

# Parameter Estimation of a Permanent Magnetic DC Motor

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**Abstract** (The identification of system parameters plays an essential role in system modeling and control. This paper presents a parameter estimation for a permanent magnetic DC motor using the simulink design optimization method. The parameter estimation may be represented as an optimization problem. Firstly, the initial values of the DC motor parameters are extracted using the dynamic model through measuring the values of voltage, current, and speed of the motor. Then, these values are used as an initial value for simulink design optimization. The experimentally input-output data can be collected using a suggested microcontroller based circuit that will be used later for estimating the DC motor parameters by building a simulink model. Two optimization algorithms are used, the pattern search and the nonlinear least square. The results show that the nonlinear least square algorithm gives a more accurate result that almost approaches to the actual measured speed response of the motor.)

**Index Terms**— PMDC Motor, Microcontroller, Rotary Encoder, Matlab Software.

## 1. INTRODUCTION

The incorrect calculation of the motor parameters values leads to the weakness in the control and its instability, so the accuracy in extracting the parameters is a real problem, which is earnest studied in [1]. The major target for the parameters estimation of the system is to improve the mathematical model, which completely describes the physical model that used to predict the system response and the design advantages that can be obtained from system modification [2]. The utility of finding a mathematical model is to know the behavior of the system and to predict its response from different inputs under different conditions [3]. A suggested method for estimating the parameter values of the model from the open loop and closed loop step responses are presented in [4]. Several methods that are applied for estimation the PMDC motor parameters found in [4-16]. Practically, various approaches used for the parameters estimation, which are: the frequency response method [5][6], algebraic parameters estimation methodology [7][8][9][10], ordinary least squares and recursive least squares algorithms [11], constraint optimization technique

[12], moment method [13], inverse theory [14], Chaotic initialized particle swarm optimization [15], and designed optimization of parameters using response surface methodology with central composite [16].

The paper is organized as follows; Section 2 studies the mathematical model of the PMDC motor. Section 3 illustrates the simulink design optimization method. Section 4 presents the parameter estimation as an optimization problem. The method used to collect the experimental data from the motor is presented in section 5. Section 6 explains all steps used for estimation of the motor's parameters.

## 2. Model of PMDC motor

The PMDC motor behavior contains both electrical and mechanical dynamics. The PMDC motor equivalent circuit is shown in Fig. (1). Equations that illustrate the relationship between rotor torque  $T_e$  and armature current  $i_a$  are illustrated as follows:

$$T_e = k_t i_a \quad (1)$$

When the motor rotates, it produced a back EMF voltage proportional to the speed of the rotor  $\omega_a$ .

$$V_c = k_v \omega_a \quad (2)$$

The dynamics of a PMDC motor can be represented using equations (3 and 4).

$$V_a = L_a \frac{di_a}{dt} + R_a i_a + V_c \quad (3)$$

$$T_e = J \frac{d\omega_a}{dt} + B \omega_a + T_l \quad (4)$$

where  $V_a$  represents the armature voltage,  $\omega_a$  the rotor angular speed, and  $T_l$  the load torque. All details of the motor parameters that are needed for estimation are shown in Table 1.

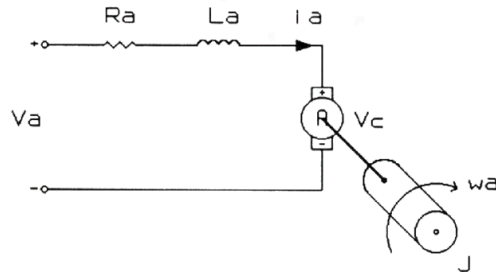


Fig. 1 Representative PMDC Motor Schematic

Table 1. PMDC motor parameters.

Parameter	Definition	Comment
$R_a$	Armature resistance	Determined how much power dissipating in the motor for a certain current level
$L_a$	Armature inductance	Determined how fast the motor is running
$k_v$	Motor voltage constant	Determined motor's steady state speed for a certain voltage
$k_t$	Motor torque constant	Determined the motor current required for a certain torque output
$B$	Damping (Viscous) Friction	A scale of dynamic friction
$J$	Motor inertia	A scale of the change in the object's resistance in relation to the rotation rate

### •The relationship between $k_v$ and $k_t$

The back emf constant of the motor was defined as:

$$k_v = \frac{V_c}{\omega_a} \quad (5)$$

Similarly, the torque constant of the motor was defined as:

$$k_t = \frac{T_e}{i_a} \quad (6)$$

The relationship between these constants can be obtained by assuming that the electromagnetic losses are equal to zero. This indicates that the electrical power dissipated in the armature by the back emf voltage is equal to the mechanical power. The equation of the mechanical power is given by:

$$P_m = T_e \omega_a \quad (7)$$

The electrical power defined as:

$$P_e = V_c i_a \quad (8)$$

So that:

$$P_m = P_e = T_e \omega_a = V_c i_a, \text{ then}$$

$$\frac{V_c}{\omega_a} = \frac{T_e}{i_a} \quad (9)$$

This implies  $k_v = k_t$ . It has become possible to define a new constant,  $k$  the motor constant as:

$$k = k_v = k_t \quad (10)$$

Using dynamic equations (1-4), enables us to calculate the practical motor parameters through measured the voltage, current, and speed of the motor. Table 2 shows the measured parameters values and units.

Table 2: the measured parameters of PMDC motor.

Parameter	Value
$R_a$	1.107 $\Omega$
$L_a$	0.120016 H
$k$	0.02497621 V.Sec/Rad
$B$	0.0007815 N.m.Sec
$J$	0.000121 Kg.m <sup>2</sup>

### 3. Simulink Design Optimization

The simulink design optimization is a numerical optimization that estimate and tune the

simulink model parameters and it provides two tasks, these are: 1) Calibrate model parameters with experimental data to increase the model accuracy 2) Calculate model parameter values that meet the design requirements. The simulink design optimization method provides a user interface called the parameter estimation tool, which is used to estimate the parameters of the simulink model.

This method uses the experimental data that collect from the physical model and later use this data to find the unknown model parameter values. If the simulink model parameters values is not match the actual parameters of the physical model, the output response of the simulink model will never match the actual measured response of the physical model. The steps that make up the parameters estimation process are:

- a- Collect experimental data from a physical model.
- b- Designated the parameters needed to estimate (including initial values, parameter limits, etc).
- c- Build a simulink model rewards the physical model and select a suitable estimated algorithm.
- d- Validate results against other experimental data sets and repeat the above steps if it is necessary.

#### 4. Parameter Estimation as an Optimization Problem

The parameter estimation tool is used to calculate the parameters of the PMDC motor model using an optimization problem, which is called the simulink design optimization. This optimization problem consists of:

- a- Design variables ( $x$ ): the parameters of the PMDC model that need to estimate.
- b- Bounds ( $x_{\min} \leq x \leq x_{\max}$ ): limits of the parameters that need to estimate. Based on the knowledge of the system, the bounds of the parameters can be adjusted. For the PMDC motor model, the parameter values must be varies between zero and infinity as: ( $0 < x < \infty$ ).
- c- Objective function ( $F(x)$ ): a function that calculates a measure of the difference between the actual and simulink response of the model. To obtain the parameter values of PMDC

motor, the software tunes these values to obtain a simulated speed response ( $y_{sim}$ ) that tracks the measured speed response from the physical model, which consider as a reference signal ( $y_{ref}$ ). The optimization algorithm minimizes the estimation error ( $e(t)$ ), which is defined as:

$$e(t) = y_{ref}(t) - y_{sim}(t) \quad (11)$$

The sum squared error (SSE or objective function) is used to calculate  $e(t)$  as:

$$SSE = F(x) = \sum_{t=0}^{t_n} [e(t)]^2 = \sum_{t=0}^{t_n} [y_{ref}(t) - y_{sim}(t)]^2 \quad (12)$$

where  $n$  is the number of samples

The simulink design optimization consists of several algorithms such as, the pattern search (PS), nonlinear least square (NLS), simplex search (SS) and gradient descent (GD). The best algorithms that used for estimate the motor parameters are the pattern search and nonlinear least square [17].

##### 4.1 Pattern Search

The pattern search (PS) method is used to search the minimum value of the objective function  $F(x)$  [18]. This method is used the latin hypercube search algorithm to generate a set of search direction iteratively and then it is compared the value of  $F(x)$  to search the minimum value. It depends on the value of  $F(x)$ , which is a sequence of  $x_0, \dots, x_n$ .

PS method involves two types of move: the exploratory move and the pattern move. The exploratory move is the initial move that looks for an improved direction to move. This is done by disturbing the current point ( $x_0$ ) with a small amount ( $p$ ) in each variable direction and observing the objective function value as follows:

$$F_+ = F(x_0 + p)$$

$$F = F(x_0) \quad (13)$$

$$F_- = F(x_0 - p)$$

The pattern search method is used to calculate the minimum objective and defines the pattern move as follows:

$$F_{\min} = \min(F_+, F, F_-) \quad (14)$$

If  $F_{\min} < F(x_0)$  then  $F(x_0)$  is replaced by  $F_{\min}$  and the initial point is updated. If  $F_{\min} > F(x_0)$  then the increment factor ( $p$ ) is decreased and the process is repeated. All this process continue repeated until the minimum value of the objective function is reached and the optimization parameters are obtained. Fig. (2) shows the flow chart for the PS algorithm.

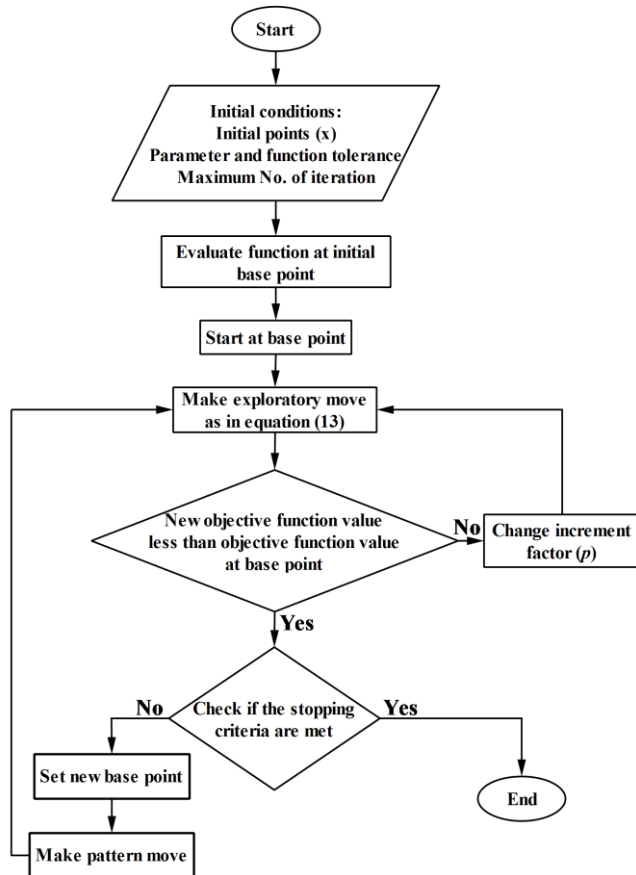


Fig. (2) The flow chart for the PS algorithm.

## 4.2 Nonlinear Least Squares

The purpose of the nonlinear least squares (NLS) method is to find a vector  $x$  that minimizes the expression in (15).

$$\min_x \sum_i f_i^2(x) = \min_x \|F(x)\|_2^2 \quad (15)$$

where  $f_i(x)$  is a scalar-valued nonlinear function of the residual of data point  $i = 1, \dots, n$  and  $x$  is a vector of variables. The trust-region-reflective least squares algorithm is used to solve equation (15) by approximate the function  $f$  to very simple function  $q$ , which rationally reflects the

behavior of the function  $f$  inside a reign around the point  $x$ , This region is called the neighborhood  $N$  (the trust region). A trial step  $s$  is calculated by minimizing  $q(s)$  over  $N$ .

$$\min_x \{q(s), s \in N\} \quad (16)$$

If  $f(x_0 + s) < f(x_0)$ , the initial point is updated  $x_0 + s$ . If  $f(x_0 + s) > f(x_0)$ ,  $x_0$  keeps unchanged, the neighborhood  $N$  is reduced in size and then the process is repeated. The key aspects of optimization the trust-region are how to approximate the function, how to choose and change the size of the neighborhood  $N$ , and how to better resolve the subproblem.

A standard trust-region method has been studied in [19], where the aim of this study was to approximate the function  $f$  by allow  $q$  be defined as the first two terms of Taylor expansion of  $f$ . The subproblem is illustrated as follows:

$$\min_x \left\{ \frac{1}{2} s^T H s + s^T g \text{ such that } \|Ds\| \leq \Delta \right\} \quad (17)$$

where  $H$  is the Hessian matrix (the symmetric matrix of second derivatives),  $g$  is the gradient of  $f$  at  $x$ ,  $D$  is scaling matrix with diagonal entries,  $\Delta$  a positive scalar, and  $\|\cdot\|$  is the 2-norm.

The best algorithms for solving equation (15) to calculate the trial step  $s$  have been studied in [20][21][22]. Fig. (3) shows the flow chart for the NLS algorithm.

The optimization stopping criterion is decided by several factors: if two successive parameter change by less than the selected parameter tolerance ( $|x_{i+1} - x_i| < \varepsilon_1$ ), or if the objective function values change by less than the selected function tolerance ( $|SSE_{i+1} - SSE_i| < \varepsilon_2$ ), or if the maximum number of iterations is reached. By adjusted these parameters, the optimization can continue searching for a more accurate solution.

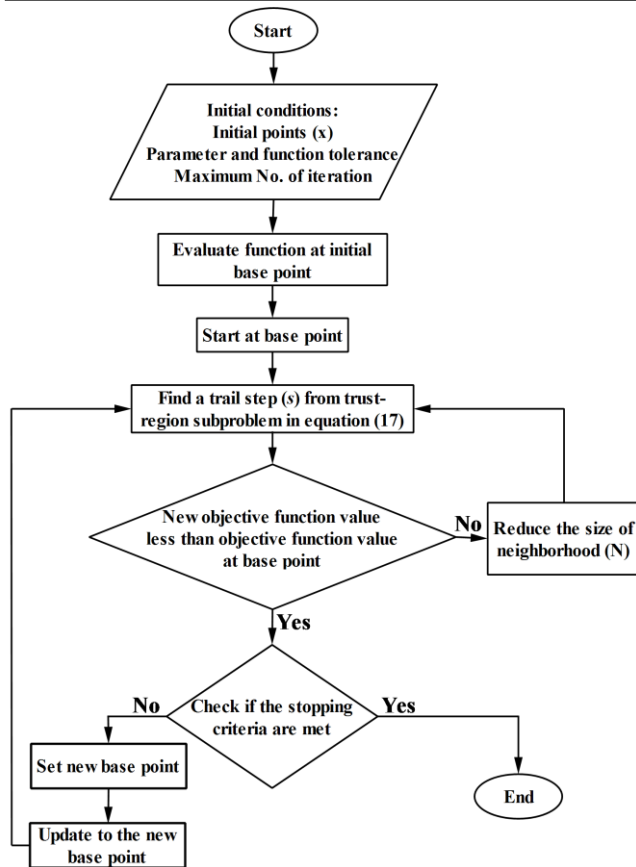


Fig. (3) The flow chart for the NLS algorithm.

## 5. Input-Output motor data estimation

The input and output data is measured by using a simple circuit base a microcontroller system as shown in Fig. (4).

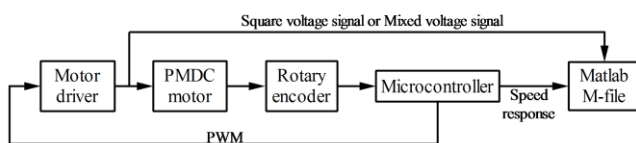


Fig. (4) Block diagram of the experimentally voltage and speed data.

The microcontroller with the help of the motor driver is used to excite the PMDC motor with two different signals (square voltage signal and mixed voltage signal) and resulting speed, which is observed for used later in the identification of the motor parameters. In order to reach a more accurate result in the identification, the PMDC motor should be excited with a signal involving different frequencies. The first experiment is done by applying the motor with a square voltage signal as in equation (18), to excite DC

components of the motor such as  $R_a$  and  $K$ . The second experiment is done by applying the motor with a mixed (fourier) voltage signal as in equation (19), which is a summation of sinusoids with different frequencies in order to obtain the other parameters such as  $L_a$ ,  $B$ , and  $J$ .

$$V_1(t) = 12 \times [u(t) - 2u(t-1) + 2u(t-2) - 2u(t-3)] \quad (18)$$

$$V_2(t) = 3.7 \times [\sin(2\pi 0.1t) + \sin(2\pi 0.2t) + \sin(2\pi 0.4t) + \sin(2\pi t)] \quad (19)$$

Figure (5) and Fig. (6) shows the square and fourier input signals.

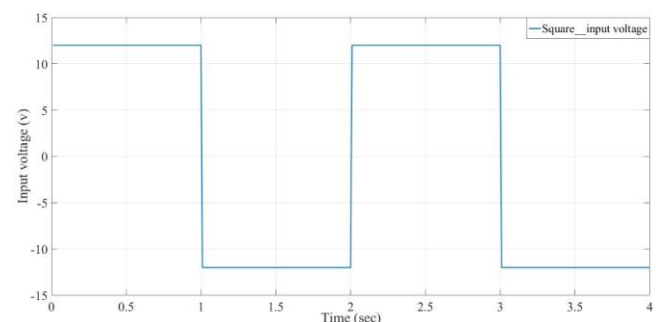


Fig. (5) Square input signal

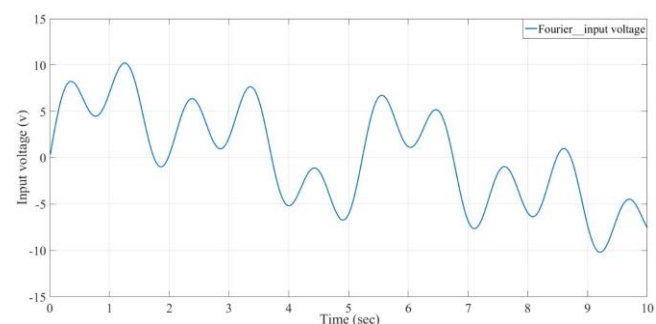
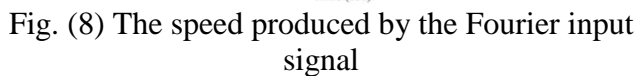
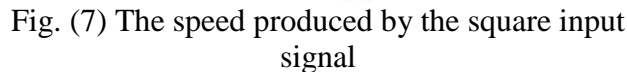


Fig. (6) Fourier input signal

The microcontroller is then used to measure the output speed response of the PMDC motor through the rotary encoder that placed on the shaft of the motor. Fig. (7) and Fig. (8) shows the speed response due to the square and fourier input signals.



## 6. The Identification Process

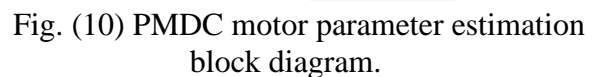
The diagram illustrates a speed feedback control system. The reference input is a constant voltage (1) and a feedback signal from the motor speed (Speed). These are combined at a summing junction (Add1). The output of Add1 is the current derivative  $d/dt(i)$ , which is integrated by Integrator1 to produce the current  $i$ . The current  $i$  is fed back to the summing junction via a gain  $K_e$  and also passes through a gain  $b$  to produce the speed output. The speed output is also fed back to the summing junction via a gain  $K_t$ . The current  $i$  is also fed back to the summing junction via a gain  $R$  (Resistance). The current  $i$  is also fed back to the summing junction via a gain  $L$  (Inductance) to produce the current derivative  $d/dt(i)$ . The current  $i$  is also fed back to the summing junction via a gain  $L$  (Inductance) to produce the current derivative  $d/dt(i)$ . The current  $i$  is also fed back to the summing junction via a gain  $L$  (Inductance) to produce the current derivative  $d/dt(i)$ .

All steps that must follow to extract system parameters are described as follows:

(b) Select the parameter to estimation. The measured parameters selected as initial values. Because none of the parameters value can be negative, adjust the boundary of the parameter value to begin from zero to infinity

(d) Add progress plots by click **Add Plot** on the **Parameter Estimation** tab, select the optimization algorithm, and run the estimation.

The above steps can be applied using the nonlinear least square algorithm once and again using the pattern search algorithm and the obtained results will be illustrated in the next section. The block diagram of the process is shown in Fig. (10).



## 7. Result and discussion

The results that obtain from using NLS and PS algorithms with ( $\varepsilon_1$  &  $\varepsilon_2 = 0.001$  and maximum number of iteration =100) are shown in Fig. (11) and Fig. (12).



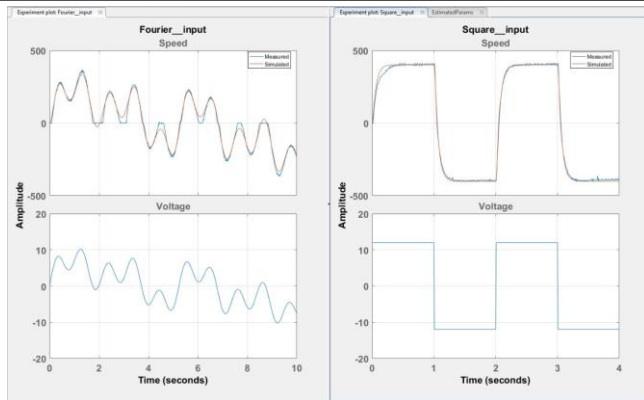


Fig. (11) Voltage and speed for the actual and estimated model using NLS algorithm.

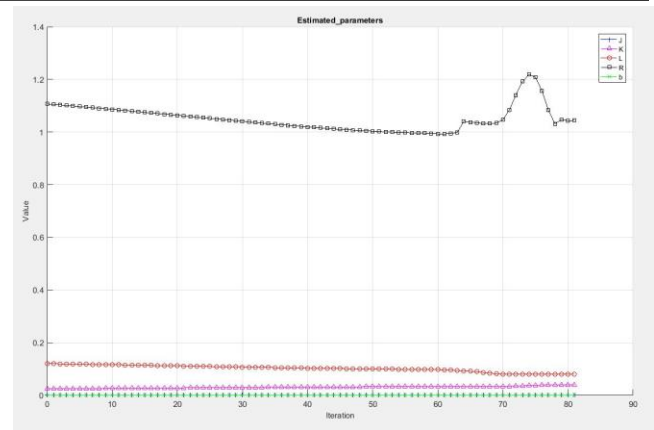


Fig. (13) Parameter trajectory using NLS algorithm

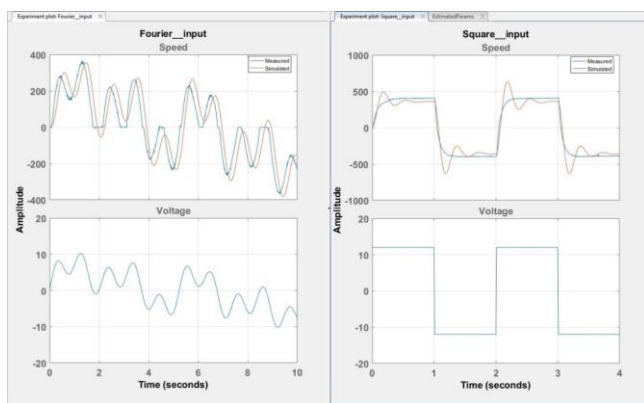


Fig. (12) Voltage and speed for the physical and simlink model using PS algorithm.

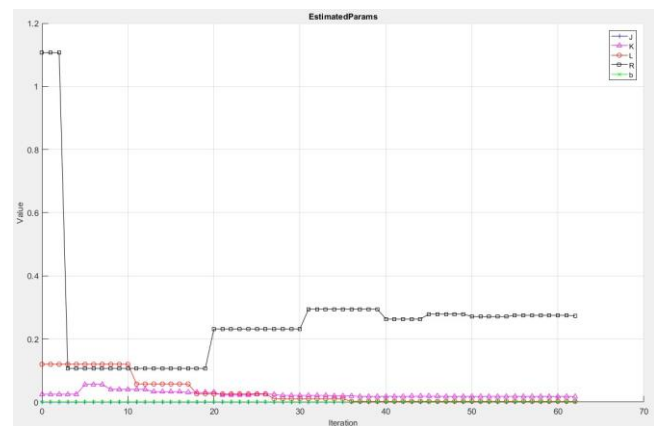


Fig. (14) Parameter trajectory using PS algorithm

The above two figures show the experimental data (blue curve) added with the simulation data (red curve), which comes from the simlink motor model with the estimated parameters. It is clear that whenever the parameter values improved, the output response of the simlink model getting closer to the actual measured speed curve. The plots in Fig. (13) and Fig. (14) shows the trajectories of the parameter for NLS and PS algorithms at each iteration of the estimation process. It is shown that the parameters stabilize to their optimum values as the estimation process converges to a solution. The result of using the NLS algorithm gives a closer result to the actual measured speed and stops at 81 iterations, while the result of using the PS algorithm keep away from the actual measured speed and stops at 62 iterations.

Table 3 contains the result for the two used algorithms. It became clear that the parameters values of NLS algorithm are approximately the same and closer to the expected values.

Table 3: The estimated parameters with NLS and PS algorithms

Parameters	NLS	PS
$R_a$	1.0591	0.27302
$L_a$	0.1	0.0028285
$k$	0.03728	0.018262
$B$	0.0011448	0.00078146
$J$	0.0009	0.00012102

## 8. Conclusion

In this work, the simlink design optimization method was used for estimating the unknown parameter values of the PMDC motor via using

two algorithms, which are pattern search and nonlinear least square. The dynamic equations of the PMDC motor were used to create a simulink model of the PMDC motor and to calculate an initial estimated for the motor parameters, which were used as an initial value in the simulink design optimization method. The parameters estimation was based on the experimental data collected from the physical PMDC motor by using two excitation signals (square and fourier signal) to measure the actual output speed response of the motor. It was observed that the results obtained by using the NLS algorithm were more accurate because the output response from the simulink modal was almost symmetric to the actual measured response of the motor, so the values of the motor parameters that resulted from using the NLS algorithm was considered as the accurate results.

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