HW4

Thursday, March 17, 2022 6:56 PM

Question 1 (6 points)

Prove Bayes' Theorem. Briefly explain why it is useful for machine learning problems.

Bayes' Law $\rightarrow P(h|D) = \frac{P(D|h) \cdot P(h)}{P(D)}$

Product Rule

Joint prob $\rightarrow P(A,B) = P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$

$\Rightarrow P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$

$\Rightarrow \boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$

Bayes' Theorem can be used to solve problems that involve conditional probabilities. Given a prior probability + some new data, a likelihood can be calculated to essentially adjust the prior to obtain a posterior probability. For example, if a person is 50% likely to go to the gym (no matter the weather) + you see that it is sunny (new data), then the probability that the person will go to the gym will likely increase. Bayes' Theorem can be used in machine learning to predict probabilities along with predicting classes, + is used in many places like the Naive Bayes' Classifier.

Question 2 (8 points):

Consider again the example application of Bayes rule in Section 6.2.1 of Tom Mitchell's textbook. Suppose the doctor decides to order a second laboratory test for the same patient and suppose the second test returns a positive result as well. What are the posterior probabilities of *cancer* and ¬*cancer* respectively following these two tests? Assume that the two tests are independent.

$P(cancer) = 0.008 \qquad P(\neg cancer) = 0.992$

$P(\oplus \mid cancer) = 0.98 \qquad P(\ominus \mid cancer) = 0.02$

$P(\oplus \mid \neg cancer) = 0.03 \qquad P(\ominus \mid \neg cancer) = 0.97$

2 positive tests   since independent

$\Rightarrow P(\oplus\oplus \mid cancer) = 0.98 \cdot 0.98 = 0.9604$

$\Rightarrow P(\oplus\oplus) = P(\oplus\oplus \mid cancer) \cdot P(cancer) + P(\oplus\oplus \mid \neg cancer) \cdot P(\neg cancer)$

$= 0.9604 \cdot 0.008 + 0.03 \cdot 0.03 \cdot 0.992 = 0.0086$

$$\Rightarrow P(cancer \mid \oplus\oplus) = \frac{P(\oplus\oplus \mid cancer) \cdot P(cancer)}{P(\oplus\oplus)} = \frac{0.9604 \cdot 0.008}{0.0086} = \boxed{0.8934}$$

$\Rightarrow P(\neg cancer \mid \oplus\oplus) = 1 - 0.8934 = \boxed{0.1066}$

Question 3 (8 points):

Section 6.9.1 of Tom Mitchell's textbook demonstrates an example using the Naïve Bayes Algorithm to predict a new instance based on a dataset with 14 examples from Table 3.2 of Chapter 3 of the book. If we only have 12 examples as shown below, what is the prediction results for the same new instance? Show your calculation.

New instance: <Outlook=sun, Temperature=cool, Humidity=high, Wind=strong>

Day	*Outlook*	*Temperature*	*Humidity*	*Wind*	*PlayTennis*
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes

Naive Bayes' Classifier $\rightarrow v_{NB} = \operatorname{argmax}_{v_j \in V} P(v_j) \prod_i P(a_i \mid v_j)$

$P(tennis=yes) = 8/12 \qquad P(tennis=no) = 4/12$

$P(sunny \mid tennis=yes) = 2/8 \qquad P(sunny \mid tennis=no) = 3/4$

$P(cool \mid tennis=yes) = 3/8 \qquad P(cool \mid tennis=no) = 1/4$

$P(high \mid tennis=yes) = 3/8 \qquad P(high \mid tennis=no) = 3/4$

$P(\text{cool} \mid \text{tennis}=\text{yes}) = 3/8$ $\quad$ $P(\text{cool} \mid \text{tennis}=\text{no}) = 1/4$

$P(\text{high} \mid \text{tennis}=\text{yes}) = 3/8$ $\quad$ $P(\text{high} \mid \text{tennis}=\text{no}) = 3/4$

$P(\text{strong} \mid \text{tennis}=\text{yes}) = 3/8$ $\quad$ $P(\text{strong} \mid \text{tennis}=\text{no}) = 2/4$

$$\Rightarrow P(\text{tennis}=\text{yes} \mid \text{data}) = P(\text{tennis}=\text{yes}) \cdot P(\text{sunny} \mid \text{tennis}=\text{yes}) \cdot P(\text{cool} \mid \text{tennis}=\text{yes}) \cdot P(\text{high} \mid \text{tennis}=\text{yes}) \cdot P(\text{strong} \mid \text{tennis}=\text{yes})$$

$$= 8/12 \cdot 2/8 \cdot 3/8 \cdot 3/8 \cdot 3/8 = \underline{0.0088}$$

$$\Rightarrow P(\text{tennis}=\text{no} \mid \text{data}) = P(\text{tennis}=\text{no}) \cdot P(\text{sunny} \mid \text{tennis}=\text{no}) \cdot P(\text{cool} \mid \text{tennis}=\text{no}) \cdot P(\text{high} \mid \text{tennis}=\text{no}) \cdot P(\text{strong} \mid \text{tennis}=\text{no})$$

$$= 4/12 \cdot 3/4 \cdot 1/4 \cdot 3/4 \cdot 2/4 = \underline{0.0234}$$

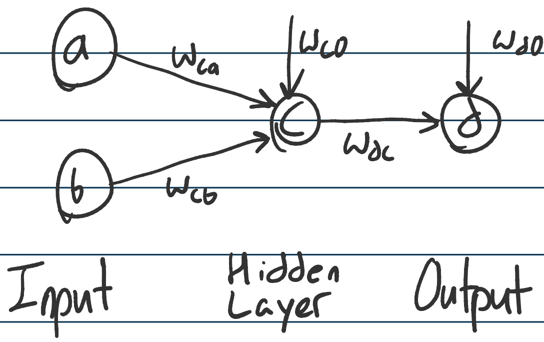
$\Rightarrow$ Since $0.0088 < 0.0234$ $\Rightarrow$ $\boxed{\text{Play Tennis} = \text{No}}$

$\rightarrow$ conditional probability of play tennis = yes $\rightarrow \dfrac{0.0088}{0.0088 + 0.0234} = \underline{0.273}$

Question 4 (14 points): Answer question 4.7 (page 125) of Tom Mitchell's textbook as quoted below:

Consider a two-layer feedforward ANN with two inputs a and b, one hidden unit c, and one output unit d. This network has five weights (w_{ca}, w_{cb}, w_{c0}, w_{dc}, w_{d0}), where w_{x0} represents the threshold weight for unit x. Initialize these weights to the values (.1, .1, .1, .1, .1), then give their values after each of the first two training iterations of the BACKPROPAGATION algorithm. Assume learning rate $\eta = .3$, momentum $\alpha = 0.9$, incremental weight updates, and the following training examples:

a	b	d
1	0	1
0	1	0



$*\ \sigma(x) = \frac{1}{1+e^{-x}}$

$*\ \sigma'(x) = \sigma(x)\cdot(1-\sigma(x))$

$y_c = \sigma(w_{c0} + w_{ca}\cdot a + w_{cb}\cdot b)$

$y_d = \sigma(w_{d0} + w_{dc}\cdot y_c) = \sigma(w_{d0} + w_{dc}\cdot\sigma(w_{c0} + w_{ca}\cdot a + w_{cb}\cdot b))$

Forward Pass

$y_c = \sigma(0.1 + 0.1(1) + 0.1(0)) = \sigma(0.2) = \frac{1}{1+e^{-0.2}} = 0.55$

$y_d = \sigma(0.1 + 0.1(0.55)) = \sigma(0.155) = \frac{1}{1+e^{-0.155}} = 0.539$

Backpropagation

$\delta y_d = y_d(1-y_d)\cdot(d-y_d) = 0.539(1-0.539)(1-0.539)$
$= 0.115$

$\delta y_c = y_c(1-y_c)\cdot(w_{dc}\cdot\delta y_d) = 0.55(1-0.55)\cdot(0.1\cdot 0.115)$
$= 0.0028$

Update Weights

$$= 0.0028$$

Update Weights

$$* \ \Delta w_{i,j}(n) = \eta \delta_j x_{i,j} + \alpha \Delta w_{i,j}(n-1)$$

learning rate   momentum

$$w_{ca} = w_{ca} + \eta \cdot \delta y_c \cdot a + \alpha \cdot \Delta w_{ca(1-1)} = 0.1 + 0.3 \cdot 0.0028 \cdot 1 + 0.9 \cdot 0$$
$$= \underline{0.10084}$$

$$w_{cb} = w_{cb} + \eta \cdot \delta y_c \cdot b + \alpha \cdot \Delta w_{cb(1-1)} = 0.1 + 0.3 \cdot 0.0028 \cdot 0 + 0.9 \cdot 0$$
$$= \underline{0.1}$$

$$w_{c0} = w_{c0} + \eta \cdot \delta y_c \cdot x_0 + \alpha \cdot \Delta w_{c0(1-1)} = 0.1 + 0.3 \cdot 0.0028 \cdot 1 + 0.9 \cdot 0$$
$$= \underline{0.10084}$$

$$w_{dc} = w_{dc} + \eta \cdot \delta y_d \cdot y_c + \alpha \Delta w_{dc(1-1)} = 0.1 + 0.3 \cdot 0.115 \cdot 0.55 + 0.9 \cdot 0$$
$$= \underline{0.119}$$

$$w_{d0} = w_{d0} + \eta \cdot \delta y_d \cdot x_0 + \alpha \Delta w_{d0(1-1)} = 0.1 + 0.3 \cdot 0.115 \cdot 1 + 0.9 \cdot 0$$
$$= \underline{0.134}$$

Repeat for second training example

Forward Pass

$$y_c = \sigma(0.10084 + 0.10084 \cdot 0 + 0.1 \cdot 1) = \sigma(0.20084) = 0.55004$$
$$y_d = \sigma(0.134 + 0.119 \cdot 0.55004) = \sigma(0.1995) = 0.5497$$

Backpropagation

$$\delta y_d = y_d(1 - y_d) \cdot (d - y_d) = 0.5497(1 - 0.5497) \cdot (0 - 0.5497)$$
$$= -0.136$$

$$\delta y_c = y_c(1 - y_c) \cdot (w_{dc} \cdot \delta y_d) = 0.55004(1 - 0.55004)(0.119 \cdot (-0.136))$$
$$= -0.004$$

Update Weights

$$W_{ca} = W_{ca} + \eta \cdot \delta y_c \cdot a + \alpha \cdot \Delta W_{ca(2-1)} = 0.10084 + 0.3(-0.004) \cdot 0 + 0.9 \cdot 0.00084 \cdot 1$$
$$= \boxed{0.1016}$$

$$W_{cb} = W_{cb} + \eta \cdot \delta y_c \cdot b + \alpha \cdot \Delta W_{cb(2-1)} = 0.1 + 0.3(-0.004) \cdot 1 + 0.9 \cdot 0.1$$
$$= \boxed{0.0988}$$

$$W_{c0} = W_{c0} + \eta \cdot \delta y_c \cdot X_0 + \alpha \cdot \Delta W_{c0(2-1)} = 0.10084 + 0.3(-0.004) \cdot 1 + 0.9 \cdot 0.00084 \cdot 1$$
$$= \boxed{0.1004}$$

$$W_{dc} = W_{dc} + \eta \cdot \delta y_d \cdot y_c + \alpha \Delta W_{dc(2-1)} = 0.119 + 0.3(-0.136) \cdot 0.55004 + 0.9 \cdot 0.019 \cdot 1$$
$$= \boxed{0.1137}$$

$$W_{d0} = W_{d0} + \eta \cdot \delta y_d \cdot X_0 \cdot \alpha \Delta W_{d0(2-1)} = 0.134 + 0.3(-0.136) \cdot 0 + 0.9 \cdot 0.034 \cdot 1$$
$$= \boxed{0.1646}$$

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