Optical Diagnostics Assignment 1

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August 4, 2024

Contents

Question 1: Equilibrium ratios

A, B & C: Different energy levels

The Boltzmann distribution was used to determine the ratio in the thermal equilibrium. Here N_2 is the higher energy level and N_1 is the lower level. The results can be seen in table 1 or figure 1.

Table 1: The ratio $\frac{N_2}{N_1}$ between the population of two energy levels in thermal equilibrium.

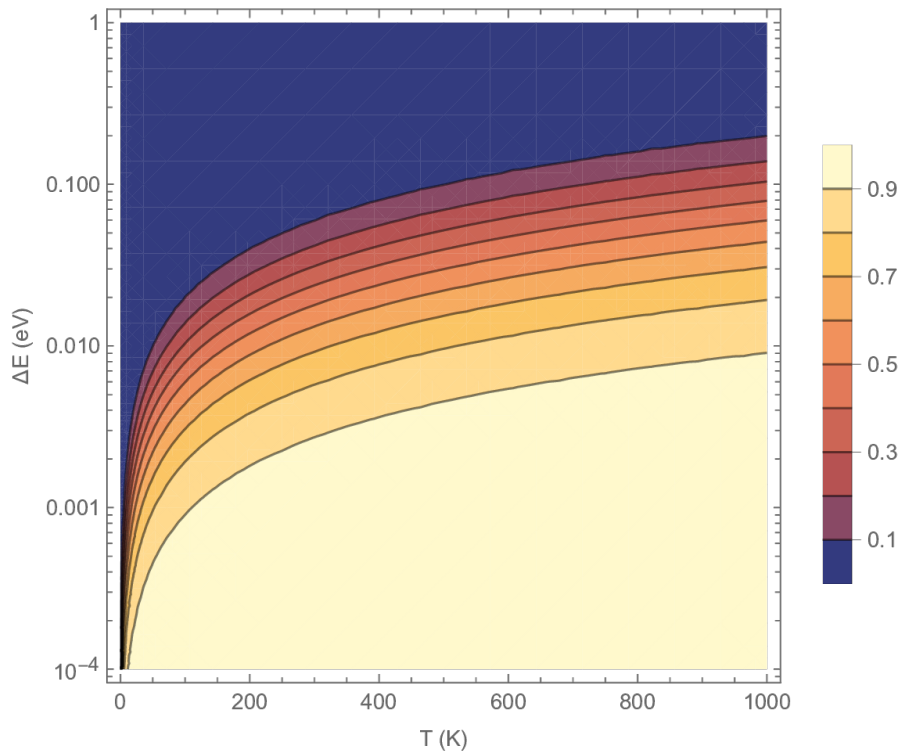| ΔE (eV)\T (K)	100	300	1000
$3 \cdot 10^{-4}$	0.9658	0.9885	0.9965
$5 \cdot 10^{-2}$	0.0030	0.1448	0.5601
3	$8.7 \cdot 10^{-152}$	$4.4 \cdot 10^{-51}$	$7.87 \cdot 10^{-16}$

Figure 1: A contour plot of the ratio $\frac{N_2}{N_1}$ between the population of two energy levels in thermal equilibrium.

D: Conclusion

From these results we can conclude that molecular rotational energy levels are almost always (even at relatively low temperatures) occupied beyond the ground state.

This also holds true for molecular vibrational energy levels, although to a lesser extent especially at lower temperatures.

However this doesn't apply to electronic excitation levels, where most particles stay in the ground state even at higher temperatures.

Question 2: Spectrometer

A: Needed properties of ICCD

To be able to measure the line shape and (absolute) intensity the ICCD need to be calibrated. Specifically the sensitivity per wavelength needs to be determined.

You can then use the spectrometer to determine the distribution/shape and the ICCD to determine the intensity.

B: Determining properties

The ICCD can be calibrated using a Tungsten ribbon lamp in combination with several filter to determine the (absolute) sensitivity across the relevant wavelengths. You use the Tungsten lamp (with a known luminance) with the filters (varying each time) to reconstruct a sensitivity curve (as a function of wavelength). Calibrated LEDs can also be used.

Question 3: Population levels

A: Rate equations

Since the degeneracy is equal the Einstein coefficients are equal $B_{12} = \frac{g_2}{g_1} B_{21} = \frac{g}{g} B_{21} = B_{21}$. So we can simply use the τ directly.

$\dot{N_2} = R_p - \frac{N_2}{\tau_2}$

$\dot{N_1} = \frac{N_2}{\tau_2} - \frac{N_1}{\tau_1}$

B: Population as a function of time

Solve the differential equation as given in A. The first one is relatively easy to solve. We assume $N_2(t = 0) = 0 = N_1(t = 0)$.

We can solve the differential equation as a super position of two different differential equations $\dot{N_2} = R_p$ and $\dot{N_2} = \frac{N_2}{\tau_2}$.

Which results in $N_2 = Ce^{-\frac{t}{\tau}} + \tau R_p$.

Solving for $N_2(t = 0) = 0$ results in $C = -\tau R_p \rightarrow N_2 = -\tau R_p e^{-\frac{t}{\tau}} + \tau R_p$

Solving for N_1 is more difficult so Wolfram Mathematica was used. We fill in the previous result into the second differential equation and obtain a result. We then again we plug in the initial conditions. $N_1 = -R_p \tau_1 e^{\frac{t}{\tau_1}} \left(1 + \frac{\tau_2}{\tau_1 - \tau_2} \right) + R_p \tau_1 e^{-\frac{t}{\tau_1}} \left(e^{\frac{t}{\tau_1}} + \frac{e^{t\left(\frac{1}{\tau_1} - \frac{1}{\tau_2} \right)} \tau_2}{\tau_1 - \tau_2} \right)$

C: Plots

We expect the amount of atoms in level two to exponentially rise/decay to the equilibrium position which is $\dot{N_2} = 0 = R_p - \frac{N_{2eq}}{\tau_2} - \rightarrow N_{2eq} = R_p \tau_2$.

Similarly we expect the atoms in level one to asymptotically approach the equilibrium position $\dot{N_1} = 0 = \frac{N_{eq}}{\tau_2} - \frac{N_{eq}}{\tau_1} \rightarrow N_{1eq} = N_{2eq} \frac{\tau_1}{\tau_2} = R_p \tau_1$.

$R_p = 5 \cdot 10^7$ particles per second is chosen (so that the highest equilibrium position will be 100 particles).
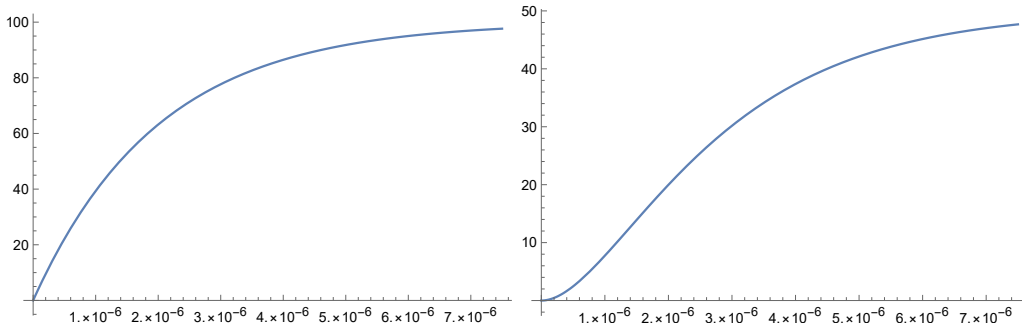
(a) Particles in level 2 versus time in seconds.   (b) Particles in level 1 versus time in seconds.

Figure 2: The amount of particles in each level for $\tau_2 = 2$ μs and $\tau_1 = 1$ μs.

(a) Particles in level 2 versus time in seconds. (b) Particles in level 1 versus time in seconds.
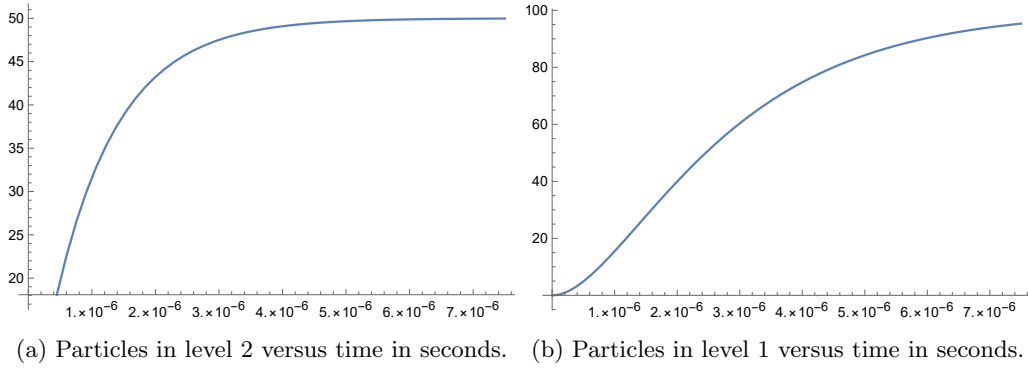
Figure 3: The amount of particles in each level for $\tau_2 = 1\ \mu$s and $\tau_1 = 2\ \mu$s.

These results are as expected. We for atoms in level two we see that it asymptotically approaches the equilibrium level. We also see a nice s-curve like shape for the particles in level 1 since those of course depend on the amount of particles in level 2.

Question 4: Cavity building

A: Cavity proposal and considerations

The proposed 2-mirror cavity is a cavity with a distance of (as close as possible to) 100 mm using the 50 mm mirrors. The laser itself has $\lambda = 2 \cdot 1064$ nm wavelength[4] and we assume a beam waist radius of (as close as possible to) 20 μm. This results in a stability criterion

$$g_1 g_2 = (1 - 100/50)(1 - 100/50) = 1 \tag{1}$$

which is at the edge of stability (and if we assume that the distance is slightly more than 100 mm than in fact it is marginally unstable).

In fact due to the (finite) beam waist $\omega_0 < 20$ μm and using equation

$$\omega_z = \sqrt{\omega_0^2 + M^4 \left(\frac{\lambda}{\pi \omega_0} \right)^2 (z - z_0)^2} \tag{2}$$

[2] one can calculate that after one distance the beam width is 846 μm and thus presumably 'leaks' to the outside of the mirror.

B: Non-linear crystals

For non-linear crystals having the mirrors being closer to each other would be better. For example $d = 75$mm.

C: Laser quality

Again using equation 2 the beam width is calculated to be 1524 μm after one distance (with $M^2 = 1.8$). This is significantly worse than before, especially considering that the cavity already was marginally stable.

Question 5: Spectrum analysis

A: Temperatures

To determine the different temperatures Specair was used. The following actions were taken:

1. The spectrum file was imported into Specair.

2. The pressure was set to 0.2 bar.

3. The slit function was set to be a trapezoidal slit function with base of 2 nm and a top width of 1 nm and a peak intensity of 1. See figure 4c.

4. Use the "Find transition" function to seek the transition (which turns out to be the N2 (2+) transition).

5. This function also does an initial fit (which fits all temperatures to be 2582 K).

6. Fit again (and set the translational temperature to 700 K as per the exercise) but fix every temperature but one. Thus that specific temperature will be fitted. See figure 4a.

7. Repeat the previous step for vibrational, rotational and electronic temperature (constantly switching which temperature is non-fixed) until the temperatures converge. These resulting temperatures are displayed in table 2. Each new fit results in a slightly better total fit (going from the initial fit with a residual of 0.7 to a final fit with a residual of 0.0255), these can bee seen in figure 5.
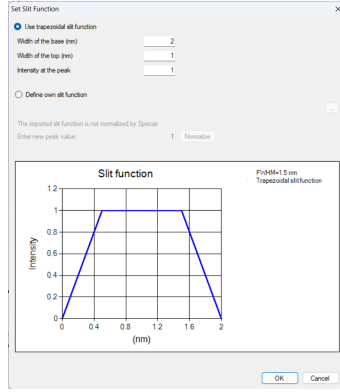
Thus it can be concluded (from table 2) that the temperatures are:

- $T_{ele} \approx 1.3 \cdot 10^4$ K.

- $T_{vib} \approx 5.1 \cdot 10^3$ K.

- $T_{rot} \approx 7.3 \cdot 10^2$ K.
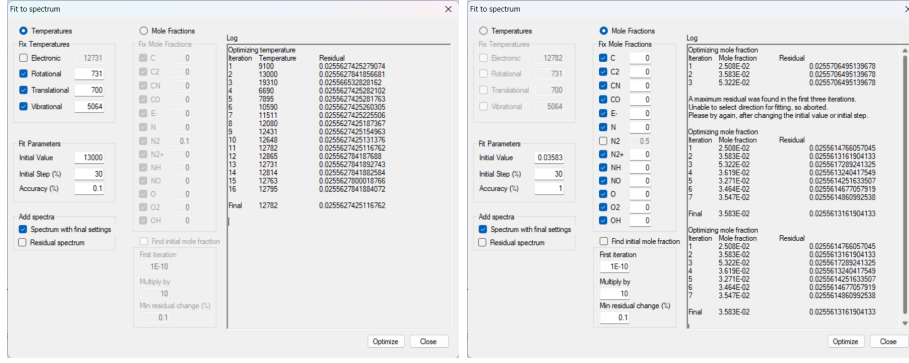
(a) The final temperature fit.

(b) The molar fit.

(c) The slit function.

Figure 4: Various settings and processes.

Table 2: The fitted temperatures after each cycle of fitting ($T_{vib} \rightarrow T_{rot} \rightarrow T_{ele}$). The translational temperature was set at $T_{tra} = 700$ K.

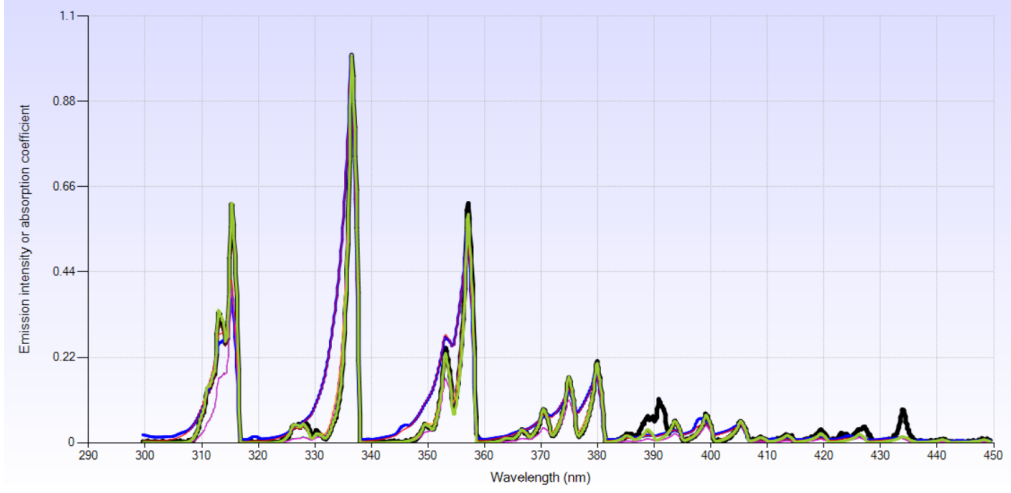|	T_{vib} **(K)**	T_{rot} **(K)**	T_{ele} **(K)**
Cycle 1	2779	988	3514
Cycle 2	4529	785	14558
Cycle 3	4964	741	13067
Cycle 4	5051	733	12731
Cycle 5	5064	731	12782

Figure 5: The original spectrum (black), the initial fit (blue), the final fit (green) and many intermediate fits.

B: Ionization degree

The ionization degree can also be determined via fitting (see figure 4b). That results in a molar fraction for N2 of $N_2 \approx 0.03583$, thus, given that $H \ll 1$, it can be concluded that the rest is (ionized) Nitrogen. So the ionization fraction is $\frac{N_2+}{N_2+N_2+} \approx 1 - 0.03583 \approx 0.964$. That is very high but given the high temperature it seems reasonable.

C: Gaussian and Lorentzian widths

In figure 5 a few unaccounted for peaks can be seen. One of them, at 434 nm [1], is the Balmer-γ peak. To determine the Gaussian and Lorentzian widths the spectrum was imported into Python (attached with the submission) where a Voigt profile was fitted to it (see figure 6). This resulted in $\sigma = 0.19 \pm 0.07$ nm and $\gamma = 0.63 \pm 0.05$ nm. That can be translated into a full width half maximum of $FWHM_{Gauss} \approx 2.355\sigma \approx 0.45$ nm and $FWHM_{Lor} = 2\gamma \approx 1.25$ nm [3].
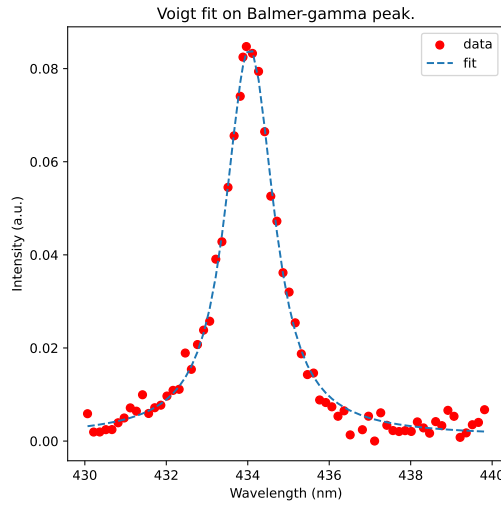

Figure 6: A Voigt fit on the 434 nm Balmer-γ peak in the spectrum.

References

[1] J. J. Balmer. Notiz über die spectrallinien des wasserstoffs, November 1884.

[2] Anthony E. Siegman. Defining, measuring, and optimizing laser beam quality. In Anup Bhowmik, editor, *Laser Resonators and Coherent Optics: Modeling, Technology, and Applications*, volume 1868, pages 2 – 12. International Society for Optics and Photonics, SPIE, 1993.

[3] Wikipedia. Full width at half maximum, 04 2024.

[4] A. Yariv. *Quantum Electronics*. Wiley, 1989.