# The generalized Maxwell-Slip model: A novel model for friction simulation and compensation

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# The Generalized Maxwell-Slip Model: A Novel Model for Friction Simulation and Compensation

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***Abstract*—A novel, multistate friction model is presented, which is obtained from the Maxwell-slip model by replacing the usual Coulomb law at slip by a rate-state law. The form of the latter state equations is arrived at by comparison with a recently developed generic friction model as well as limiting behavior cases. The model is particularly suitable for quick simulation and control purposes, being both easy to implement and of high fidelity. This communication situates the model, by outlining its development background and structure, and highlights its basic characteristics.**

***Index Terms*—Friction modeling, friction compensation, mechanical systems, presliding regime, sliding regime.**

## I. Introduction

The past few decades have witnessed an increasing preoccupation with friction modeling for the purpose of understanding, simulation and control in a variety of disciplines ranging from geophysics to electromechanical systems. The simple classical, static models of Coulomb, Stribeck, etc. have given way gradually to more sophisticated, dynamical models with due attention to presliding hysteresis and time-lag effects. In the field of mechanical systems and control, motivated by the needs for fast and accurate positioning, the past decade or so has seen the development and introduction of several models, based on the rate-state phenomenology, which was already developed in the geophysics community.

The subject of friction was reintroduced in the systems and control community with renewed zest in 1991 by Armstrong [1]. The development of various models and control methods, as discussed in [2], culminated in a first milestone with the publication of the LuGre model [3], which is essentially based on the rate-state law, stipulated by, among others, Rice and Ruina [4]. The LuGre model both benefitted from and related itself well to the most important developments of the time in friction modeling (e.g., [4]–[6]y ). (We will further show in this note that there is a general model structure to which the various models belong). It was, moreover, a very easy model to implement, which is witnessed to by its current popularity.

As far as we are concerned, the LuGre model had one essential shortcoming, namely that the hysteresis behavior, in the presliding regime, was not endowed with the nonlocal memory character, a blemish, which could have some important implications around velocity reversals. Let us remark here that presliding is marked physically by the dominance of adhesive forces (at asperity contacts) such that the friction force, in that regime, appears to be predominantly a hysteresis function (with nonlocal memory) of the displacement rather than the velocity. The presence of this type of hysteresis in mechanical systems, e.g., rolling element guideways, is apt to lead to complex dynamical behavior, as has been shown recently [7]–[9].

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Realizing this shortcoming of the LuGre model, many authors [10]–[12] proposed modifications and extensions to overcome it, or at least certain aspects thereof. Notably, the Leuven model [10], [11] succeeded in incorporating the nonlocal memory hysteresis property at the expense of more difficult implementation. Here, we must add that models are generally judged on two basic criteria: their fidelity (in simulating realistic friction behavior) and their ease of implementation. (With some users, the latter seems to take priority over the former).

Recently, the present authors developed a generic friction model based on the physical aspects of the phenomenon [13]. Analysis of the results obtained from that model led them to the synthesis of the model presented in this communication, which combines high fidelity with ease of implementation. In essence, the model is a generalization of the Maxwell-slip formulation, whereby the Coulomb slip law is replaced by a rate-state law. In this way both pre-sliding and gross sliding could be correctly modeled in one simple structure.

The name we gave to the new model was the generalized Maxwell-slip (GMS) model. However, we realized later that this name was not unique. It turned out that the term "generalized" (Maxwell-slip) had been used by several authors before [14]–[17] to denote the usual Maxwell-slip system that comprises *more than one element* (each element being called a Maxwell-slip model).

In the following, Section II outlines a general model structure for friction, while Section III shows how the various existing heuristic friction models relate to it. Section IV discusses the novel, easy-to-implement friction model, our GMS friction model, and its properties. Finally, appropriate conclusions are drawn.

## II. General Heuristic Friction Model Structure

Most of the existing heuristic friction models that are successfully used for identification and compensation purposes correspond to a generalized friction model structure, which consists in a friction force equation and a state equation. The friction force $F_f$ is a generalized function of an internal state vector, $\mathbf{z}$, the velocity $v$ and the position $x$ of the moving object

$$F_f = \mathcal{F}(\mathbf{z}, v, x). \tag{1}$$

The state equation describes the dynamics of the internal state vector $\mathbf{z}$, which can be written as a first-order differential equation of a general form

$$\frac{d\mathbf{z}}{dt} = \mathcal{G}(\mathbf{z}, v, x). \tag{2}$$

In the previous, $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$ are general nonlinear functions. In particular, $\mathcal{G}(\cdot)$ may be discontinuous so that both pre-sliding and gross sliding could be represented (see later).

This generic characterization of dry friction force dynamics, known today as the rate-state model, is originally due to, among others, Rice and Ruina [4]. They showed, in particular, that $\mathcal{F}(\mathbf{z}, v, x) = F_1(\mathbf{z}, v, x) + F_2(v)$, where $F_1$ is responsible for the transient response (in the velocity), while $F_2$ represents the instantaneous response to velocity change. However, their formulation corresponded to sliding and did not therefore allow for presliding or for passing through zero velocity.

Heuristic friction modeling consists then in finding suitable expressions for the generalized functions $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$, such that the resulting model would simulate faithfully all observed types of friction behavior.

Two generic conditions apply, which provide limiting conditions on the functions $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$, namely, the following.

1) For constant velocities, the steady-state $(d\mathbf{z}/dt = 0)$ friction force is a function of the velocity $v$ only. This friction behavior imposes a first condition on the general functions

$$\begin{aligned}\text{if } v = \text{constant, then} \quad & \mathcal{G}(\mathbf{z}, v, x) = 0 \\ \text{and} \quad & \mathcal{F}(\mathbf{z}, v, x) = F(v) = s(v) + F_2(v) \end{aligned} \tag{3}$$

where $s(v)$ is the velocity-weakening curve, which is often (erroneously) referred to as the Stibeck curve, while $F_2(v)$ is the viscous (or velocity-strengthening) curve[1]

2) The second condition on the general functions is determined by the frictional behavior in the presliding regime, at small displacements. The friction force is then a hysteresis function of the position, with nonlocal memory characteristics [10]

$$F_f = \mathcal{F}(\mathbf{z}, v, x) = F_h(x). \tag{4}$$

From (3) and (4), we can see that $\mathcal{G}(\cdot)$ is generally not analytic.

Let us add here that the generic model described at the end of Section III also shows that there will be a systematic variation in the normal force with tangential relative motion. This should entail at least one extra state equation for the normal force, which is coupled to (1) and (2). Such a refinement of the model structure could lead to a very complex dynamics (partly through the addition of an extra d.o.f.) and is considered outside the scope of this note.

## III. How the Existing Friction Models Relate to This Model Structure

In this section, we shall try to situate the various (empirical, heuristic) friction models used in the literature with the model structure outlined in the previous section.

A first set of friction models are the so-called 'classical models of friction', such as the Coulomb model (with or without viscous friction). Those models describe a static relationship between the friction force and the velocity, corresponding to (1) and the first of the conditions (3), i.e., without a state equation or presliding behavior. Although such models may suffice for general purposes, highly accurate positioning, e.g., within the micrometer range in machine tool applications, requires more advanced modeling.

The Dahl model [5] provided an answer to the presliding problem, which appears only as a jump discontinuity in the Coulomb model. It approximated the friction force, in presliding, by steady state hysteresis loops, corresponding to (4), though now often formulated as a first order differential equation, in the force and the displacement, which yields the hysteresis curve. It is worthy of mention that the Dahl model does not include any means of modeling the nonlocal memory behavior, and is thus suitable only for the analysis of steady-state periodic motion with only two motion reversals per cycle.

The LuGre model [3] developed the Dahl (static differential) equation into a (dynamic) state equation (2), where the state corresponds to the average deformation of surface asperities or "bristles," and supplemented it with the friction force equation (1) in a way that satisfies condition (3) but not condition (4). In fact, in the latter case, the dynamic state equation reduces to the quasistatic Dahl equation, which has no provision for nonlocal memory effects, as mentioned above. This shortcoming leads to the phenomenon of spurious drift (see next paragraph), when an oscillatory force, smaller than the breakaway threshold, is applied to a friction block.

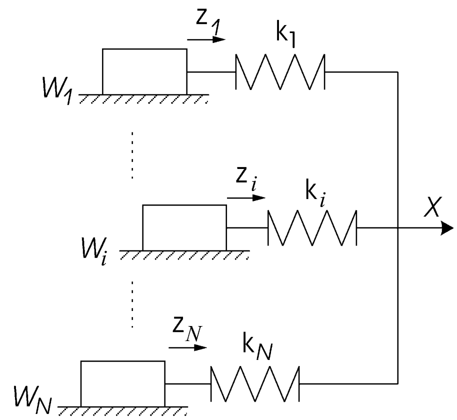


Fig. 1. Representation of the Maxwell-slip friction model using $N$ elementary models.

Having shown that neither the Dahl model nor the LuGre model are capable of realizing the nondrifting property in presliding,[2] Dupont *et al.* [12] proposed an extension of the LuGre model which captures that property. This so-called elasto-plastic friction model introduces a modification to the state equation of the LuGre model, such that an amplitude and rate dependent transition is affected, from pure elastic bristle deformation to the plastic deformation state corresponding to the usual LuGre model. Although this model ensures indeed the elimination of spurious drift below a certain presliding displacement limit, it does not solve the problem of lack of nonlocal memory.

The integrated friction model by Swevers *et al.* [10], called the Leuven model, elaborated the LuGre model further by including the presliding hysteresis with nonlocal memory explicitly. This type of hysteresis proves to be an essential behavioral characteristic of friction, which determines the model's performance in pre-sliding. However, incorporating the (analytic) hysteresis function $F_h$ while maintaining the LuGre formulation was not without implementation difficulties, which made the model less attractive. To overcome those problems, Lampaert *et al.* [11] proposed to replace $F_h$ by a Maxwell-slip model representation, which works as follows.

Referring to Fig. 1, the idea is to assemble $N$ elasto-plastic (Maxwell or Jenkin) elements in parallel, which all have one common input displacement $x$. Each of the elements $i$ has an output force $F_i$; the element is characterized by a stiffness $k_i$, a slip (or saturation) force limit $W_i$ and a state variable $z_i$ (representing the spring deflection). Since the elements are assumed to have no mass, there will be a static relationship between the force $F_i$ and the deflection, $z_i$, so that $F_i = k_i z_i$. The blocks stick each time there is a velocity reversal.

The hysteresis force is equal to the sum of hysteresis forces $(F_i)$ of each element

$$F_h(x) = \sum_{i=1}^{N} F_i. \tag{5}$$

Like LuGre, the Leuven model can account accurately for experimentally obtained friction characteristics: Stribeck effect in sliding, frictional lag, varying break-away forces, stick-slip behavior and, furthermore shows hysteretic behavior with nonlocal memory in presliding, which is not shown by the LuGre model.

[1]Strictly speaking, the Stribeck curve contains both velocity weakening and velocity strengthening, i.e., $s + F_2$. However, velocity weakening is often referred to as the Stribeck *effect*, which may be the source of the misuse. We must note however, that the two effects, emanating from different mechanisms, should be clearly distinguished from one another.

[2]Dupont *et al.* stated that a mass undergoing an arbitrary force smaller than the break-away force will not drift away from its pre-sliding regime. They called this property stiction.

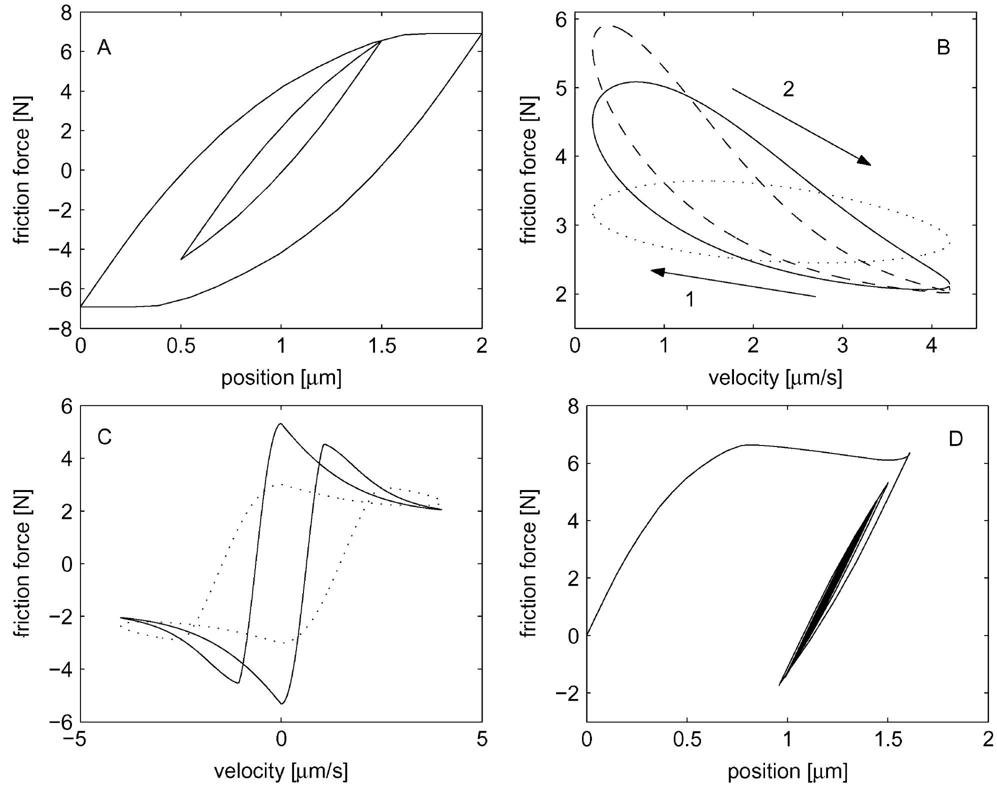


Fig. 2. Friction behavior of the GMS friction model. Panel A shows the (rate-independent) hysteresis with nonlocal memory in the presliding, panel B shows frictional lag with increasing acceleration $(\text{dashed} = \text{ref.},\ \text{solid} = \times 16, \text{dotted} = \times 100)$, Panel C shows periodic motion friction with sliding, frequency changing by factor 10 between solid and dotted, Panel D shows the nondrifting property, which is the response to an oscillatory force input, after sliding, whose amplitude remains under the break-away limit.

Finally, the present authors have recently developed a generic friction model at asperity level [13], [18], [19], which is based on physical mechanisms behind friction. It considers a contact scenario involving a large population of interacting asperities subject to such phenomenological mechanisms as normal creep, adhesion between contacting asperities, deformation of asperities, hysteresis losses in materials and impact of asperity masses.

The generic model proved capable of faithfully simulating all the known macroscopically measured friction force dynamics, from presliding, rate-independent hysteresis with nonlocal memory to frictional lag and stick-slip phenomena in sliding. However, owing to its elaborateness and complexity, it is not suitable for quick simulation or online applications.

Finally, inspired by the generic model and seeking to improve on the Maxwell modification to the Leuven model, it was not difficult to conceive of the Generalized Maxwell-slip friction model, which we then developed as an easy-to-implement version of the generic model.

## IV. GENERALIZED GMS FRICTION MODEL

A faithful friction model should fulfil the two conditions mentioned in Section II and, moreover, should have the ability to properly simulate other possible friction properties such as frictional lag, nondrifting, break-away force dependence on the rate of applied tangential force, etc. Those properties, which are not imposed by the two conditions, can be seen as 'accidental' results of the formulation of the equations for the discussed friction models in the previous section. For instance, the LuGre model captures two different friction properties by one and the same state equation: The presliding hysteresis effect and the frictional lag. (As a consequence, those two effects cannot be altered separately by independent parameters [2]).

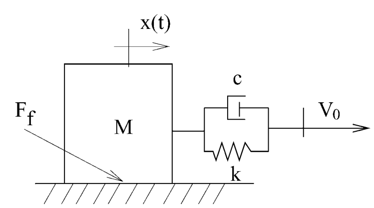


Fig. 3. Scheme to study the stick-slip phenomenon.

Furthermore, it is still possible to find numerous other models that satisfy the two former properties. For example, other possible forms of the state equation for the Leuven model could be

$$\frac{dz}{dt} = f_1(v,z) f_2\left(1 - \frac{F_h(z)}{s(v)}\right), \quad \text{with } f_2(0) = 0.$$

This state equation still obeys the two conditions and also captures implicitly the frictional lag and break-away force.

The reason why most models are fundamentally based on only two properties lies perhaps in the difficulty of measuring accurately the other phenomena. Based on the form and the results of the generic friction model discussed briefly in the previous section, the challenge is now to formulate a heuristic model without leaving out any essential phenomenon that is useful for control purposes.

### A. Formulation of the Generalized Maxwell-Slip Friction Model

The developed model is based explicitly on three friction properties: i) a Stribeck curve for constant velocities, ii) a hysteresis function with nonlocal memory in the presliding regime, and iii) frictional lag in the sliding regime. Whereas the Leuven model tries to fit the hysteresis property (e.g., by the Maxwell-slip implementation) into the

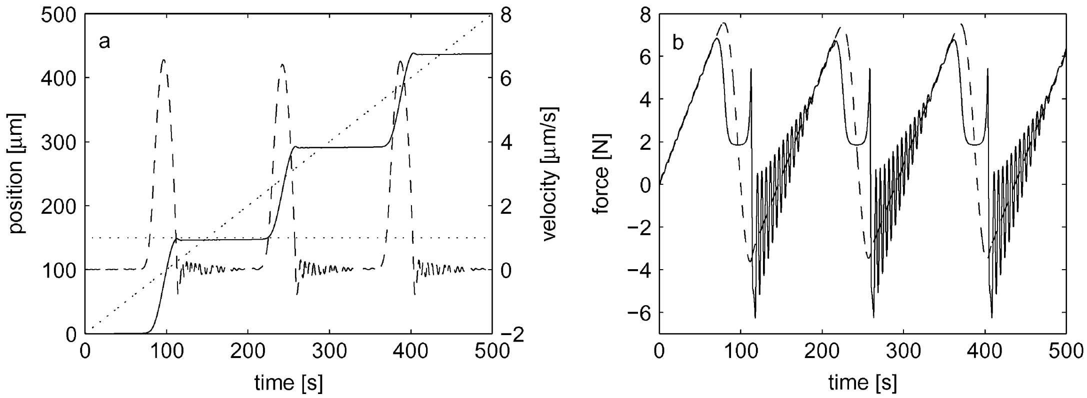


Fig. 4. Stick-slip behavior. (a) Desired position and velocity in dotted lines, the real position of the mass in full line and the real velocity as a function of time in dashed line. (b) Friction force (full line) and applied spring force (dashed line) as a function of time.

steady state sliding property, the novel model follows the more natural way, in line with the physics of the problem, of imposing sliding dynamics onto the slip phase of the Maxwell-slip implementation of the hysteresis. That is why we have named it the Generalized Maxwell-slip (GMS) friction model. As in the usual Maxwell-slip model, Fig. 1, the developed model is also a parallel connection of $N$ single state friction models, all having the same input (namely the displacement or the velocity) and the same dynamics model (but with different sets of parameter values). Each of these single state friction models has a logic state which indicates whether the element is sticking or slipping. Let $v$ be the velocity input to the system and $z_i$, $(1 \leq i \leq N)$ be the $i$th element of the state vector $\mathbf{z}$, the dynamics of each elementary model is determined by the following rules.

- If the element sticks, the state equation is given by:

$$\frac{dz_i}{dt} = v \tag{6}$$

  and the element remains sticking until $z_i = s_i(v)$.
- If the element slips, the state equation is given by:

$$\frac{dz_i}{dt} = \mathrm{sgn}(v) C_i \left(1 - \frac{z_i}{s_i(v)}\right) \tag{7}$$

  and the element remains slipping until the velocity goes through zero. We have obtained this form of the state equation by considering best fits to simulation runs of the generic friction model.

In the last equation, $C_i$ is the attraction parameter, (which is a *gain* that determines how fast $z_i$ converges to $s_i$) and $s_i(v)$ is the velocity-weakening (Stribeck) function for element $i$. Note that one can show easily that if $s_i(v)$ is replaced by the constant (Coulomb, slip) value $W_i$, the GMS model will reduce to the usual Maxwell-slip model.

The friction force is given as the summation of the outputs of the $N$ elementary state models plus two extra terms to account for unmodeled effects

$$F_f(t) = \sum_{i=1}^{N} (k_i z_i(t) + \sigma_i \dot{z}_i(t)) + f(v).$$

The first term under the summation is the elasto-sliding friction force, the second represents possible "viscoelastic" behavior. Finally, $f(v)$ is the velocity-strengthening, or "viscous" component that is usually set to be proportional to $v(t)$.

### B. Identification of the GMS Friction Model

The number of unknown parameters in the model depends on the number of Maxwell elements comprized. Each element is characterized by a stiffness, $k_i$, a viscoelastic coefficient, $\sigma_i$, an attraction parameter $C_i$, and a velocity weakening function $s_i(v)$. This latter could be described usually by three or more parameters, e.g., as: $s_i(v) = s_i(0) + (s_i(0) - s_i(\infty))(\exp(-\alpha_i v) - 1)$, where $\alpha_i$ is the reciprocal of the Stribeck velocity for element $i$. In this case, we need to identify five unknown parameters per element. However, we could reduce this number, without sacrificing the essence of this model, by assuming one common form of the velocity weakening curve for all elements, i.e., $s_i = \nu_i S$, where $\nu_i$ is a scaling parameter and $S$ is described by three (or more parameters). The same could be done in regard to the attraction parameter, i.e., $C_i = \nu_i C$, see [20] or the viscoelastic coefficient.

Finally, the identification of the unknown parameters of the GMS model can best be carried out by any optimization method that is suitable for nonlinear problems, e.g., the Downhill–Simplex algorithm [21]. The friction-displacement-time data which is used for identification should be obtained in such a way as to contain the effects of the different friction regimes. Using one common $S(v)$ curve, [21] shows that as few as four Maxwell elements could provide a very good fit to the data.

### C. Properties and Characteristics of the GMS Friction Model

The standard properties, which a well-behaved friction model is supposed to possess, i.e., that i) the friction force be a continuous function of time; ii) the nonviscous part of the friction force be bounded by the static friction force, and iii) the GMS model be dissipative, could all be proved in straightforward manner, as shown in [20]. In that reference, also the friction behavior predicted by the GMS model was compared with that of LuGre, Leuven and the generic models, in the different regimes, i.e., i) presliding, ii) frictional lag, iii) transition between pre-sliding and sliding, and iv) the nondrifting property. The GMS proved to be the only model that is consistent with the generic model in regard to all properties. The obtained behavior is shown in Fig. 2. In particular, the GMS model was the only one that rendered the nondrifting property correctly.

For the sake of completeness, a word must be said about the simulation of the stick-slip phenomenon by the GMS model. Although the LuGre model, the elastoplastic model and the Leuven model are all able to simulate this phenomenon, the result obtained from the GMS model is qualitatively different, and is in close agreement both with the experimental results obtained in [22] and with the generic model results [13] (see also [18]).

Fig. 3 shows the scheme used to examine stick–slip motion. A constant velocity is applied to a mass-spring-damper system where the friction in the contact is simulated by the GMS model.

Fig. 4 shows typical results obtained. Remarkable is that each stick period is preceded by a characteristic spike in the force followed by high frequency damped vibration. The high frequency is evidently the result of the increased stiffness as more Maxwell elements (physically, more surface asperities) transit from a slip to a stick state, while the damping is the result of the hysteresis (or so-called "structural") dissipation.

## V. Conclusion

This note presented a novel, heuristic friction model, called the GMS model, which is obtained by assigning rate-state dynamics to the slip phase of the conventional Maxwell-slip blocks. While being able to simulate accurately presliding hysteresis with nonlocal memory, frictional lag, and the nondrifting property, it compares most favorably with a generic, physics-based model as well as with experimental results, in particular significantly, regarding stick–slip simulation. By virtue of its simplicity, this model is very suitable for quick simulation and (online) control purposes.

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# A New Phase-Lead Design Method Using the Root Locus Diagrams

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***Abstract*—A new phase-lead design method using the root locus diagrams is proposed. In the traditional phase-lead root locus design procedure, the designer has no direct control to the steady-state error constant of the resulting closed-loop system. In order to obtain a satisfactory steady-state error constant, the designer has to try different locations of the compensator's zero and the desired dominant roots. However, the procedure does not indicate to the designer how to alter the locations to obtain the desired steady-state error constant. With the new method presented in this note, the applicability of the phase lead compensation network is determined once the desired dominant roots are given, and the maximum "reachable" steady-state error constant can be directly determined. If the desired steady-state error constant rely within the "reachable" limit, the desired steady-state error constant together with the desired dominant roots are directly used to obtain the compensator. If the desired stead-state error constant exceeds the reachable limit, the note proposes a way to find out the proper locations of the desired dominant roots.**

***Index Terms*—Design method, phase-lead compensator, phase-lead design, root-locus diagrams, steady-state error constant.**

## I. Introduction

Among many compensation configurations, the phase-lead networks are designed by using the Bode plot method and the root locus method to introduce extra phase-lead at the cross-over frequency. With the Bode diagram method, the frequency response of the uncompensated open-loop system is plotted with the desired gain to allow an acceptable steady-state error. Then, the phase margin and the expected maximum value of the frequency response are examined to see whether they satisfy the specification. If the phase margin is not sufficient, phase-lead can be added to the phase angle curve by placing the zero and pole of the compensator in a suitable location. To obtain maximum additional phase-lead, the frequency at which the maximum phase-lead occurs is located at the frequency where the magnitude of the compensated system crosses the 0-dB axis. This method works well except that the relationship among the phase margin and the maximum magnitude to the system's transient response is not direct.

When the time domain performance indexes, such as the settling time, the rise time, the peak time, and the percentage overshoot for

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