HW1

Saturday, February 13, 2021 9:56 PM

## Part 2:

deterministic; noise (stochastic/random)

$X_t = 0.01 + 0.2X_{t-2} + a_t$

where $\{a_t\}$ has mean 0 & variance 0.02.

(a) Mean: $E[X_t] = E[0.01 + 0.2X_{t-2} + a_t]$

$\mu = E[0.01] + E[0.2X_{t-2}] + E[a_t]$

$\mu = 0.01 + 0.2E[X_{t-2}] + E[a_t]$

$\mu = 0.01 + 0.2\mu + E[a_t]$ since stationary

$\mu = 0.01 + 0.2\mu + 0$

$0.8\mu = 0.01 \rightarrow \mu = \frac{0.01}{0.08} \rightarrow \boxed{\mu = 0.0125}$

Variance: $(X_t - \mu) = 0.2(X_{t-2} - \mu) + a_t$

$E[X_t - \mu]^2 = E[0.2(X_{t-2} - \mu) + a_t]^2$

$= E[0.2^2(X_{t-2} - \mu)^2] + E[a_t^2] + E[2 \cdot a_t \cdot 0.2(X_{t-2} - \mu)]$ → 0

$Var = 0.2^2 Var + \sigma_a^2$

$0.96 Var = \sigma_a^2 \rightarrow \boxed{\sigma_a^2 = 0.021}$

(b) Lag-1 autocorrelation: $\boxed{\rho_1 = 0}$

Lag-2 autocorrelation: $\rho_2 = 0.2\rho_0 = 0.2(1) \rightarrow \boxed{\rho_2 = 0.2}$

(c) $X_{100} = -0.01$, $X_{99} = 0.02$.

One step ahead forecast: $\hat{X}_{101} = E[X_{101} / X_{100}, X_{99}]$

$= 0.01 + 0.2X_{t-1}$

$= 0.01 + 0.2 \cdot 0.02$

$\boxed{\hat{X}_{101} = 0.014}$

Two step ahead forecast: $\hat{X}_{102} = E[X_{102} / X_{100}, X_{99}]$

$= 0.01 + 0.2X_{t-1}$

$= 0.01 + 0.2 \cdot (-0.01)$

$\boxed{\hat{X}_{102} = -0.0021}$