two coins $\{C_0, C_1\}$

k: prob of choosing C_0

p_0: prob of C_0 showing head

p_1: prob of C_1 showing head

init: $k = 0.5$, $p_0 = 0.6$, $p_1 = 0.1$

observe: $X = \{HHH, HHT, TTT\}$

E - step:

$$P(C_0 \mid X_i) = \frac{P(X_i \mid C_0) P(C_0)}{P(X_i)}$$

$$\Rightarrow \begin{cases} P(X_1 \mid C_0) = 0.6^3 \\ P(C_0) = 0.5 \\ P(X_1 \mid C_1) = 0.1^3 \\ P(C_1) = 0.5 \end{cases} \Rightarrow P(X_1) = 0.5 \times 0.6^3 + 0.5 \times 0.1^3$$

$$= 0.1085$$

$$\Rightarrow \begin{cases} P(C_0 \mid X_1) = \dfrac{0.6^3 \times 0.5}{0.1085} = 0.995 \\ P(C_1 \mid X_1) = \dfrac{0.1^3 \times 0.5}{0.1085} \approx 0.005 \end{cases}$$

$$P(X_2) = 0.5 \times 0.6^2 \times 0.4 + 0.5 \times 0.1^2 \times 0.9$$

$$= 0.072 + 0.0045$$

$$= 0.0765$$

$$\Rightarrow \begin{cases} P(C_0 | X_2) = \dfrac{0.072}{0.0765} = 0.941 \\ P(C_1 | X_2) = \dfrac{0.0045}{0.0765} = 0.058 \end{cases}$$

$$P(X_3) = 0.5 \times 0.4^3 + 0.5 \times 0.9^3$$

$$= 0.032 + 0.3645$$

$$= 0.3965$$

$$\Rightarrow \begin{cases} P(C_0 | X_3) = \dfrac{0.032}{0.3965} = 0.08 \\ P(C_1 | X_3) = \dfrac{0.3645}{0.3965} = 0.92 \end{cases}$$

M - step :

$$k = \frac{1}{N} \Sigma P(C_0) = (0.995 + 0.941 + 0.08) \div 3$$

$$= 0.672$$

$$P_0 = \frac{E[H]}{E[C_0]} = \frac{0.995 \times 3 + 0.941 \times 2 + 0.08 \times 0}{0.995 \times 3 + 0.941 \times 3 + 0.08 \times 3}$$

$$= 0.804$$

$$P_1 = \frac{E[H]}{E[C_1]} = \frac{0.005 \times 3 + 0.058 \times 2 + 0.92 \times 0}{0.005 \times 3 + 0.058 \times 3 + 0.92 \times 3}$$

$$= 0.045$$

$$\Rightarrow \theta^{(1)} = \{k^{(1)}, P_0^{(1)}, P_1^{(1)}\} = \{0.672, 0.804, 0.045\}$$