HW2

Saturday, February 20, 2021 6:06 PM

1) (a) Stationary AR(1) time series $x(t)$, $x(t)$ is uncorrelated to $x(t-l)$ for $l >= 2$.

$AR(1): x(t) = a_0 + a_1 X_{t-1} + \varepsilon_t$
$Lag\ 2: x(t-2) = a X_{t-3} + \varepsilon_{t-2}$

$\Rightarrow x(t) = a[a X_{t-2} + \varepsilon_{t-1}] + \varepsilon_t$
$= a^2 X_{t-2} + a\varepsilon_{t-1} + \varepsilon_t$

$\Rightarrow x(t) = a^3 \underline{x(t-3)} + a^2 \underline{\varepsilon_{t-2}} + a\varepsilon_{t-1} + \varepsilon_t$
$x(t-2) = a\underline{x(t-3)} + \underline{\varepsilon_{t-2}}$

Since these terms are common in both equations $\rightarrow$ $\boxed{x(t) + x(t-2) \text{ are correlated.} \Rightarrow \text{False}}$

(b) Stationary MA(1) time series $x(t)$, coefficient diff after time lag $l >= 1$ in ACF plot.

$x(t) = \theta\varepsilon_{t-1} + \varepsilon_t + \mu$
$x(t-1) = \theta\varepsilon_{t-2} + \varepsilon_{t-1} + \mu$
$x(t-2) = \theta\varepsilon_{t-3} + \varepsilon_{t-2} + \mu$

$\rightarrow$ No common terms exist when comparing $x(t) + x(t-2)$ for MA(1).
$\Rightarrow$ The equations are uncorrelated + covariances are 0 for $l > 1$.
$\Rightarrow$ ACF is also 0 for $l > 1$. $\Rightarrow$ False