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Portfolio Risk Management Enhancement using Var and CVar

The expected shortfall, or conditional value at risk, or CVaR, is a risk assessment metric that expresses how much tail risk an investment portfolio contains. A weighted average of the "extreme" losses in the tail of the possible return distribution, above the value at risk (VaR) cutoff point, is used to calculate CVaR. In order to effectively manage risk, conditional value at risk is employed in portfolio optimization. In general, an investment's value at risk may be adequate for risk management in a portfolio that includes it if it has demonstrated stability over time. VaR, however, is agnostic to anything that falls outside of its own threshold, so the less stable the investment, the more likely it is that it will not provide a complete picture of the risks. The VaR model is a statistical technique used to measure the amount of financial risk within a firm or an investment portfolio over a specified period of time. Conditional Value at Risk, or CVaR, aims to address the shortcomings of the VaR model. Whereas CVaR is the expected loss in the event that the worst-case threshold is ever crossed, VaR represents the worst-case loss related to a probability and a time horizon. Stated differently, CVaR measures the predicted losses that happen above the VaR breakpoint. VaR is rarely much higher than that of safer investments, such as investment-grade bonds or large-cap U.S. stocks. Derivatives, emerging market stocks, small-cap U.S. equities, and other more volatile asset classes can show CVaRs that are significantly higher than VaRs. Investors are ideally searching for small CVaRs.

Nonetheless, the most lucrative investments frequently have high CVaRs. VaR is frequently used extensively in financially engineered investments because it is not affected by outlier data in models. But there have been instances where, had CVaR been preferred, engineered goods or models might have been built more skillfully and used more carefully. There are numerous historical examples, such as Long-Term Capital Management, which relied on VaR to assess its risk profile but nevertheless failed due to improper accounting for losses that exceeded the VaR model's prediction. In this instance, CVaR would have directed the hedge fund's attention away from the VaR cutoff and toward the actual risk exposure. VaR vs CVaR for effective risk management is a topic of constant discussion in financial modeling.

Portfolio optimization is a very important aspect of financial management, targeted to put together an optimal investment portfolio all while minimizing risks and maximizing returns. Usually, traditional portfolio optimization often involves relying on a variety of metrics that include banking on variance and expected return. In this research report, I plan to dive deeper into a more healthy and strong approach to portfolio optimization, involving minimizing Conditional Value-at-Risk (CVAR). The reference “Portfolio optimization by minimizing conditional Value-at-risk via nondifferentiable optimization” by Churlzu Lim, Hanif D. Sherali, and Stan Uryasev gives us more insight into a complete methodology for addressing this complex problem.

The main goal of the reference is to minimize CVaR to find an optimal portfolio composition while under price uncertainty. Using the vector x, the CVaR is able to be manipulated and ultimately minimize F(x). F(x) is what quantifies the risk in the portfolio considering the unpredictable price distribution. In the reference, they frame it as:

- Objective: Minimize the CVaR function $F(x)$ of portfolio x, given a probabilistic distribution of prices.
- Constraints:
    - The sum of portfolio weights equals 1
    - Non-negative weights: $x \geq 0$

Establishing these objectives and constraints ensures the portfolio's budget is invested across different financial instruments without borrowing.

The reference goes into detail about portfolio optimization using Value-at-Risk vs Conditional Value-at-Risk with different risk criteria being employed. Value-at-risk gained traction as an effective risk measure representing the percentile of loss distribution at different specified confidence levels. Although, the authors say that the sub-additivity and nonconvexity pose challenges for optimization. Conditional Value-at-Risk on the other hand has shown to be the best option giving us sub-additivity, convexity, and coherence as risk measures. The linear programs Although CVaR reduction issues can be written as linear programs (LPs) for linear loss functions, there is a computational penalty associated with the large dimensionality of these LPs as the number of scenarios increases.

The Conditional Value-at-Risk is shown to be nondifferentiable. To characterize the function's behavior at nondifferentiable points subgradients are used. A key finding is that if no scenario results in a loss equal to Value-at-Risk, the subdifferential at a point only contains one subgradient which is crucial for developing a perturbed differentiable solution for the algorithm.

The reference introduces a two-phase method to solve the Conditional Value-at-Risk minimization problem efficiently.

Phase 1: Perturbation Technique (PT): This is where the nondifferentiable of the Conditional Value-at-Risk function is addressed by perturbing nondifferentiable iterates to nearby differentiable points. This allows for techniques like the steepest descent method, quasi-Newton methods, and other similar differentiable optimization techniques.

- Objective: Use conventional optimization techniques that work around nondifferentiable points.
- Process:
  - Initialization: Starting with an initial vector in x then check if the Conditional Value-at-Risk function is differentiable at this point. If it turns out not to be then a perturbation technique is used to get to the next differentiable point.
  - Differentiable Techniques: Gradient methods are employed to iteratively improve the solution.
  - Line Search: A quadratic-fit line search is employed to determine the best step size for each step. The new point must stay the same within the defined portfolio constraints.
- Key Idea: This phase aims to find the most optimal solution quickly which sets up Phase 2 which is more complicated.

Phase 2: Modified Variable Target Value Method: During this phase, a modified variable target value method is employed. This convergent NDO algorithm helps refine solutions obtained during Phase I. In this phase, the modified variable target value method uses a subgradient search direction and a step size determined by a target value. The algorithm then approaches an optimal solution.

- Objective: Use a method designed for nondifferentiable problems to refine the solution gathered during Phase 1.
- Process:
    - Variable Target Value Method: This method takes the target value which helps in converging towards an optimal solution by manipulating the step size.
    - Subgradients: Because they are not clear at nondifferentiable points, subgradients are used.
    - Sequential Projections: Certain points are projected onto cutting planes that gradually refine areas toward an optimal solution using the algorithm.
- Key Idea: Subgradients combined with an efficient step-size strategy, Phase 2 helps guide you to the optimal solution regardless of a nondifferentiable function. This phase takes the most optimal solution from Phase 1 and further hones in on the true optimum.

(Optional) Phase 3: This optional phase is introduced to further optimize the near-optimal solutions from Phase 1 and Phase 2. The simplex algorithm is applied to the starting point from the end of Phase 2.

The effectiveness of the Phase 1, Phase 2, and Phase 3 are evaluated through numerical study in the reference as well.

Table 3 Computational results for real-world examples

Problem (Optimal CVaR)		STOCK-PR (0.0481902)	STOCK-PO (0.0481902)	BOND-PR (0.00489605)	BOND-PO (0.00467892)
CPU	TPM-SD	0.77	0.77	10.37	8.1
	Phase I–II-SD	0.38	0.38	4.22	2.07
	TPM-SU	0.68	0.72	10.28	7.84
	Phase I–II-SU	0.3	0.33	3.71	2.07
	TPM-MBFGS	0.73	0.59	10.29	7.5
	Phase I–II-MBFGS	0.35	0.29	3.98	2.12
	PBA-SIM	3.38	3.24	8.86	7.93
	PBA	1.25	1.22	0.04	0.04
	SIMPLEX	3	3.66	57.42	57.52
	BARRIER	1.29	1.3	12.03	16.37
ITR	TPM-SD	35	34	5	5
	TPM-SU	35	34	5	5
	TPM-MBFGS	35	8	5	7
	PBA-SIM	779	748	127	128
	SIMPLEX	1854	2000	7541	7510

Table 3 shows the results of real-world examples. The performance of the Three-Phase Method with different gradient strategies (SD, SU, MBFGS) is compared. The Proximal Bundle Algorithm, the Simplex (PBA-SIM), the Barrier method, and the standalone Simplex algorithm are all above. CPU and the number of simplex interactions are the performance metrics.

Table 4 Computational results for examples with target return constraint

Problem (Optimal CVaR)		STOCK-PR (0.0497455)	STOCK-PO (0.0499127)	BOND-PR (0.0079878)	BOND-PO (0.00695251)
CPU	TPM-SD	0.96	1.25	22.14	23.56
	TPM-SU	0.94	1.21	23.1	10.01
	TPM-MBFGS	0.94	1.36	21.27	23.68
	PBA-SIM	2.63	3.45	20.91	24.92
	SIMPLEX	6.28	5.02	72.36	62.09
	BARRIER	1.34	3.22	13.47	13.95
ITR	TPM-SD	154	259	955	1583
	TPM-SU	154	259	887	150
	TPM-MBFGS	154	270	955	1599
	PBA-SIM	551	789	1364	1861
	SIMPLEX	2172	2124	8125	7537

Table 4 adds constraints on the target return along with the same computational results for the real-world examples discussed above. This table shows that the Three Phase Method-SU method outperforms other methods in terms of CPU time.

These computational results show the advantages of the proposed Two-Phase and Three-Phase Methods. During random tests, the Two-Phase Method produced near-optimal results. These near-optimal results included minimal optimality gaps which shows the Two-Phase method's efficiency. The Three-Phase Method further refined those already near-optimal results.

In Conclusion, this reference shows us an approach to portfolio optimization that is both comprehensive and minimizes the Conditional Value-at-Risk efficiently. Respectively, the two-phase and three-phase methods successfully address the negatives of the Conditional Value-at-Risk function posed by the nondifferentiable aspects of it. The numerical studies in Table 3 and Table 4 show the efficiency and accuracy levels of these methods for large-scale problems. This approach marked a huge advancement in the field of optimization. Financial Managers can achieve a complete risk assessment to better help their investment decisions.

During my research, I also dug into another paper on Value-at-Risk and Conditional Value-at-Risk by Sergey Sarykalin, Gaia Serraino, and Stan Uryasev titled, "Value-at-Risk vs. Conditional Value-at-Risk in Risk Management and Optimization". This resource's purpose is to explain the strengths and weaknesses of Value-at-Risk and Conditional Value-at-Risk. The paper compares the two in terms of practical applications in risk management, ease of optimization, and their mathematical properties. This paper defines Value-at-Risk by saying that "Value-at-Risk estimates the maximum loss a portfolio might experience over a given period with a certain confidence level. For example, a 95% VaR of $1 million means there's a 95% chance the portfolio won't lose more than $1 million in a given timeframe" and explains Conditional Value-at-Risk by saying, "Conditional Value-at-Risk, also known as Expected Shortfall, measures the average loss exceeding the VaR threshold. For instance, if the 95% VaR is $1 million, Conditional Value-at-Risk would tell you the average loss if the portfolio experiences losses greater than $1 million". Value-at-Risk is mostly used because of its simplicity in it being a single number that represents risk while Conditional Value-at-Risk addresses some of the limits Value-at-Risk has. Deviation is also a concept that this paper goes deeper into. Deviation quantifies the dispersion of a distribution. Conditional Value-at-Risk deviation is said to be a good alternative to standard deviation and the author further explains the generalized one-fund theorem along with the Capital Asset Pricing Model. This is how they showed how Conditional Value-at-Risk deviation can be integrated into those financial frameworks.

In terms of practical applications, Value-at-Risk is more popular for being simple and a lot more easier to understand. Although it may underestimate risk at times it's useful for reporting and regulatory compliance. Conditional Value-at-Risk is preferred where acknowledging tail-end risk is crucial. Bigger scale problems involving planning and risk management are where Conditional Value-at-Risk is very helpful. In terms of optimization, due to its nonconvexity, Value-at-Risk optimization is rather difficult. Conditional Value-at-Risk on the other hand can be optimized using linear programming methods like the first resource/paper explained. For their mathematical properties, Value-at-Risk is nonconvex and discontinuous while Conditional Value-at-Risk is continuous and captures the risk of extreme losses. This is why Conditional Value-at-Risk provides a more accurate risk assessment.

Overall this paper comes to the conclusion that depending on your risk management, both Value-at-Risk and Conditional Value-at-Risk have their own respective places. Value at risk is simple which makes it more attractive for more basic risk assessments. Conditional Value-at-Risk on the other hand is much more comprehensive. Conditional Value-at-Risk is the better option for internal risk management and optimization when dealing with big portfolios that have the possibility of huge losses. This paper highlights the strengths and weaknesses of both.

In the comparison of the two papers, both go over the positives and negatives of Value-at-Risk and Conditional Value-at-Risk but approach the comparison from different perspectives. The first paper goes over the specific applications of Conditional Value-at-Risk in terms of portfolio optimization while the second paper gives a more broad overview of Value-at-Risk and Conditional Value-at-Risk going into detail on their various financial contexts. The second paper emphasizes the importance of choosing the correct one for your situation. The first paper targets those who practice and are in the quantitative finance field, specifically those in portfolio optimization and risk management. The second paper has a much bigger audience that includes everything between students along financial professionals. The second paper serves more as an educational report detailing concepts rather than a specific methodology found in the first paper.

Works Cited

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