Homework #0 Solution B

Spring 2020, CSE 446/546: Machine Learning Jiahui Xi Due: 4/8/20 11:59 PM A: 37 points. B: 3 points

April 2020

B.1 Solution:

$$E[Y] = \frac{n}{1+n}$$

B.2 Solution:

AB is an $n \times n$ matrix and BA is an $m \times m$ matrix. $(AB)_{ii} = \sum_{j=1}^{m} A_{ij}B_{ji}$ and $(BA)_{ii} = \sum_{j=1}^{n} B_{ij}A_{ji}$ For the trace,

$$(AB)_{ii} = \sum_{j=1}^{m} A_{ij} B_{ji}$$
 an

$$(BA)_{ii} = \sum_{i=1}^{n} B_{ij} A_{ji}$$

$$Tr(AB) = \sum_{i=1}^{n} \sum_{j=1}^{m} A_{ij} B_{ji}$$

For the trace,

$$Tr(AB) = \sum_{i=1}^{n} \sum_{j=1}^{m} A_{ij}B_{ji}$$

$$Tr(BA) = \sum_{i=1}^{m} \sum_{j=1}^{n} B_{ij}A_{ji}$$
So Tr(AB) = Tr(BA).

So
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.

B.3 Solution:

- a. min(rank)=1 and max(rank)=d
- b. $\min(\operatorname{rank})=1$ and $\max(\operatorname{rank})=\min\{n,d\}$
- c. min(rank)=0 (if Av=0) and max(rank)=D
- d. min(rank)=0 and $max(rank)=min\{n, D\}$ if V is of rank d, min(rank)=1 beacuase AV can't be 0, max(rank)=d.