Pre-Calculus

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Contents

1 Trigonometry

1.1 Coterminal Angles

Example 1: Find an angle between 0 and 2π that is coterminal with $\theta = -\frac{11\pi}{6}$.

- Two angles are coterminal if their starting and ending sides are the same.
- An angle is in standard position when its initial side is on the positive x-axis.
- For negative angles, rotate clockwise.
- $-2\pi = -\frac{12\pi}{6}$, therefore we're $\frac{\pi}{6}$ short.
- $-\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$
- **Answer:** The coterminal angle is $\frac{\pi}{6}$

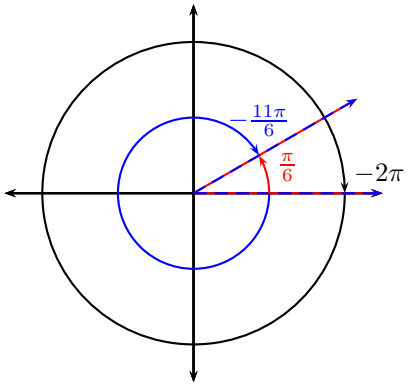

Figure 1: Coterminal angles in radians

Example 2: Let $\theta = 600°$.
a) Find an angle between $0°$ and $360°$ that is coterminal with θ.
b) Find an angle between $0°$ and $-360°$ that is coterminal with θ.

- $360° + 180° = 540°$
- $540° + 60° = 600°$
- $180° + 60° = 240°$
- **Answer a):** $240°$
- $180° - 60° = 120°$
- **Answer b):** $120°$

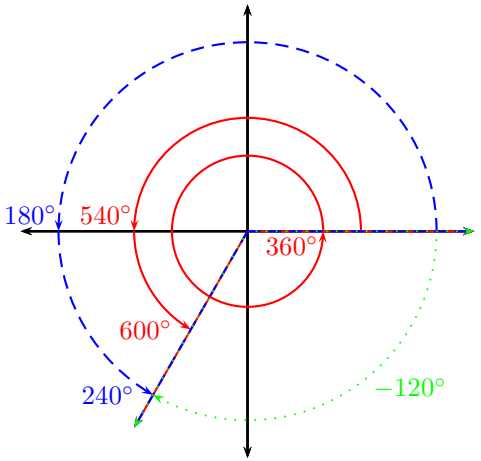

Figure 2: Coterminal angles in degrees

1.2 Reference Angles

Example 1: Find the reference angle for $\theta = \frac{5\pi}{6}$.

- The reference angle is the acute positive angle formed by the terminal side of θ and the x-axis.
- Reference angle $= \pi - \frac{5\pi}{6} = \frac{\pi}{6}$
- **Answer:** $\frac{\pi}{6}$

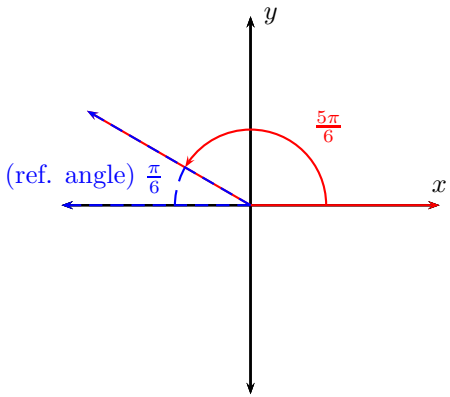

Figure 3: Reference angle in radians

Example 2: Find the reference angle for $\theta = -156°$.

- Reference angle $= 180° - 156° = 24°$
- **Answer:** $24°$

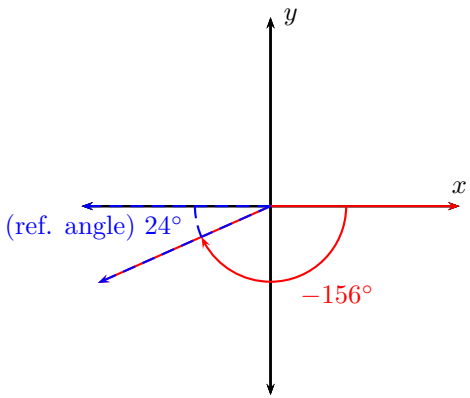

Figure 4: Reference angle in degrees

Example 3: Find the reference angle for $\theta = \frac{17\pi}{9}$.

- Reference angle $= 2\pi - \frac{17\pi}{9} = \frac{\pi}{9}$
- **Answer:** $\frac{\pi}{9}$

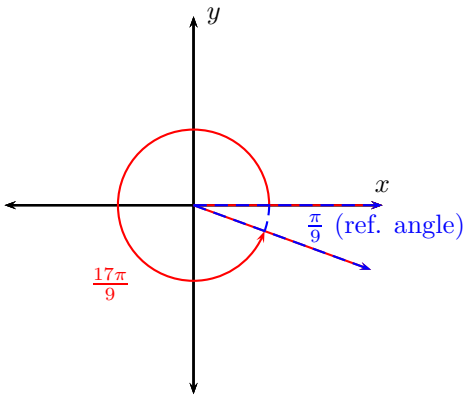

Figure 5: Reference angle in radians

1.3 Sketching an Angle in Standard Position

Example 1: Sketch $\theta = \frac{2\pi}{3}$ in standard position.

- An angle is in standard position if its initial side is on the positive x-axis.
- If $\theta > 0$, rotate counter-clockwise. If $\theta < 0$, rotate clockwise.
- One complete rotation measures 2π radians.
  - The radian measure of an angle is defined as the length of the corresponding arc on the unit circle.
  - Since the circumference of the unit circle is 2π, then the radian measure of one complete revolution is 2π.
  - $\therefore$ half a rotation would measure π radians.
- If π is split into thirds, $\frac{2\pi}{3}$ would span two of them.

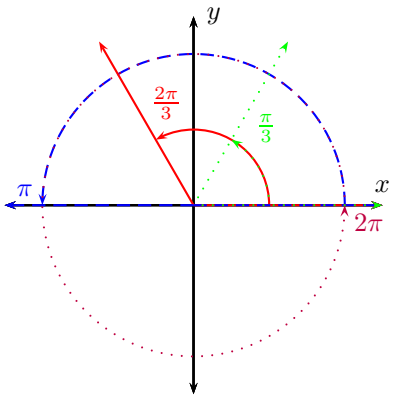

Figure 6: θ in standard position

Example 2: Sketch $\theta = -\frac{5\pi}{4}$ in standard position.

- If -2π is split into fourths, $-\frac{5\pi}{4}$ would span five of them going clockwise.

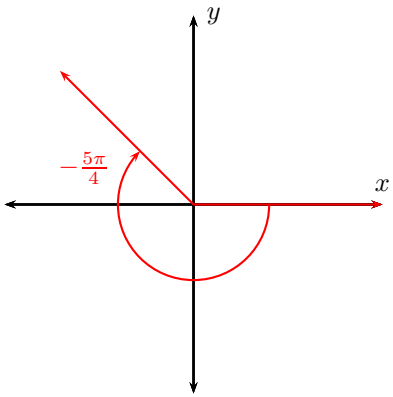

Figure 7: θ in standard position (negative)