likelihood : $$p(y|w) \propto \exp\left(-\frac{\beta}{2}(y-\Phi w)^T(y-\Phi w)\right)$$

$$\Rightarrow \ln P(y|w) = -\frac{\beta}{2}\left(y^Ty - 2y^T\Phi w + w^T\Phi^T\Phi w\right) + C$$

$$= -\frac{1}{2}w^T(\beta\Phi^T\Phi)w + w^T(\beta\Phi^Ty) + C$$

prior : $$P(w) \propto \exp\left(-\frac{1}{2}(w-\mu_0)^T\Lambda_0(w-\mu_0)\right)$$

$$\Rightarrow \ln P(w) = -\frac{1}{2}w^T(\Lambda_0)w + w^T(\Lambda_0\mu_0) + C$$

posterior : $$\ln P(w|y) = \ln P(y|w) + \ln P(w)$$

$$= -\frac{1}{2}w^T(\beta\Phi^T\Phi + \Lambda_0)w + w^T(\beta\Phi^Ty + \Lambda_0\mu_0) + C$$

$$\ln P(w|y) = \ln N(w|\mu_N, \Lambda_N^{-1})$$

$$= -\frac{1}{2}(w-\mu_N)^T\Lambda_N(w-\mu_N) + C$$

$$= -\frac{1}{2}w^T\Lambda_N w + w^T\Lambda_N\mu_N + C$$

二次项： $\Lambda_N = \Lambda_0 + \beta \Phi^T \Phi$

一次项： $\Lambda_N \mu_N = \Lambda_0 \mu_0 + \beta \Phi^T y$

$$\therefore \quad \Lambda_N = \Lambda_0 + \beta \Phi^T \Phi$$

$$\Rightarrow \begin{cases} \mu_N = \Lambda_N^{-1} (\Lambda_0 \mu_0 + \beta \Phi^T y) \\ S_N = \Lambda_N^{-1} = (\Lambda_0 + \beta \Phi^T \Phi)^{-1} \end{cases}$$