Aktorik und Sensorik mit intelligenten Materialsystemen 4

ASIM 4 Final Exam Assignment: SMA Actuated Wing

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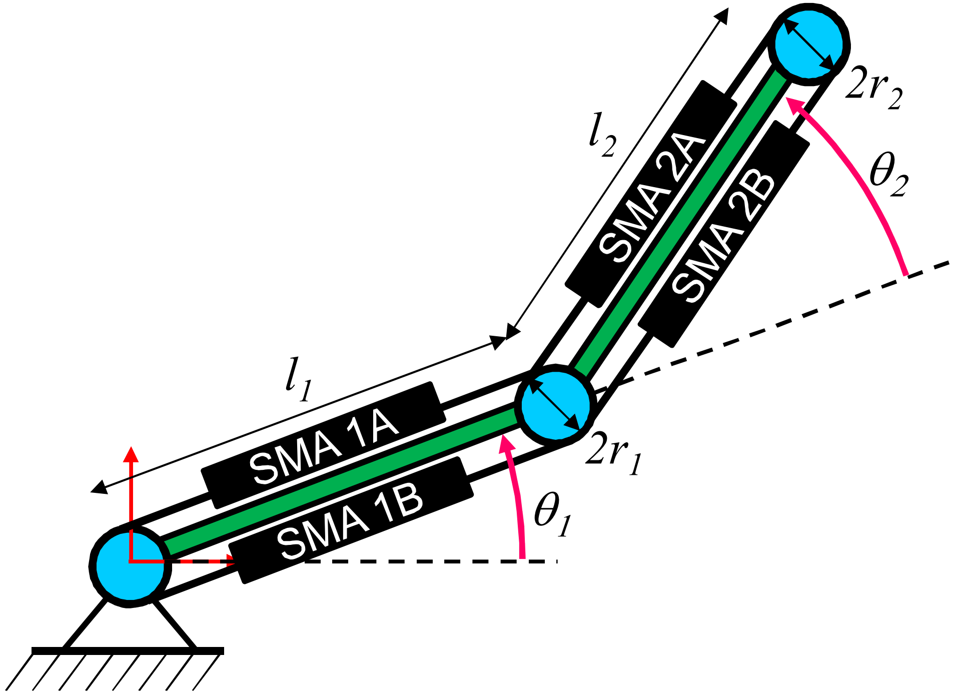
- System layout: two rigid rods of lengths l_1 and l_2 are connected through two rotational joints of radii r_1 and r_2, and actuate by bundles of SMA wire actuators
- By controlling the Joule heating of the SMA wires, rotation angles of both joints θ_1 and θ_2 can be controlled



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- Project guidelines:
  - Study the effects of control inputs $J_{SMA,1A}$, $J_{SMA,1B}$, $J_{SMA,2A}$, and $J_{SMA,2B}$ on the actuator performance, in terms of θ_1 and θ_2, with all Joule heating signals limited between 0 and 2 W;
  - Define control inputs u_1 and u_2, such that
    if $u_1 \geq 0$ then $J_{SMA,1A} = |u_1|$, $J_{SMA,1B} = 0$
    if $u_1 < 0$ then $J_{SMA,1A} = 0$, $J_{SMA,1B} = |u_1|$
    if $u_2 \geq 0$ then $J_{SMA,2A} = |u_2|$, $J_{SMA,2B} = 0$
    if $u_2 < 0$ then $J_{SMA,2A} = 0$, $J_{SMA,2B} = |u_2|$
  - Find a linear approximation of the responses between u_2 and θ_2 (for $u_1 = 0$),
  - Design and compare linear controllers for position regulation of θ_2
  - Design and compare linear controllers for trajectry tracking of θ_2
  - Compare linear and nonlinear controllers for θ_2
  - Repeat the above identification/control steps for the $u_1 - \theta_1$ response (with u_2 = 0)
  - Test the performance of both controllers when they work together to control θ_1 and θ_2 at the same time
  - Compare continuous-time and discrete time controllers obtained with different sampling times and discretization methods, as well as the effects of anti-windup
  - Test the control system response when disturbanced τ_d are affecting the actuators
  - Test the control system response when environmental temperature T_E is changing
  - Compare sensor-based and sensorless control based on resistance signals

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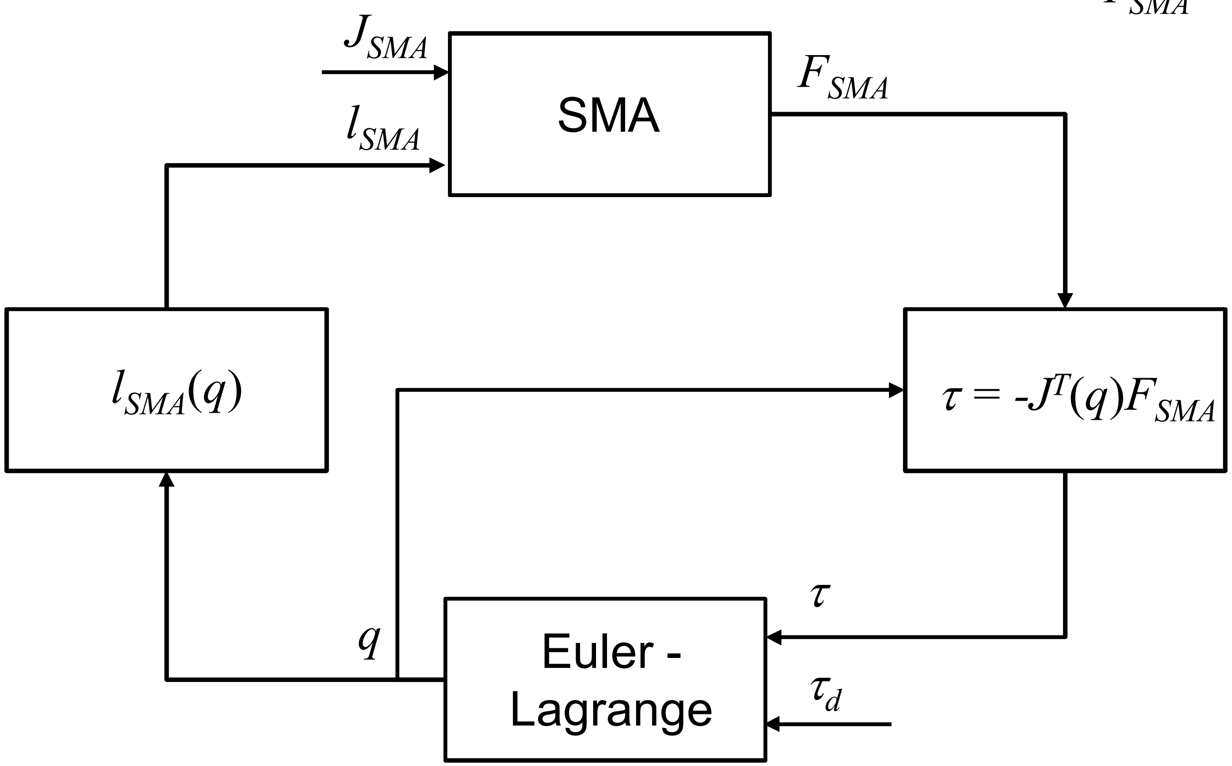
- Structure model



$$F_{SMA} = \begin{bmatrix} F_{SMA,1A} \\ F_{SMA,1B} \\ F_{SMA,2A} \\ F_{SMA,2B} \end{bmatrix} \quad l_{SMA} = \begin{bmatrix} l_{SMA,1A} \\ l_{SMA,1B} \\ l_{SMA,2A} \\ l_{SMA,2B} \end{bmatrix}$$

$$J_{SMA} = \begin{bmatrix} J_{SMA,1A} \\ J_{SMA,1B} \\ J_{SMA,2A} \\ J_{SMA,2B} \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_{\theta_1} \\ \tau_{\theta_2} \end{bmatrix} \quad \tau_d = \begin{bmatrix} \tau_{d,\theta_1} \\ \tau_{d,\theta_2} \end{bmatrix}$$

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

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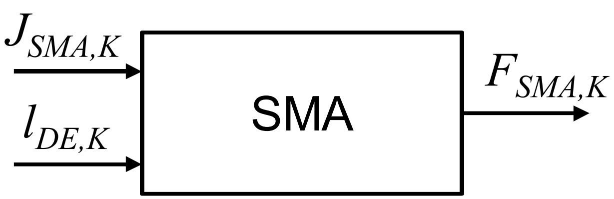
- SMA bundle model

$$
\begin{cases}
\dot{T}_{SMA,K} = -\dfrac{\lambda A_s}{mC_v}\left(T - T_E\right) + \dfrac{1}{mC_v N} J_{SMA,K} + \dfrac{VH_+}{mC_v}\left(-x_{+,SMA,K}\, p^{+A} + \left(1 - x_{+,SMA,K}\right) p^{A+}\right) \\
\dot{x}_{+,SMA,K} = -x_{+,SMA,K}\, p^{+A} + \left(1 - x_{+,SMA,K}\right) p^{A+} \\
F_{SMA,K} = N\pi r_0^2 \dfrac{\dfrac{l_{SMA,K} - l_0}{l_0} - \varepsilon_T x_{+,SMA,K}}{\dfrac{x_{+,SMA,K}}{E_M} + \dfrac{1 - x_{+,SMA,K}}{E_A}}
\end{cases}
$$



$K = \{1A, 2B, 2A, 2B\}$

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- Kinematic model

$$l_{SMA}(q) = \begin{bmatrix} l_{SMA,1A}(q) \\ l_{SMA,1B}(q) \\ l_{SMA,2A}(q) \\ l_{SMA,2B}(q) \end{bmatrix} = \begin{bmatrix} l_0 + \Delta - r_1\theta_1 \\ l_0 + \Delta + r_1\theta_1 \\ l_0 + \Delta - r_2\theta_2 \\ l_0 + \Delta + r_2\theta_2 \end{bmatrix}$$

- Jacobian matrix

$$J(q) = \frac{\partial l_{SMA}(q)}{\partial q} = \begin{bmatrix} \frac{\partial l_{SMA,1A}}{\partial \theta_1} & \frac{\partial l_{SMA,1A}}{\partial \theta_2} \\ \frac{\partial l_{SMA,1B}}{\partial \theta_1} & \frac{\partial l_{SMA,1B}}{\partial \theta_2} \\ \frac{\partial l_{SMA,2A}}{\partial \theta_1} & \frac{\partial l_{SMA,2A}}{\partial \theta_2} \\ \frac{\partial l_{SMA,2B}}{\partial \theta_1} & \frac{\partial l_{SMA,2B}}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -r_1 & 0 \\ r_1 & 0 \\ 0 & -r_2 \\ 0 & r_2 \end{bmatrix}$$

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- Kinetic energy

$$\begin{aligned}
\mathcal{T}(q,\dot{q}) &= \frac{1}{2}m_1\left[\left(-\frac{l_1}{2}\dot{\theta}_1\sin(\theta_1)\right)^2+\left(\frac{l_1}{2}\dot{\theta}_1\cos(\theta_1)\right)^2\right]+\frac{1}{2}I_1\dot{\theta}_1^2 \\
&\quad+\frac{1}{2}m_2\left[\left(-l_1\dot{\theta}_1\sin(\theta_1)-\frac{l_2}{2}(\dot{\theta}_1+\dot{\theta}_2)\sin(\theta_1+\theta_2)\right)^2+\left(l_1\dot{\theta}_1\cos(\theta_1)+\frac{l_2}{2}(\dot{\theta}_1+\dot{\theta}_2)\cos(\theta_1+\theta_2)\right)^2\right]+\frac{1}{2}I_2\dot{\theta}_2^2 \\
&= \frac{1}{2}m_1\frac{l_1^2}{4}\dot{\theta}_1^2+\frac{1}{2}I_1\dot{\theta}_1^2+\frac{1}{2}m_2\left(l_1^2\dot{\theta}_1^2+\frac{l_2^2}{4}(\dot{\theta}_1+\dot{\theta}_2)^2+l_1l_2\left(\dot{\theta}_1^2+\dot{\theta}_1\dot{\theta}_2\right)\cos(\theta_2)\right)+\frac{1}{2}I_2\dot{\theta}_2^2
\end{aligned}$$

- Potential energy

$$\mathcal{V}(q)=0$$

- Lagrangian

$$\mathcal{L}(q,\dot{q})=\mathcal{T}(q,\dot{q})-\mathcal{V}(q)=\frac{1}{2}m_1\frac{l_1^2}{4}\dot{\theta}_1^2+\frac{1}{2}I_1\dot{\theta}_1^2+\frac{1}{2}m_2\left(l_1^2\dot{\theta}_1^2+\frac{l_2^2}{4}(\dot{\theta}_1+\dot{\theta}_2)^2+l_1l_2\dot{\theta}_1\left(\dot{\theta}_1+\dot{\theta}_2\right)\cos(\theta_2)\right)+\frac{1}{2}I_2\dot{\theta}_2^2$$

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- Euler-Lagrange model

$$M(q)\ddot{q} + C(q,\dot{q}) = \tau - B\dot{q} + \tau_d$$

- Definition of above quantities

$$M(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + I_1 + m_2 l_1^2 + m_2 \frac{l_2^2}{4} + m_2 l_1 l_2 \cos(\theta_2) & m_2 \frac{l_2^2}{4} + \frac{1}{2} m_2 l_1 l_2 \cos(\theta_2) \\ m_2 \frac{l_2^2}{4} + \frac{1}{2} m_2 l_1 l_2 \cos(\theta_2) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_2 \left(\dot{\theta}_1 \dot{\theta}_2 + \frac{\dot{\theta}_2^2}{2} \right) \sin(\theta_2) \\ \frac{1}{2} m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2) \end{bmatrix} \qquad B = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}$$

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- Complete model in compact form

$$\begin{cases} \dot{T}_{SMA,K} = -\dfrac{\lambda A_s}{mC_v}(T - T_E) + \dfrac{1}{mC_v N} J_{SMA,K} + \dfrac{VH_+}{mC_v}\left(-x_{+,SMA,K}\, p^{+A} + \left(1 - x_{+,SMA,K}\right) p^{A+}\right), \;\; K \in \{1A, 1B, 2A, 2B\} \\ \dot{x}_{+,SMA,K} = -x_{+,SMA,K}\, p^{+A} + \left(1 - x_{+,SMA,K}\right) p^{A+}, \;\; K \in \{1A, 1B, 2A, 2B\} \\ \dot{q} = \upsilon \\ \dot{\upsilon} = M^{-1}(q)\left(-G(q,\upsilon) - J^T(q) F_{SMA}\left(l_{SMA,K}(q), x_{+,SMA,K}\right) - B\upsilon + \tau_d\right) \end{cases}$$

- Inverse of inertia matrix

$$M^{-1}(q) = \frac{1}{\det(M(q))}\begin{bmatrix} m_2 \dfrac{l_2^2}{4} + I_2 & -m_2 \dfrac{l_2^2}{4} - \dfrac{1}{2} m_2 l_1 l_2 \cos(\theta_2) \\ -m_2 \dfrac{l_2^2}{4} - \dfrac{1}{2} m_2 l_1 l_2 \cos(\theta_2) & m_1 \dfrac{l_1^2}{4} + I_1 + m_2 l_1^2 + m_2 \dfrac{l_2^2}{4} + m_2 l_1 l_2 \cos(\theta_2) \end{bmatrix}$$

$$\det(M(q)) = m_1 m_2 \frac{l_1^2 l_2^2}{16} + m_2^2 \frac{l_1^2 l_2^2}{4}\left(1 - \cos^2(\theta_2)\right) + I_1 m_2 \frac{l_2^2}{4} + m_1 I_2 \frac{l_1^2}{4} + I_1 I_2 + m_2 I_2 l_1^2 + m_2 I_2 \frac{l_2^2}{4} + m_2 I_2 l_1 l_2 \cos(\theta_2)$$