HW2_4

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4) Vector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, Matrix $A = \begin{bmatrix} a_1, a_2 \\ a_3, a_4 \end{bmatrix}$

$f(x) = 0.5 \cdot x^T A x \rightarrow \nabla f(x) = Ax$

$\rightarrow f(x) = 0.5 \cdot \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Fill in x & A

$\rightarrow f(x) = 0.5 \cdot \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 2a_1+3a_2 \\ 2a_3+3a_4 \end{bmatrix}$ Simplify

$\rightarrow f(x) = 0.5 \cdot [4a_1+6a_2+6a_3+9a_4]$

if $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\rightarrow f(x_1, x_2) = 0.5 \cdot [a_1 x_1^2 + a_2 x_1 x_2 + a_3 x_1 x_2 + a_4 x_2^2]$

$\rightarrow \nabla f = \begin{pmatrix} \frac{df}{dx_1} \\ \frac{df}{dx_2} \end{pmatrix} = \begin{bmatrix} 0.5(2a_1 x_1 + a_2 x_2 + a_3 x_2) \\ 0.5(a_2 x_1 + a_3 x_1 + 2a_4 x_2) \end{bmatrix}$ find partial derivatives

$= \begin{bmatrix} a_1 x_1 + 0.5 a_2 x_2 + 0.5 a_3 x_2 \\ 0.5 a_2 x_1 + 0.5 a_3 x_1 + a_4 x_2 \end{bmatrix}$ Simplify

$= \begin{bmatrix} a_1 x_1 + (0.5 a_2 + 0.5 a_3) x_2 \\ (0.5 a_2 + 0.5 a_3) x_1 + a_4 x_2 \end{bmatrix}$

$= \begin{bmatrix} a_1 + (0.5 a_2 + 0.5 a_3) \\ (0.5 a_2 + 0.5 a_3) + a_4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ factor out x_1 & x_2

Since matrix A is symmetric $\rightarrow a_2 = a_3$

$\Rightarrow \nabla f = \begin{bmatrix} a_1 + a_2 \\ a_3 + a_4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Ax$

$\Rightarrow$ Gradient of $f(x) = 0.5 \cdot x^T \cdot A \cdot x$ with vector x & symmetric matrix A is equivalent to $A \cdot x$.