Literature Review (Group 15)

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1   Introduction

For the Final Project, Our group has decided to do an orbit simulation for the James-Web Space Telescope (JWST) Halo orbit at the L2 point in the Earth-Sun system. After that, we will simulate the transfering orbit from Earth Surface to the L2 point using a feedback control with simulated noisy measurement data of the orbit.
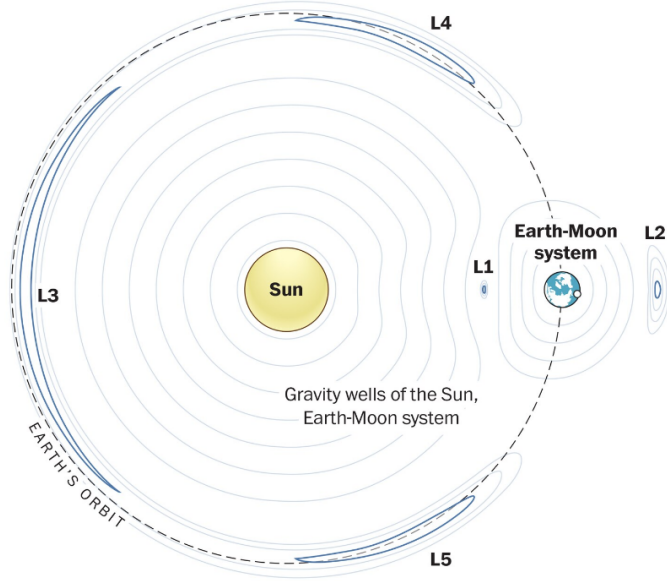

Figure 1: The 5 Lagrange points for the Sun-Earth system (1)

After some literature reviews, we decided to use the Circular Restricted Three Body Problem to describe the motion of JWST at the L2 point. As shown in Fig. 2, assume the third mass is negligibly small $m << m_1, m_2$. The other two finite masses move in circular orbits about their common center of

gravity COG. We define COG to be the origin of the coordinate system, x-axis be parallel to the separation r_{12} pointing from the COG to m_2, and y-axis perpendicular to x-axis in the orbit plane. Lastly, assume this frame is rotating about the z-axis at constant rate ω relative to a stationary frame, the universe. The system is normalized by letting the sum of the masses be 1, the constant separation between m_1 and m_2 be 1, and the gravitational constant $G = 1$. Since the sum of masses is 1, it is reasonable to assign $m_1 = 1 - \mu$, $m_2 = \mu$, with $\mu \leq 0.5$.
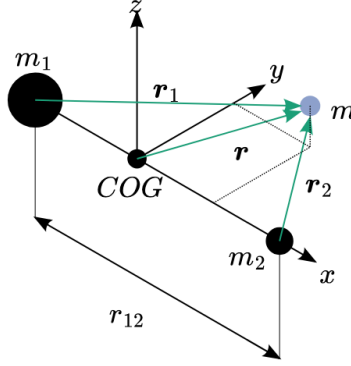

Figure 2: The three-body system (2)

The equation of motion of the negligible mass (JWST) can be derived as:

$$\ddot{x} - 2\dot{y}\omega - x\omega^2 = \frac{1-\mu}{r_1^3}(x_1 - x) + \frac{\mu}{r_2^3}(x_2 - x)$$

$$\ddot{y} + 2\dot{x}\omega - y\omega^2 = \frac{1-\mu}{r_1^3}(y_1 - y) + \frac{\mu}{r_2^3}(y_2 - y) \quad (1)$$

$$\ddot{z} = \frac{1-\mu}{r_1^3}(z_1 - z) + \frac{\mu}{r_2^3}(z_2 - z)$$

In fact, one can show that the L2 point for the Earth-Sun system is unstable when the amplitude of the motion is small. However, when the amplitude of the motion is large, there exists periodic orbits around L2 point. In fact, The Halo Orbit in our simulation is a periodic orbit (3). We will show this in the final report.

Claim: All contents are hand-typed, no use of AI/Chatbot.

References

[1] Neff, W. (2022, January 21). What is a Lagrange point, the final destination for the james webb telescope? - The Washington Post. What is a Lagrange point, the final destination for the James Webb Space Telescope? https://www.washingtonpost.com/science/2022/01/21/lagrange-point-explainer-webb-telescope/.

[2] Weber, B. (n.d.). Circular restricted three-body problem. Circular Restricted Three-Body Problem - Orbital Mechanics Astrodynamics. https://orbital-mechanics.space/the-n-body-problem/circular-restricted-three-body-problem.html.

[3] J. E. PRUSSING AND B. A. CONWAY, *Orbital mechanics*, Oxford University Press, 1993.