Risk Parity VS MVO

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Abstract

This paper presents a comparative analysis between a Risk Parity investment strategy and Mean Variance Optimization, otherwise known as Markovitz with an Equally weighted portfolio as a benchmark. I investigate their performances over 20 years using Walk Forward Backtesting. The Risk Parity strategy, known for allocating assets based on risk contribution rather than market capitalization, aims to deliver enhanced risk returns and more consistent returns across various market conditions. This strategy employs a sophisticated model that adjusts asset weights based on the volatility of individual assets, enhancing the portfolio's ability to manage risk and reduce exposure to market behaviors. MVO on the other hand constructs an optimal portfolio by maximizing the expected return for a given level of risk, based on historical returns, volatility, and covariance of the portfolio's assets. The goal of the model is to get the best risk-adjusted returns otherwise known as Sharpe Ratio. I constructed two basic models to test my hypothesis, with risk parity focusing on risk expectations and balancing between all assets and MVO focusing on the highest risk-adjusted returns. Our analysis uses various metrics including cumulative returns, Sharpe Ratio, maximum drawdown, beta, and alpha along with annualized returns and volatility, providing a comprehensive view of the risk-adjusted performance of each strategy.

Risk Parity VS MVO

The foundation of the risk parity strategy is centered around assets being allocated based on their risk contributions, rather than their market value. It aims to allocate risk evenly across various assets, enhancing the risk-adjusted returns of the portfolio. Under MVO, diversification is achieved through spreading the amount of capital allocated across asset classes. With a risk parity strategy, diversification is achieved by equalizing the amount of total portfolio risk attributed to each asset class. While both require the prediction of future quantities, risk parity's estimation of a future covariance matrix is easier to deal with than MVO's estimation of both expected returns and a future covariance matrix. Additionally, since volatilities are more stable than prices it makes more sense to use historical data to predict volatility over returns. Expected returns and risk may also be subject to behavioral biases, like overestimation of returns by expecting strong momentum in asset prices and underestimating risk through personal experience and familiarity with an asset class. Due to risk parity's reliance on the covariance matrix as the sole input, the strategy is more mechanical and there is no clear evidence suggesting that the quality of covariance estimates impacts returns in a biased manner. Lastly, it has been suggested that risk parity is more efficient compared to mean-variance when noise is high and distributions exhibit "fatter tails", and because of this, theoretically it makes sense that risk parity outperforms MVO. Therefore, risk parity is a favored strategy among investors seeking to reduce exposure to market volatility and downturns due to its ability to provide consistent returns across different market conditions compared to traditional investment strategies that rely heavily on a single asset class.

Hypothesis

Risk Parity's strategy of equalizing risk contributions among assets tends to stabilize portfolio performance during volatile and unpredictable market periods compared to MVO, where its reliance on historical returns and covariances has been argued to not capture market movements in times of economic stress or changes and is therefore not as robust

Implementation of Risk Parity Strategy

Pre-processing and beginning Calculations

My strategy begins with data fetching, where I retrieve historical financial data for a set of assets using Bloomberg. Focusing on the cumulative total returns gross dividends over 20 years, I incorporate replacing infinite values, dropping NA's, and clipping extreme values to prevent any skewing of results. Following my data preparation, my strategy continued to calculate returns as percentage changes from one day's closing prices to the next and drops empty rows. Through these basic steps, I was able to prepare my data for all future models in my strategy. We also solidified a rebalancing period of quarterly to make sure our data stays accurate.

Volatility and Correlation

In terms of volatility, working with accurate measurements of the risk associated with each asset is crucial for the risk parity approach. Volatility, representing the degree of variation in asset prices over time, is a key indicator of risk. For my volatility calculation, I employed the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model to estimate the conditional volatility of each asset. The GARCH model allows us to analyze the changing levels of volatility over time within financial markets and estimate the conditional volatility of each asset, which is needed for understanding the inherent risk of investment choices over time. It is useful for capturing volatility clustering, where high-volatility events follow high-volatility events, and low-volatility events follow low-volatility events. By incorporating past volatilities and returns, the GARCH model provides a responsive measure of risk and can adapt to changes in the market. The output from this modeling process is a time series of estimated volatilities for each asset, which are then used to determine how much weight each asset should have in the portfolio to achieve risk parity.

In terms of correlation, it is important to our model as it helps us understand the relationships between our assets and, in turn, structure a portfolio where risk contributions from different assets offset each other, thereby minimizing overall portfolio risk. I employ a rolling window approach where correlations between the returns of the assets are calculated regularly, ensuring our estimates are updated frequently to reflect the latest market conditions. This helps us maintain a balanced, accurate risk distribution. Correlation, combined with our volatility estimates, helps in creating a portfolio where the risk is distributed evenly across all assets, therefore ensuring that risk is balanced through all assets.

By Combining the estimated GARCH volatilities with the historical correlation data, the covariance matrix is calculated. This matrix serves as the foundation for portfolio optimization and integrates both the volatility and correlation to determine the weights of each of the assets.

Risk Parity Weight Calculation

My approach to calculating weights leverages the volatility estimates derived from the GARCH models and the correlation matrix to achieve a balanced portfolio. As mentioned above, the volatility is calculated from the GARCH models, which provide conditional volatility for each asset. In a risk parity strategy, an asset with higher volatility should have a lower weight in the portfolio because it represents a higher risk. To achieve this, the inverse of each asset's volatility is used. This inversion ensures that assets with higher volatility are assigned lower weights, aligning risk parity principles. In addition to volatility, a rolling correlation matrix is employed in order to reflect the relationships between assets over time. The correlation matrix is then used alongside the GARCH volatilities to create the covariance matrix. This matrix incorporates both the volatilities and correlations of assets. Then we can calculate initial weights and later normalize weights to make sure the percentages add up to 1.

$\mathbf{W} = \mathbf{W}\mathrm{RP}$

Where:

$\mathrm{WRP} = \mathrm{IC} \cdot \mathrm{IV} / \sum(\mathbf{IC} \cdot \mathbf{IV})$

- **IC** is the inverse of the correlation matrix & **IV** is the vector of inverse volatilities

MVO Implementation

The objective function in MVO is typically designed to either maximize the portfolio's Sharpe ratio or minimize its variance for a given level of expected return. This involves solving an optimization problem where the expected return and covariance matrix are used to find the optimal asset weights. The objective function drives the optimization process, guiding the model to achieve the desired risk-return trade-off.

Constraints

In our model, we utilized two constraints. One was a sum of weights constraint which ensured that the total weights sum up to 1. Second, we added weight bounds so one asset could not have 50% of the portfolio allocation and each asset allocation had to be at least equal to 1%. This helps us ensure diversification in our portfolio and is up to investors' risk and diversification preference.

Core Components

Expected returns are a core component of MVO and represent each asset's expected average return. We calculated this using our return data, and the expected return of each asset creates a base for calculating the expected return of our portfolio.

In terms of volatility, to make our models more comparable we similarly employed a GARCH model to risk parity. Lastly, in MVO we use covariance to also capture the relationships between all pairs of assets. By employing both here we were able to get a deeper understanding of the portfolio's risk structure, this time with the goal for highest risk-adjusted returns.

MVO Weight Calculation

The primary factors I use to optimize my portfolio as mentioned above include expected returns, the covariance matrix, and optimization constraints. My first step is to calculate all of my variables mentioned above including GARCH. Once I have set my constraints and constructed my covariance matrix I can use quadratic optimization to find the weights that would either maximize our Sharpe ratio or minimize our variance. By finding the sweet spot between the max

Sharpe and minimum variance, I can get optimal portfolio weights. Similar to risk parity I normalize my weights and adjust them proportionally.

Backtesting

Backtesting is a necessity when understanding the robustness of a strategy over time. It involves creating a historical simulation of the strategy to understand its potential performance under past market conditions. I backtested my Risk Parity strategy using various performance metrics against MVO and an equally weighted portfolio to prove the validity and robustness of my strategy and hypothesis.

I employed a walk-forward backtest which involves training a model on a rolling window of historical data, otherwise known as the optimal lookback. Then applying the weights derived from various models on out-of-sample periods, rebalancing the portfolio quarterly. This strategy is great to see how our different portfolios would have performed in real-time accounting for changing markets. By repeatedly retraining and testing the model, walk-forward backtesting helps identify strategies that perform consistently across different market conditions, indicating robustness. It also tests the strategy's ability to adapt and is helpful in overfitting. Below are the performance metrics I used to compare my strategies.

- *Cumulative Returns*: The total returns generated by the portfolios over the testing period

- *Sharpe Ratio*: This ratio helps assess the risk-adjusted returns, factoring in the volatility of the portfolio returns relative to a risk-free rate (0)
- *Maximum Drawdown*: This metric measures the largest single drop from peak to trough in the portfolio's value, indicating the potential risk of losses
- *Beta*: Measures the portfolio's sensitivity to market movements
- *Alpha:* how much value a strategy adds or detracts from a portfolio compared to the benchmark index after adjusting for risk

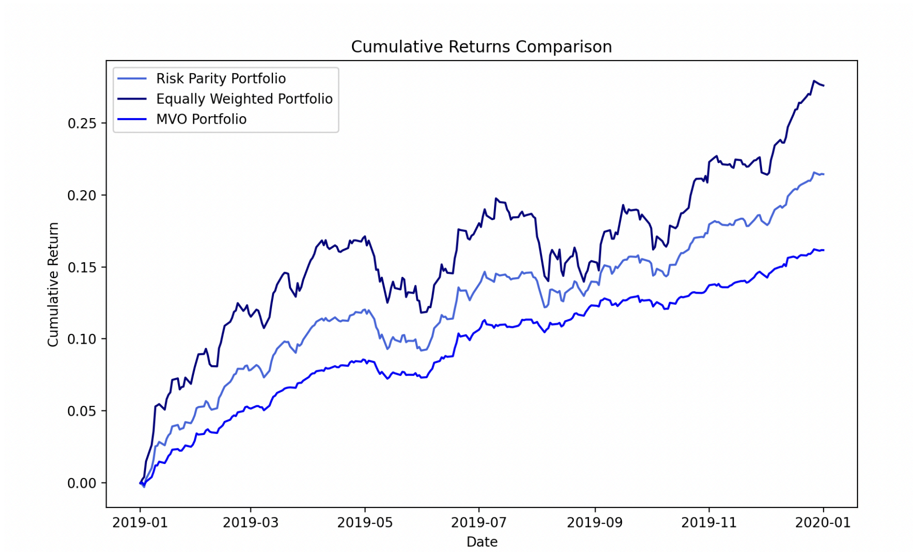
Results

Lookback

1 year Lookback

RP Sharpe: 4.17

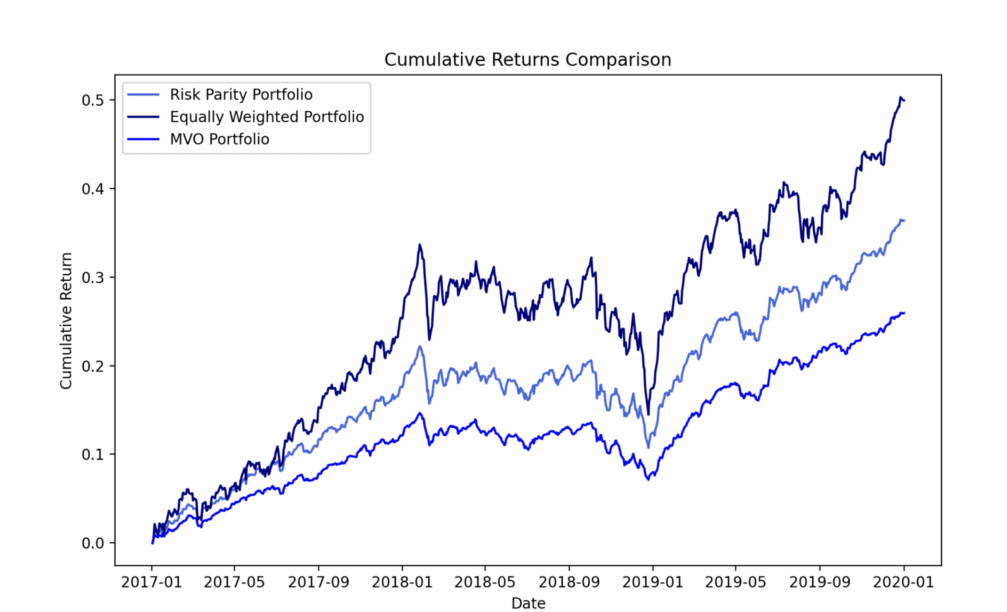
MVO Sharpe: 5.65



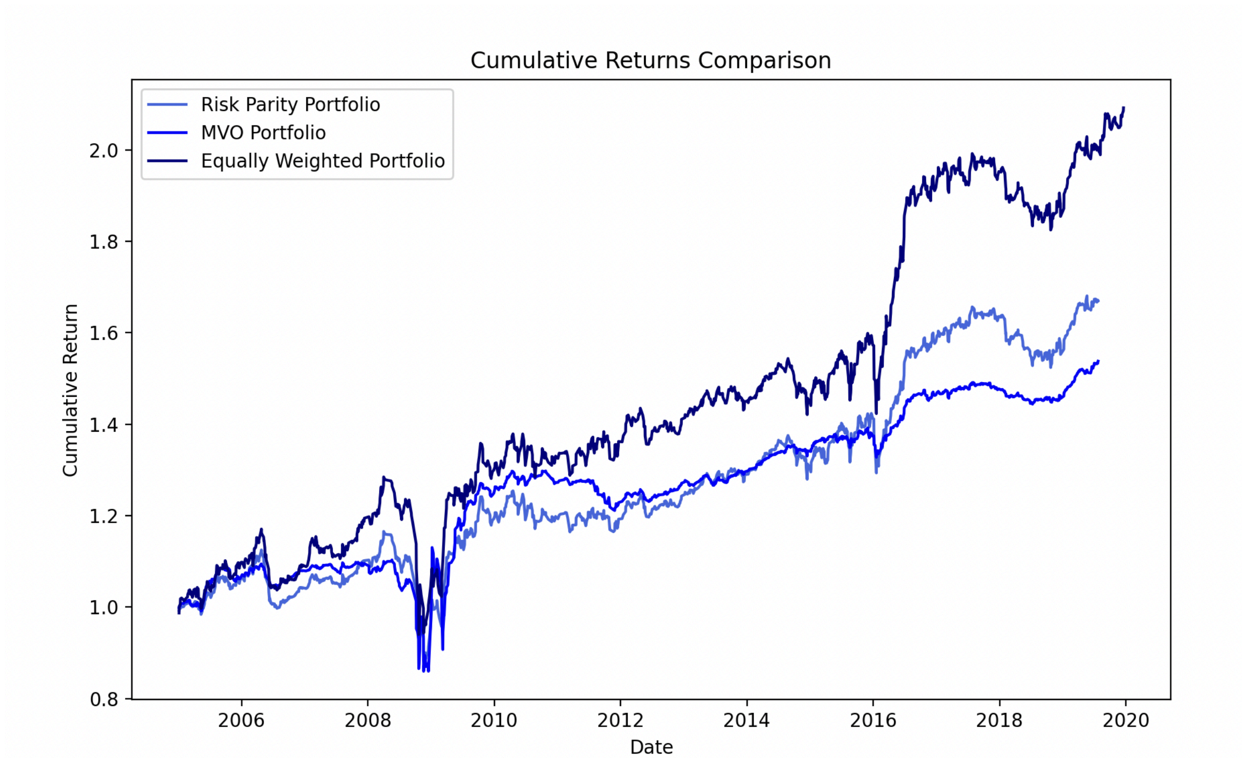
3 Year Lookback

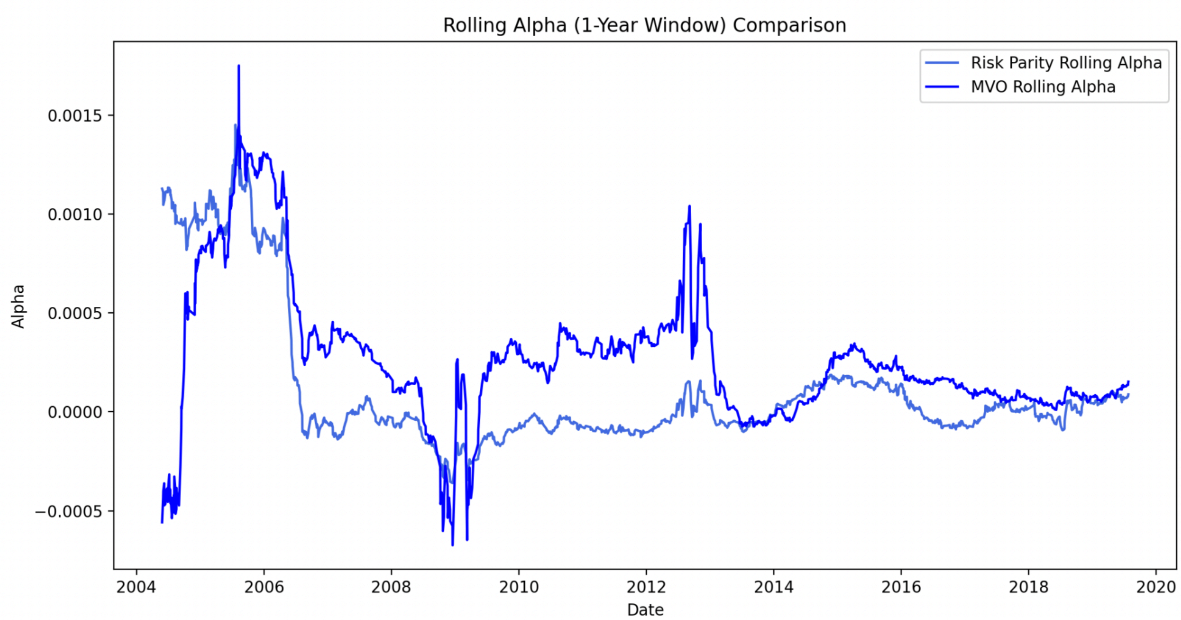
RP Sharpe: 2.10

MVO Sharpe: 2.53



WF Backtest (1y lookback)





	Sharpe Ratio	Ann. Ret	Ann. Vol	Beta	Max Drawdown
Risk Parity	1.17	16.64%	14.27%	0.43	-25.08%
MVO	0.51	8.11%	16.05%	0.24	-37.83%
EQ weight	0.89	22.62%	25.35%	1.00	-41.25%

Stress Testing

2007-2010 march (2006 in code for training)

	Sharpe Ratio	Ann. Ret	Ann. Vol	Beta	Max Drawdown
Risk Parity	1.24	33.74%	27.25%	0.43	-22.31%
MVO	0.15	4.26%	29.19%	0.80	-20.77%
EQ weight	1.21	27.78%	22.88%	1.00	-26.84%

Discussion

In my strategies, the assets I used were a diversified group of equities and bonds from major parts of the world from 2000-2020. The list of assets includes :

SPX Index: S&P 500 Index

LBUSTRUU Index: Bloomberg Barclays U.S. Aggregate Bond Index

SPGSCI Index: S&P GSCI Index

RMSG Index: Russell Midcap Growth Index

LP01TREU Index: Bloomberg Barclays U.S. Treasury: 1-3 Year Index

TWSE Index: Taiwan Stock Exchange Weighted Index

IBOV Index: Bovespa Index (Brazil)

SASEIDX Index: Tadawul All Share Index (Saudi Arabia)

NKY Index: Nikkei 225 Index (Japan)\MXDK Index:

MSCI Denmark Index

SMI Index: Swiss Market Index

VNINDEX Index: Vietnam Ho Chi Minh Stock Index

In terms of 1-year look back, I tested 3 years, 2 years, 5 years, and 10 years. The results for 5, 2, and 10 years were not near the results for the other two and therefore I quickly got rid of them. As we can see in the Sharpe Ratios above, 1 year did much better than 3 years but I was worried that 1 year could lead to overfitting, and may not be long enough to be trained. Therefore I tested both in the backtesting process. In 3 years backtesting Risk Parity Sharpe ratio decreased by .07 and MVO Sharpe was 1. Since WF is good at combatting overfitting, Risk Partity Sharpe went down, and 1 year did do much better, I stuck with 1-year lookback as we can see in the table above. In terms of Sharpe ratio, my portfolio did much better than both EQ weighted and MVO whose Sharpe ratio was .38 less than equally weighted, even though it was higher in lookback showing it is not a robust strategy. We can see that MVO's volatility is double its return compared to risk parity whose return is higher than volatility. In terms of stability, our claims were proven true as the Max Drawdown is the lowest for Risk Parity. The second graph shows the alpha of each portfolio and as we can see while MVO does go higher than Risk parity it is highly volatile and also goes much lower. Risk Parity on the other hand provides consistent, stable results which is shown in its alpha as well. Lastly, I tested my model through different periods like 2007-2010 which involved the financial crash of 2008 to see how strategies would hold up. As we can see Risk Parity did the best with EQ doing nearly as good and MVO completely failing under economic stress.

Final Thoughts

Risk Parity's strategy of equalizing risk contributions among assets tends to stabilize portfolio performance during volatile and unpredictable market periods compared to MVO, where its reliance on historical returns and covariances has been argued to not capture market movements in times of economic stress or changes and is therefore not as robust

Returning to my hypothesis of Risk Parity tending to stable portfolio performance during volatile and unpredictable market periods, as we can see through the results, was proven true. With Risk Parity having the smallest drawdown and through the cumulative returns we can see the stability Risk Parity presents. In terms of robustness, as we look at the lookback results MVO has a much higher Sharpe than RP but during backtesting failed. MVO's Sharpe ratio was below EQ weighted while Risk Parity's was much above, almost double of MVO. This proves the second part of our hypothesis that risk parity can capture market movements and therefore is more robust. Risk Parity was the only model whose returns were greater than volatility and was still able to get the most stability as seen through drawdown proving its ability to capture market movements and its robustness through a large period.

Sources

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