# HW1

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# 1  HW1 Problem 1 Solution

## 1.1  Connor DePalma

```
[1]: import numpy as np
     import sympy as sym
     import numpy.linalg as linear
     from plotly.subplots import make_subplots
     import plotly.graph_objects as go
     import plotly.figure_factory as ff
```

```
[2]: # Set given prior probabilities
     Pw1 = 3/5
     Pw2 = 2/5
     # need to generate vector of x values for plot generation
     # ensure x=5 exists in vector
     x = np.linspace(-7, 7, 141)
     # Set given conditional densities
     pxw1 = (1/(np.sqrt(2*np.pi)))*np.exp(-0.5*x**2)
     pxw2 = (1/(2*np.sqrt(2*np.pi)))*np.exp(-0.5*((x-2)/2)**2)
```

### 1.1.1  Question 1: Sketch likelihood function for each class & calculated likelihood value at x = 5

```
[3]: fig = go.Figure([go.Scatter(x=x, y=pxw1, name="w1")])
     fig.add_trace(go.Scatter(x=x, y=pxw2, name="w2"))

     fig.update_layout(title="Likelihood of p(x/wj)")
```

## Likelihood of p(x/wj)
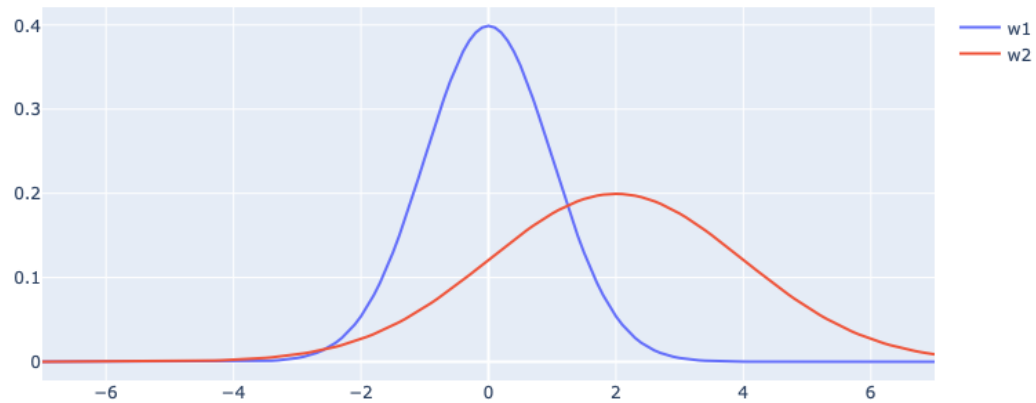


```
[4]: # Find likelihood value when x=5
     print("Likelihood values when x=5:\n")
     print("p(5/w1) = ", pxw1[np.where(x==5)][0])
     print("p(5/w2) = ", pxw2[np.where(x==5)][0])
```
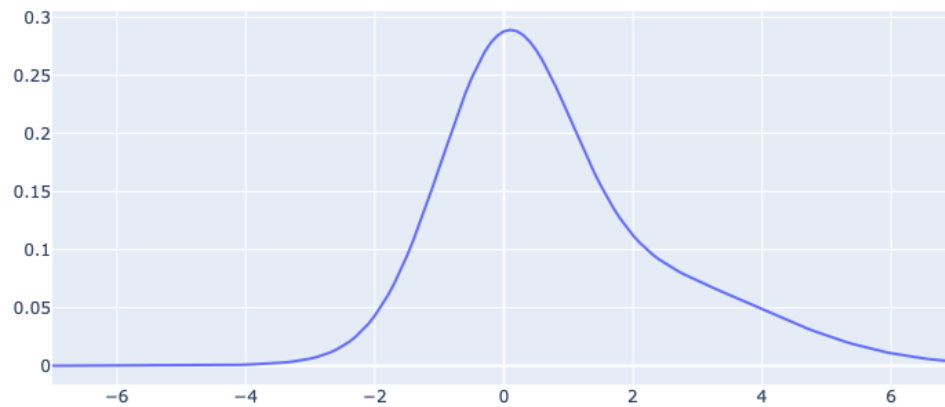
```
Likelihood values when x=5:

p(5/w1) =  1.4867195147342977e-06
p(5/w2) =  0.06475879783294587
```

### 1.1.2 Question 2: Sketch evidence distribution and find value at $x = 5$

```
[5]: # Calculate find evidence distribution
     px = pxw1*Pw1 + pxw2*Pw2
```

```
[6]: fig = go.Figure([go.Scatter(x=x,y=px)])
     fig.update_layout(title="Evidence Distribution p(x)")
```

Evidence Distribution p(x)



```
[7]: # Find evidence value when x=5
     print("Evidence value when x=5:\n")
     print("p(5) = ", px[np.where(x==5)][0])
```

Evidence value when x=5:

p(5) =  0.02590441116488719

### 1.1.3  Question 3: Sketch posterior probability functions and find values at $x = 5$

```
[8]: # Calculate posterior probabilities for each class
     Pw1x = (pxw1*Pw1)/px
     Pw2x = (pxw2*Pw2)/px
```

```
[9]: fig = go.Figure([go.Scatter(x=x, y=Pw1x, name="w1")])
     fig.add_trace(go.Scatter(x=x, y=Pw2x, name="w2"))
     fig.update_layout(title="Posterior Probabilities P(wj/x)")
```

Posterior Probabilities P(wj/x)
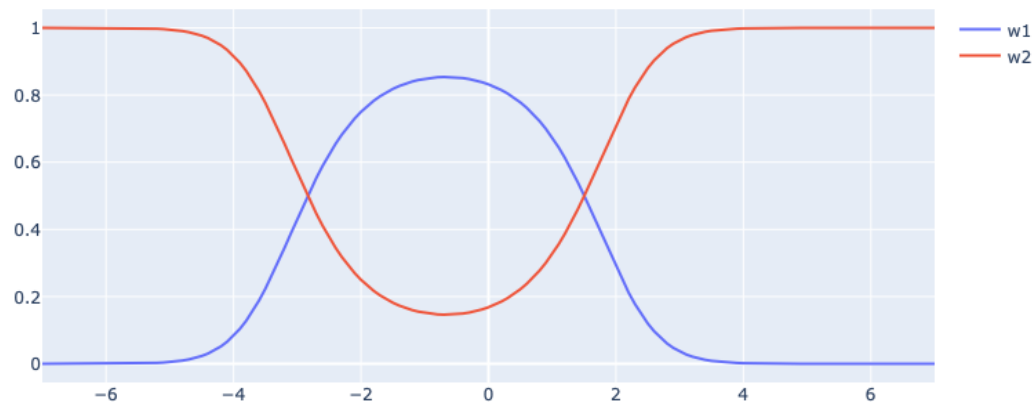


```
[10]: # Find Posterior probability values when x=5
      print("Posterior probability values when x=5:\n")
      print("P(w1/5) = ", Pw1x[np.where(x==5)][0])
      print("P(w2/5) = ", Pw2x[np.where(x==5)][0])
```

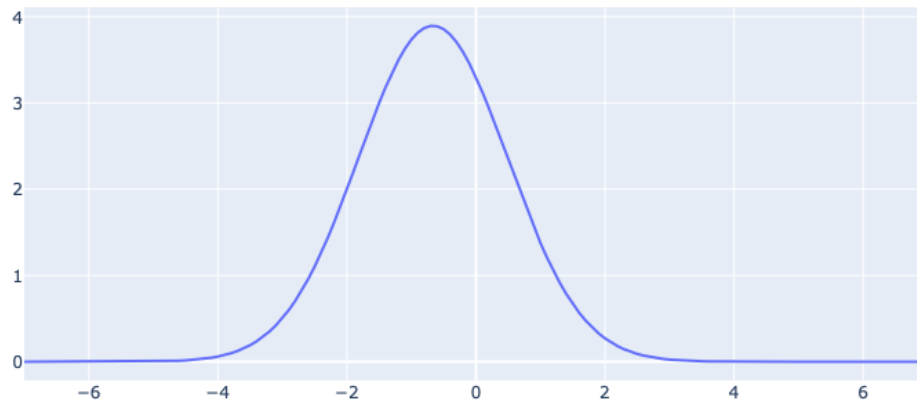Posterior probability values when x=5:

P(w1/5) =  3.443551382668394e-05
P(w2/5) =  0.9999655644861734

### 1.1.4   Question 4: Sketch likelihood ratio function and find value at $x = 5$

```
[11]: # Calculate likelihood ratio values
      likeRatio = pxw1/pxw2
```

```
[12]: fig = go.Figure([go.Scatter(x=x,y=likeRatio)])
      fig.update_layout(title="Likelihood Ratio p(x/w1)/p(x/w2)")
```

Likelihood Ratio p(x/w1)/p(x/w2)



[13]:
```python
# Find likelihood ratio value when x=5
print("likelihood ratio value when x=5:\n")
print("p(5/w1)/p(5/w2) = ", likeRatio[np.where(x==5)][0])
```

likelihood ratio value when x=5:

p(5/w1)/p(5/w2) =  2.2957799781421097e-05

### 1.1.5 Questions 5 & 6: See Additional HW1 Solutions Document

### 1.1.6 Questions 7: Sketch the Bayes Risk for each class and find the values at $x = 5$

[14]:
```python
# Find Bayes Risk values using risk matrix
Ra1x = 4*Pw2x
Ra2x = 2*Pw1x
```

[15]:
```python
fig = go.Figure([go.Scatter(x=x, y=Ra1x, name="w1")])
fig.add_trace(go.Scatter(x=x, y=Ra2x, name="w2"))
fig.update_layout(title="Bayes Risk R(ai/x)")
```

## Bayes Risk R(ai/x)
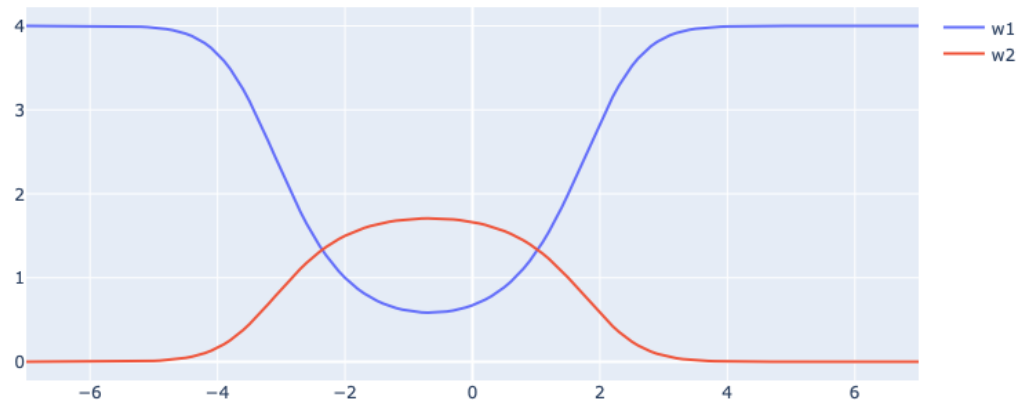


```
[16]: # Find Bayes risk values when x=5
      print("Bayes risk values when x=5:\n")
      print("R(a1/5) = ", Ra1x[np.where(x==5)][0])
      print("R(a2/5) = ", Ra2x[np.where(x==5)][0])
```

```
Bayes risk values when x=5:

R(a1/5) =  3.9998622579446934
R(a2/5) =  6.887102765336788e-05
```