data. This property is useful for data compression and visualization.

3.2 b

PCA starts with an nxm data set and computes the nxn covariance matrix. An eigenvalue decomposition is performed on the covariance matrix to identify the n dimensional vectors over which the most variance is explained. These eigenvectors are then sorted by eigenvalues to get a set of PCs sorted by the amount of variance in the original data they explain. These PCs indicate directions through the n dimensional space through which have iteratively less variance when projecting the data onto them in an orthogonal fashion.

3.3 c

Independent Component Analysis (ICA) is also a data reconstruction technique with the most obvious deviation from PCA being that the restriction that our new vectors be orthogonal is relaxed. Instead it is restricted that each of our components is indenendent, defined as having 0 mutual information. At the same time, mutual information between our independent components and our original data is maximized.

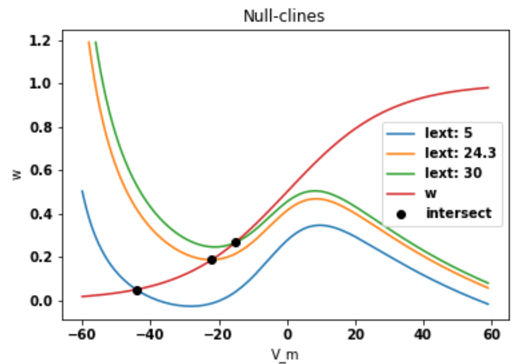

Figure 1: Null-clines in V - ω space

3.4 d

PCA requires all components to be orthogonal, and allows for perfect reconstruction of the original signal while ICA requires that components to be independent (which can be far from orthogonal) and do not necessarally allow for signal reconstruction.

3.5 e

PCA is useful if you want to represent the most possible variance in the data set in the fewest possible dimensions. If you want to visualize how separable two groups are when accounting for only two or three dimensions it is instrumental. PCA is also useful in data compression as lower ranked eigenvalues can be thrown away without losing much information. ICA is useful at source localization as it can pick up features that are independent but over partially redundant dimensions. ICA is useful for removing obvious features like eyeblinks from EEG as well as attempts at solving the source localization problem in EEG/MEG.

3.6 f

PCA can be used for non-linear dimesionallity reduction.

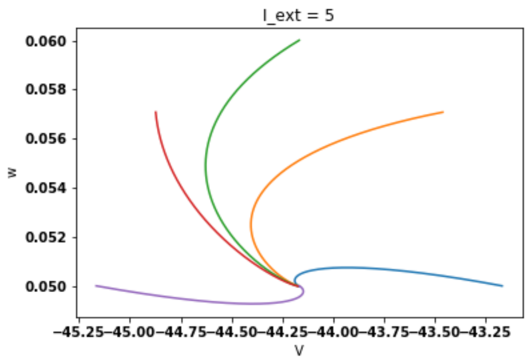

Figure 2: Traj. around $I_{ext} = 5$

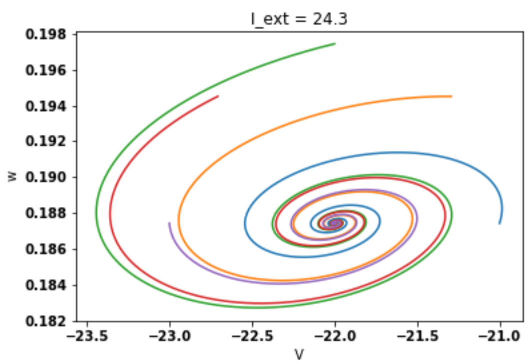

Figure 3: Traj. around $I_{ext} = 24.3$

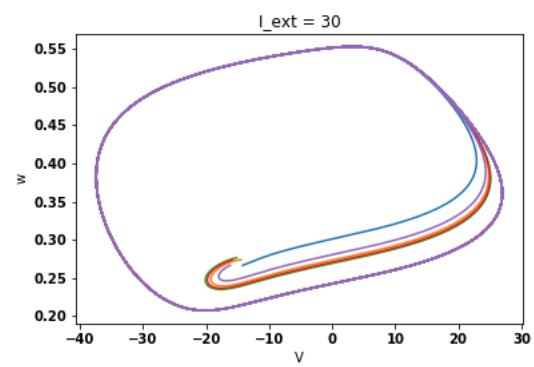

Figure 4: Traj. around $I_{ext} = 30$

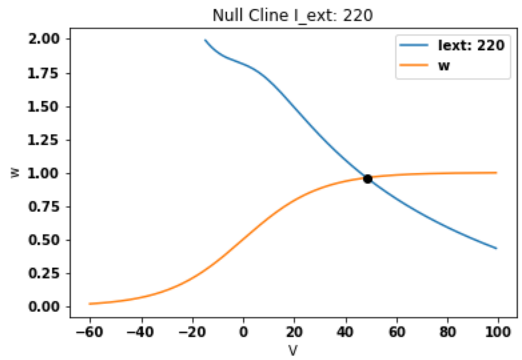

Figure 5: Null-clines $I_{ext} = 220$

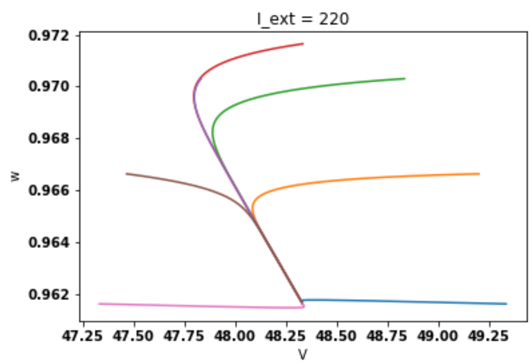

Figure 6: Traj. around $I_{ext} = 220$

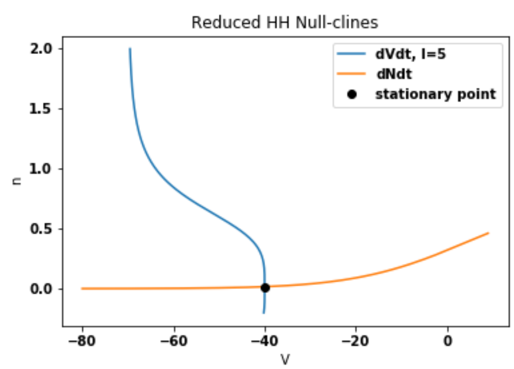

Figure 7: Reduced HH Null-clines

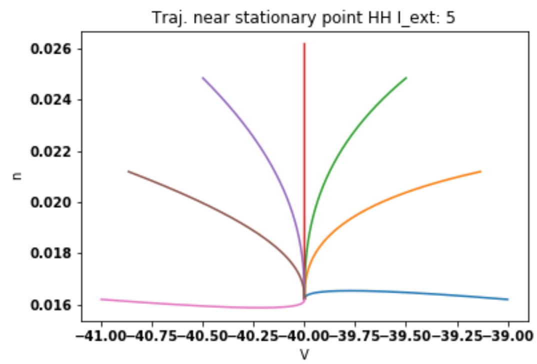

Figure 8: Traj. Reduced HH around $I_{ext} = 5$

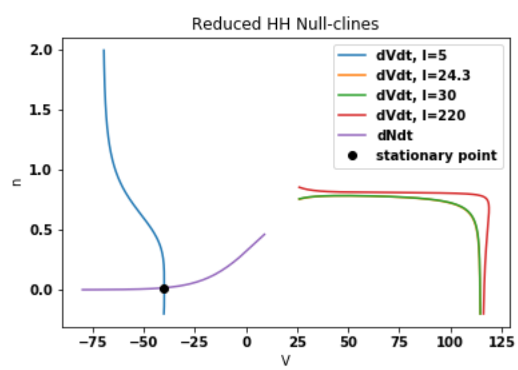

Figure 9: Reduced HH Null-clines All I_{ext}