Eindhoven University of Technology

Magnetic confinement and MHD of fusion plasmas

3MF110

Assignment 1 Computing particle orbits

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December 6, 2024

Contents

Question 1: Algorithm

The full algorithm including equations can be found in the code. The equations/steps can also be found here:

Orange block 1

$r = \sqrt{(R-R_0)^2 + Z^2}$
$\tau = \frac{q\Delta t}{2m}$
$\begin{pmatrix} E_R \\ E_\phi \\ E_Z \end{pmatrix} = \begin{pmatrix} E_0(R-R_0)/r \\ 0 \\ E_0 Z/r \end{pmatrix}$ (code is modulair and can be replaced by any other function)
$\begin{pmatrix} V_{min,R} \\ V_{min,\phi} \\ V_{min,Z} \end{pmatrix} = \begin{pmatrix} V_R + \tau E_R \\ V_\phi + \tau E_\phi \\ V_Z + \tau E_Z \end{pmatrix}$
$\begin{pmatrix} B_R \\ B_\phi \\ B_Z \end{pmatrix} = \begin{pmatrix} B_{p0} Z R_0/(rR) \\ B_0 R_0/R \\ B_{p0}(R_0 - R)R_0/(rR) \end{pmatrix}$ (code is modulair and can be replaced by any other function)
$\begin{pmatrix} B^*_R \\ B^*_\phi \\ B^*_Z \end{pmatrix} = \begin{pmatrix} B_R \\ B_\phi \\ B_Z + m(1.5V_\phi - 0.5V_{old,\phi})/(qR) \end{pmatrix}$
$c_1 = \frac{4}{4+(B^*_R)^2+(B^*_\phi)^2+(B^*_Z)^2}$
$c_2 = 2c_1 - 1$
$c_3 = 0.5c_1(V_{min,R}B^*_R + V_{min,\phi}B^*_\phi + V_{min,Z}B^*_Z)$
$\begin{pmatrix} part1_R \\ part1_\phi \\ part1_Z \end{pmatrix} = \begin{pmatrix} c_1(v_{min,\phi}B^*_Z - v_{min,Z}B^*_\phi) \\ -c_1(v_{min,R}B^*_Z - v_{min,Z}B^*_R) \\ c_1(v_{min,R}B^*_\phi - v_{min,\phi}B^*_R) \end{pmatrix}$
$\begin{pmatrix} part2_R \\ part2_\phi \\ part2_Z \end{pmatrix} = \begin{pmatrix} c_2 V_{min,R} + c_3 B^*_R \\ c_2 V_{min,\phi} + c_3 B^*_\phi \\ c_2 V_{min,Z} + c_3 B^*_Z \end{pmatrix}$
$\begin{pmatrix} V_{plus,R} \\ V_{plus,\phi} \\ V_{plus,Z} \end{pmatrix} = \begin{pmatrix} part1_R + part2_R \\ part1_\phi + part2_\phi \\ part1_Z + part2_Z \end{pmatrix}$
$\begin{pmatrix} V_{old,R} \\ V_{old,\phi} \\ V_{old,Z} \end{pmatrix} = \begin{pmatrix} V_R \\ V_\phi \\ V_Z \end{pmatrix}$
$\begin{pmatrix} V_R \\ V_\phi \\ V_Z \end{pmatrix} = \begin{pmatrix} V_{plus,R} + \tau E_R \\ V_{plus,\phi} + \tau E_\phi \\ V_{plus,Z} + \tau E_Z \end{pmatrix}$

Orange block 2

$R_{old} = R$
$\begin{pmatrix} R \\ \phi \\ Z \end{pmatrix} = \begin{pmatrix} R + \Delta t V_R \\ \phi + \Delta t V_\phi/(0.5R + 0.5R_{old}) \\ Z + \Delta t V_Z \end{pmatrix}$ (the $0.5R + 0.5R_{old}$ is due to the fact that V_R is the velocity in between the current and next step, to approximate the value of R there a average is used for small time steps this is very accurate)

Question 2: Parameter determination

The gyration frequency of the alpha particle is approximately

$$\begin{aligned} m =&4 \text{ u } \approx 4 \cdot 1.660 \cdot 10^{-27} \text{ kg} \\ q =&2 \text{ e } \approx 2 \cdot 1.602 \cdot 10^{-19} \text{ C} \\ \Omega_c =&\frac{qB}{m2\pi} \approx \frac{2 \cdot 1.602 \cdot 10^{-19} \cdot 5}{2\pi \cdot 4 \cdot 1.660 \cdot 10^{-27}} \approx 38.4 \text{ MHz} \end{aligned} \tag{1}$$

its velocity is

$$\begin{aligned} E =&3.5 \text{ MeV } \approx 5.61 \cdot 10^{-13} \text{ J} \\ v_{initial} =&\sqrt{\frac{2E}{m}} \approx \sqrt{\frac{2 \cdot 5.61 \cdot 10^{-13}}{4 \cdot 1.660 \cdot 10^{-27}}} \approx 1.30 \cdot 10^{7} \text{m/s} \end{aligned} \tag{2}$$

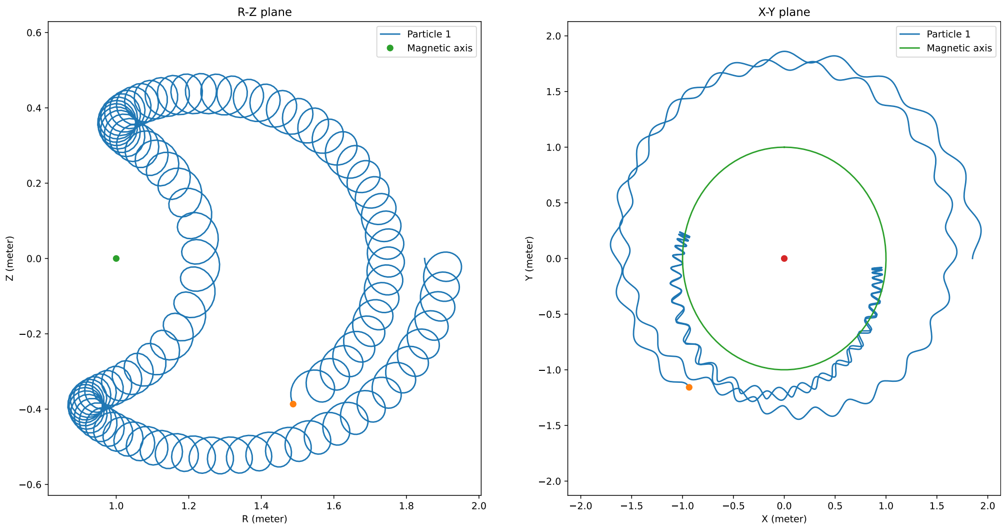
this is way below the speed of light and has a gamma of 1.001[1] so relativity doesn't need to be taken into account. This results in a gyro-radius of

$$\rho = \frac{mv}{qB} \approx \frac{4 \cdot 1.660 \cdot 10^{-27} \cdot 1.30 \cdot 10^{7}}{2 \cdot 1.602 \cdot 10^{-19} \cdot 5} \approx 5.39 \text{ cm} \tag{3}$$

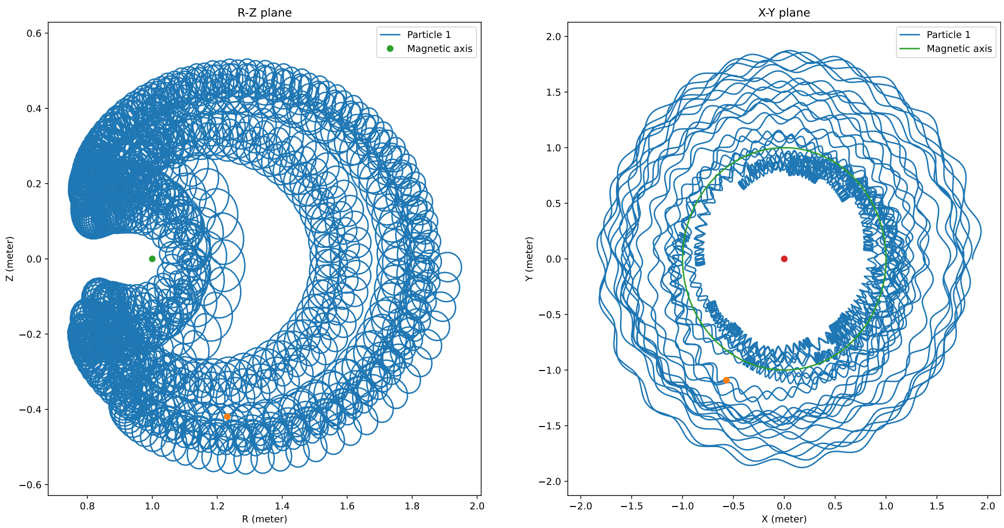
Question 3: Particle 1

Below the results of the first simple simulation can be seen. Figure 1a show the simulation with $B_0 = 5$ T and $B_{p0} = 1$ T with starting conditions $R = 1.85R_0$ and $v_r = 0$m, $v_z = -0.6v_{initial}$, $v_\phi = +0.8v_{initial}$. This was done with $\Delta t = 0.05\Omega_c^{-1}$.

[1] `https://www.wolframalpha.com/input?i=3.5+MeV+alpha+particle+velocity`



(a) Approximately a single banana orbit.



(b) Many orbits.

Figure 1: The orbit(s) of particle 1.

Next it is investigated until what level the time steps can be increased. In the figure 2 several multiples of the gyration time are used as a step size. In the subjective opinion of the author even $\Delta t = 0.05\Omega_c^{-1}$ is too high (due to the variation in R after each poriodal revolution), only $\Delta t = 0.005\Omega_c^{-1}$ has results he would be satisfied with (the deviation in R is significantly smaller than the gyro radius). But with lower standards a value of $\Delta t = 0.1\Omega_c^{-1}$ can still be acceptable since there the poriodal frequency still is somewhat accurate (see figure 2). At higher step sizes this also becomes very inaccurate and at even higher values the particle is not enclosed anymore (see $\Delta t = 0.25\Omega_c^{-1}$). The gyrations stay accurate for longer but at $\Delta t = 0.5\Omega_c^{-1}$ even the gyrations become inaccurate.

This is as expected since (according to the assignment) the following condition is required $\Delta t \ll \Omega_c^{-1}$. 0.05 is “only” 20 times smaller than 1 so while it should be decent lower would be better. Similarly 0.25 is not much less than 1 so bad results are to be expected then.

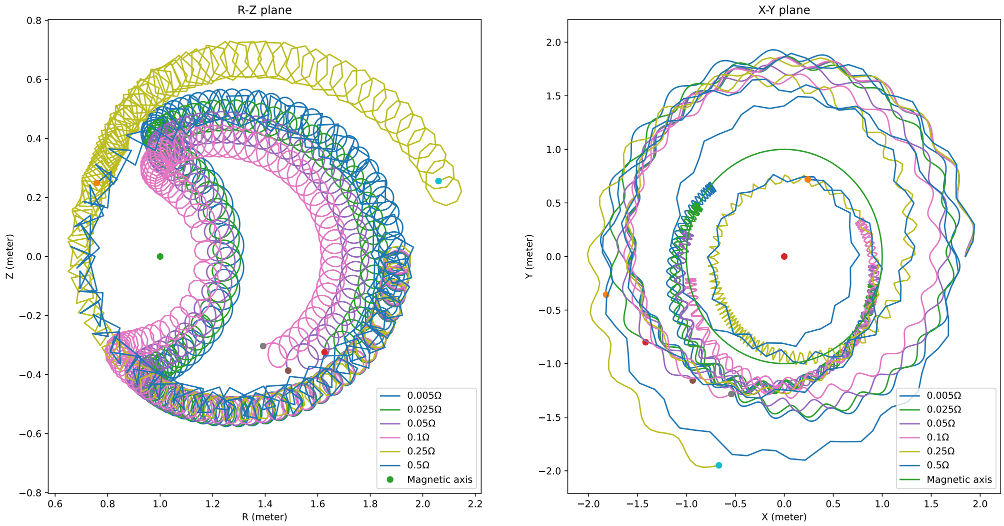


Figure 2: The orbit of particle 1 for various values of Δt.

Question 4: Particle 2

A second particle is added with initial conditions $R = 1.454R_0$, $v_r = 0.48v_{initial}$, $v_z = -0.8v_{initial}$ and $v_\phi = 0.36v_{initial}$. The results can be seen in the figure 3. The starting position of the particle is already closer to the magnetic axis than the closest point of the first particle. This results in a smaller and closer banana.

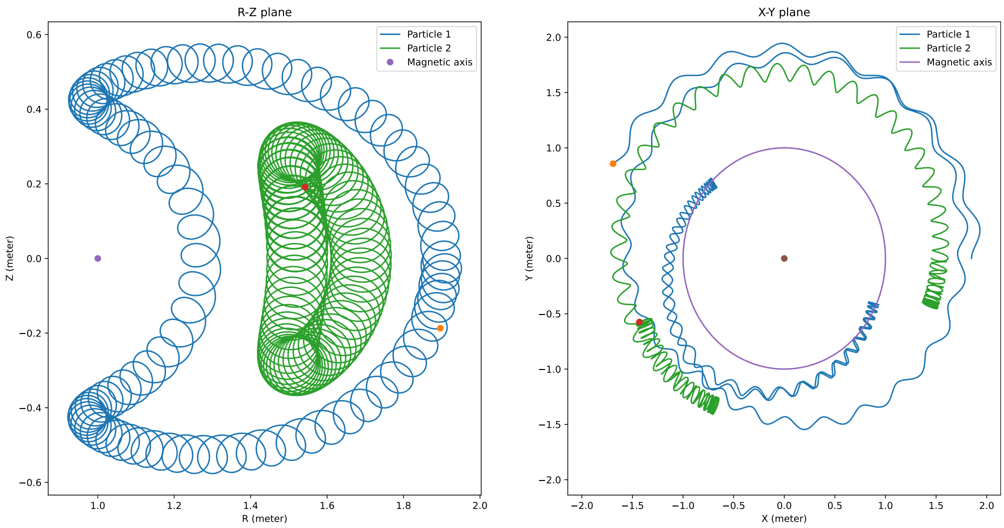


Figure 3: The orbits of both particle 1 and 2.

By running the simulation for a long time it was seen that both particle 1 and 2 are trapped. Neither particles passes to the other side (even with longer simulations). It can also be calculated with

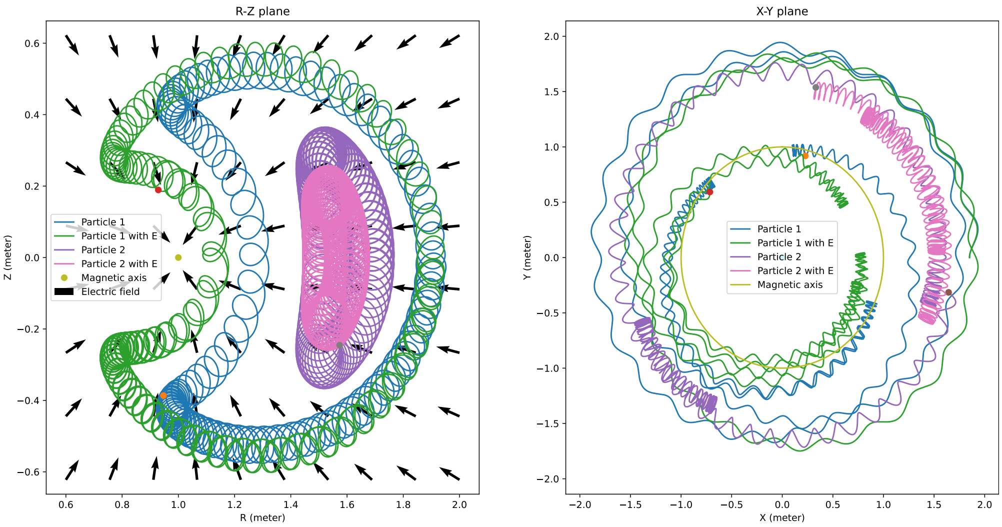
$$\frac{v_{\parallel,0}}{v_{\perp,0}} \leq \sqrt{\frac{B_{max}}{B_{min}} - 1} \tag{4}$$

however the execution of this equation presumably went wrong since the results obtained are $2.1 \leq 1.1$ for particle 1 which implies that this particle is not trapped (while in the simulation is most certainly seems to be trapped). For particle 2 the results were $0.20 \leq 0.50$ which is true but should not be seen as a reliable proof due to the earlier failure.

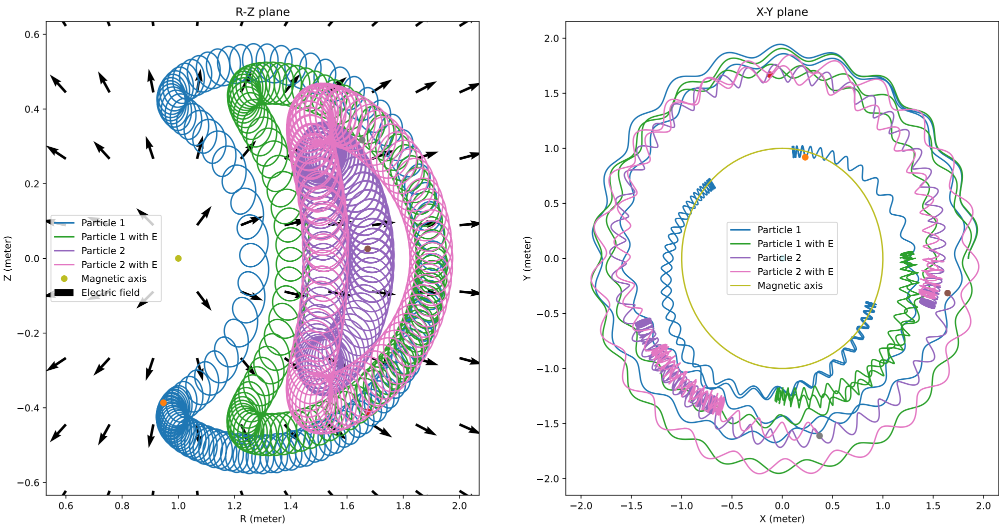
The calculated banana width is $\Delta r = 0.36$ meter for the first particle and $\Delta r = 0.13$ meter for the second particle. These approximately match the results of the simulation (0.45 meter, 0.18 meter). It is unknown what causes the discrepancy.

Question 5: Electric field

An electric field is added (see figure 4). This causes the orbits to move towards the outside of the tokamak. The effect is most pronounced in the "tips" of the bananas. When the electric field is overlaid it becomes clear why. The electric field points radially away from the magnetic axis around which the banana forms. Since the electric effects of the Lorentz force are additive and the charge of the particle is positive this means that (besides the complex magnetic forces) the particle constantly gets pushed to the outside.



(a) The orbits with $E_0 = -1000$ kV/m.

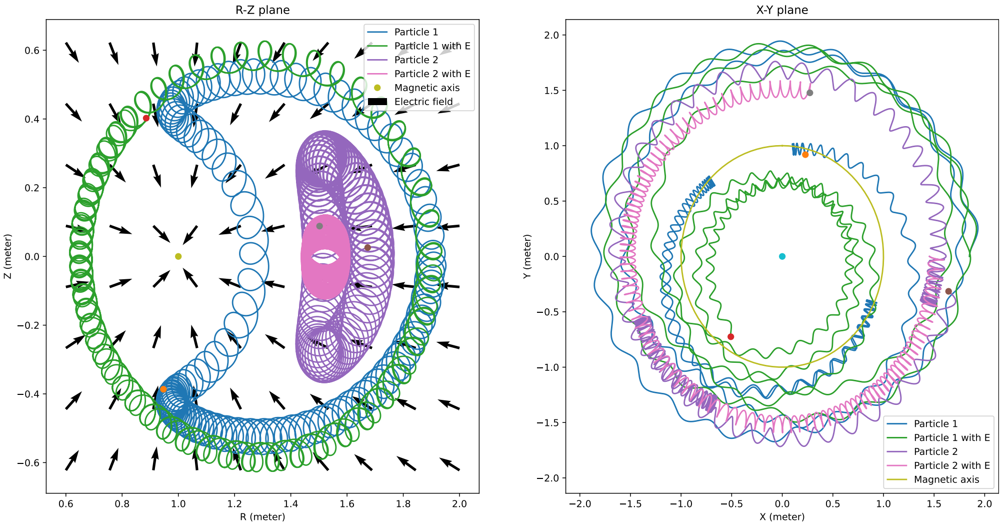


(b) The orbits with $E_0 = 1000$ kV/m.

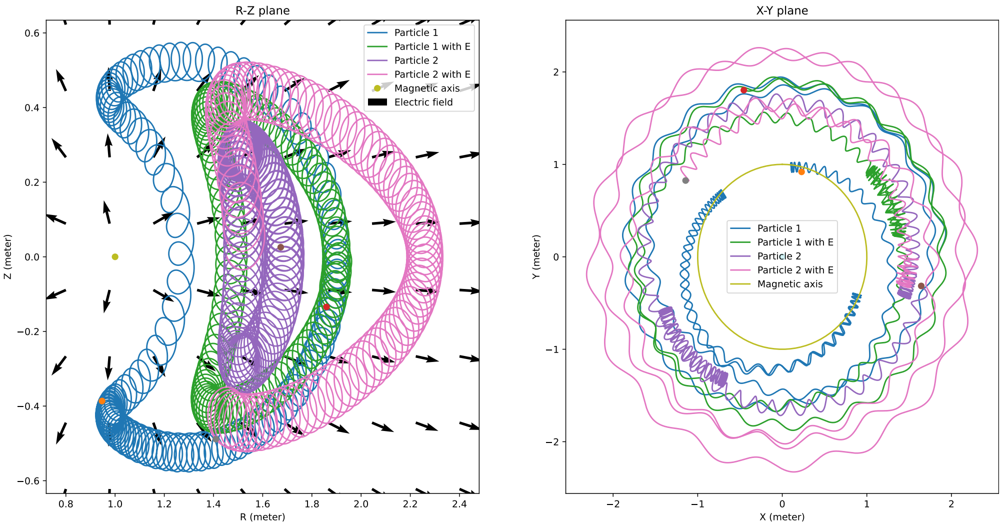
Figure 4: The orbit of particle 1 and 2 with an electic field.

An attempt to find a value of E_0 for which the precession of the particles is canceled was done by doing a parameter scan from -5000 kV/m to 1500 kV/m in steps of 50 kV/m. Sadly in none of the results the electric field caused the precession of both particles to stop.

However at -2400 kV/m particle 2, which normally starts at the inboard of its orbit, has a very low amount of precession (see figure 5a). At the same time particle 1 suddenly stops being a trapped particle. The attempt to stop particle 1 by increasing the magnetic field to positive values didn't have the results that were hoped for (see figure 5b), even higher electric field strength resulted in the particles being ejected out of their orbits.



(a) The orbits with $E_0 = -2400$ kV/m.



(b) The orbits with $E_0 = 1500$ kV/m.

Figure 5: The orbit of particle 1 and 2 with an electric field.

Question 6: ITER

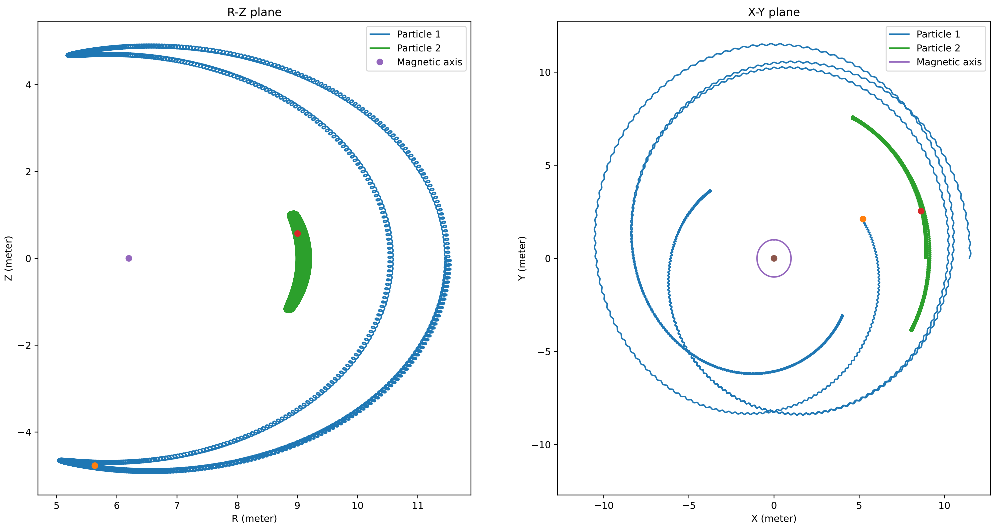


Figure 6: The orbits of both particle 1 and 2 in a ITER-like Tokamak.

In figure 6 particle 1 and 2 in a ITER-like Tokamak can be seen. The Gyrations are very small and thus barely can be seen. For particle 2 the banana width even is on the order of the gyrations.