Stevens Institute of Technology
Department of Electrical and Computer Engineering

Spring Semester 2022

CpE 646 Pattern Recognition and Classification

Midterm Exam

Mar 11, 12:00 – 4:00 PM

Instructions:

- Provide all necessary intermediate steps in your work. You will get zero credit if you only provide the final result without justification or explanation.
- **All steps and intermediate results should be composed into one single PDF or DOC file**, which should be submitted through Canvas Assignments as your midterm solutions.
- You may use Matlab or Python for math calculations. Only the basic math functions are allowed, which include probability density functions, matrix inversion, eigen-decomposition. You may not use other built-in functions or library functions, such as PCA or LDA etc.
- If you are using Matlab, you can copy your command window script into a text file. If you are using Python, you can save your Python script in a text file. Your saved Matlab or Python script file should be submitted through Canvas Assignments separately, which will only be used for reference and will not be graded.

Name (print): ____________________________

In a 3-dimensional 2-class classification problem, the following training sample vectors are given:

$$\boldsymbol{D}_1 = \left\{ \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix}, \begin{bmatrix} 12 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 6 \\ 12 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix} \right\} \textbf{ and } \boldsymbol{D}_2 = \left\{ \begin{bmatrix} 0 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 3 \end{bmatrix} \right\}$$

Following are the steps to be taken to process this dataset.

Step 1. (20 points) Use Fisher linear discriminant analysis method to calculate **a)** calculate a transform vector $\boldsymbol{w}$ to reduce the data dimension to 1 dimension, and then **b)** calculate the projections of all the data samples in the resulting 1-dimensional space. **(Show your steps and immediate results.)**

Step 2. (20 points) Continue from Step 1. In the resulting **1-dimensional** space of Step 1, assume **class 1** and **class 2** data samples follow two specific distributions as shown.

$$\textbf{class 1:} \quad p(x \mid \omega_1) = \begin{cases} \theta_1^2 (x+1) e^{-\theta_1 (x+1)} & x \geq -1 \\ 0 & \textbf{otherwise} \end{cases}$$

$$\textbf{class 2:} \quad p(x \mid \omega_2) = \frac{1}{\sqrt{2\pi}\,\sigma_2} \exp\left[-\frac{1}{2} \left(\frac{x - \mu_2}{\sigma_2} \right)^2 \right]$$

For the **class 1** distribution $p(x|\omega_1)$, given N independent and identically distributed training samples $x_1, x_2, ..., x_N$, use maximum likelihood estimation to estimate the parameter θ_1. **(This derivation should be done by hand.)** (ML estimation of **class 2** distribution parameters can be found in lecture notes CPE646-4)

Step 3. (20 points) Use the maximum likelihood estimation formula from **Step 2** and lecture notes, estimate the parameters of these two distributions $\boldsymbol{p(x|\omega_1)}$ **and** $\boldsymbol{p(x|\omega_2)}$ based on the 1-diementional projections of the training samples from **Step 1.**

Step 4. (20 points) Based on the results from **Step 3**, derive the Bayesian decision boundary with the following risk matrix. Assume the two classes have equal prior probabilities. **(Simplify your expression as much as you can, you will be graded accordingly.)**

	ω_1	ω_2
$\boldsymbol{a}_1$	0	3
$\boldsymbol{a}_2$	1	0

Step 5. (20 points) Given a test data sample $\begin{bmatrix} \mathbf{5} \\ \mathbf{1} \\ \mathbf{2} \end{bmatrix}$, **a)** calculate its posterior probabilities $\boldsymbol{P(\omega_1|x)}$ and $\boldsymbol{P(\omega_2|x)}$. **b)** If the risk matrix in Step 4 is used, calculate the associated Bayesian risk, and **c)** classify this data sample. **(Show your steps and immediate results.)**