**Applied Modeling and Optimization**

**Exam 1**

Closed book/notes. Answer all the questions clearly showing all the steps and assumptions. Upload your answer as a single pdf file on Canvas using the exam link.

1.

Show that

$$f(\mathbf{x}) = (x_2 - x_1^2)^2 + x_1^5$$

has only one stationary point which is neither a minimizer or a maximizer.

2. Check if the following functions are convex. Show your computations.

$(a)\ f(\mathbf{x}) = e^{x_1} + x_2^2 + 5$
$(b)\ f(\mathbf{x}) = 3x_1^2 - 5x_1x_2 + x_2^2$
$(c)\ f(\mathbf{x}) = \frac{1}{4}x_1^4 - x_1^2 + x_2^2$
$(d)\ f(\mathbf{x}) = 50 + 10x_1 + x_2 - 6x_1^2 - 3x_2^2$

3. Dr. C wants to invest in two projects, A and B. The total investment budget is $100. He does not want to invest more than $40 in project A. The investment goal is the maximization of satisfaction measured as the product of the amount invested in projects A and B. (a) Does this problem have a maximum? Give an argument to support your answer and (b) compute the optimal solution, if it exists.