Halo Orbit Simulation of JWST at L_2

CSE 6730 Milestone 2 (Group 15)

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GitHub Repository: https://github.com/TimHu-0728/CSE-6730-Final-Project

Abstract

This project investigates the Circular Restricted Three-Body Problem (CR3BP) [1] to model and simulate the orbital dynamics of the James Webb Space Telescope (JWST) around the L_2 Lagrange point in the Sun–Earth system. Building upon classical formulations, we treat JWST as a negligible-mass third body and work in a rotating reference frame with normalized parameters to simplify the equations of motion and analysis. Our study aims to numerically reproduce the halo orbit near L_2, a periodic solution in the CR3BP. In Milestone 2, we focus on replicating established results from the CR3BP literature: implementing the normalized rotating-frame dynamics and validating periodic motion about L_2 as a baseline for correctness. In Milestone 3, we move to innovation, pursuing (i) an optimal transfer trajectory from Earth to the L_2 vicinity and (ii) a feedback control system to guide the spacecraft along the transfer and maintain the halo orbit under noisy measurements. Together, these milestones progress from verified modeling to practical trajectory design and closed-loop station-keeping at L_2.

Summary of Current Results

1. Normalized System Parameters and Initial Conditions

Table 1 lists all parameters used for the Sun–Earth–JWST model prior to the normalization steps.

All integrations are performed in a non-dimensional CR3BP framework. Using the characteristic length r_{12}, time t_C, and velocity V_C from Table 1, dimensional variables are mapped to their non-dimensional counterparts as

$$x = \frac{x^{\text{phys}}}{r_{12}}, \qquad t = \frac{t^{\text{phys}}}{t_C}, \qquad \dot{x} = \frac{\dot{x}^{\text{phys}}}{V_C},$$

with analogous scalings for y, z, $\dot{y}$, and $\dot{z}$.

2. Third Body Non-Dimensional Equations of Motion

Using the normalized parameters listed in Table 1, we adopt the non-dimensional formulation of the Circular Restricted Three-Body Problem (CR3BP). In this normalized system, the Sun–Earth separation is taken as the unit length (so $r_{12} = 1$), the combined mass is scaled to unity ($m_1 + m_2 = 1$), and the rotating-frame angular speed is set to one ($\omega = 1$). Physical variables are converted via the scaling relations $x = x^{\text{phys}}/r_{12}$, $t = t^{\text{phys}}/t_C$, and $\dot{x} = \dot{x}^{\text{phys}}/V_C$, so all quantities appearing

Symbol	Description	Value / Expression (Units)
m_1	Sun mass	1.98847×10^{30} kg
m_2	Earth mass	5.9722×10^{24} kg
G	Gravitational constant	6.674×10^{-20} $\text{km}^3/(\text{kg} \cdot \text{s}^2)$
r_{12}	Sun–Earth mean distance (1 AU)	1.495978707×10^8 km
π_1	Mass fraction of Sun	$\pi_1 = \dfrac{m_1}{m_1 + m_2}$ (unitless)
π_2	Mass fraction of Earth	$\pi_2 = \dfrac{m_2}{m_1 + m_2}$ (unitless)
μ_G	Gravitational parameter	$\mu_G = G(m_1 + m_2)$ (km^3/s^2)
t_C	Characteristic time	$t_C = \sqrt{\dfrac{r_{12}^3}{\mu_G}}$ (s)
V_C	Characteristic velocity	$V_C = \dfrac{r_{12}}{t_C}$ (km/s)
ω	Rotating-frame angular speed	1 (non-dimensional)
x, y, z	Rotating-frame position	non-dimensional
$\dot{x}, \dot{y}, \dot{z}$	Rotating-frame velocity	non-dimensional
u_x, u_y, u_z	Control accelerations	non-dimensional

Table 1: Physical constants and normalized parameters of the Sun–Earth–JWST system.

below are dimensionless. One advantage to use the non-dimensional form is avoiding numerical instabilities, because the relative distances and velocities of celestial bodies are often in different scales.

The non-dimensional distances from the spacecraft to the two primaries are

$$\sigma = \sqrt{(x + \pi_2)^2 + y^2 + z^2}, \qquad \psi = \sqrt{(x - 1 + \pi_2)^2 + y^2 + z^2},$$

where σ and ψ represent the normalized distances from JWST to the Sun and Earth, respectively.

The resulting non-dimensional equations of motion are

$$\ddot{x} - 2\dot{y} = x - \frac{(1 - \pi_2)(x + \pi_2)}{\sigma^3} - \frac{\pi_2(x - 1 + \pi_2)}{\psi^3} + u_x, \tag{1}$$

$$\ddot{y} + 2\dot{x} = y - \frac{(1 - \pi_2)\, y}{\sigma^3} - \frac{\pi_2\, y}{\psi^3} + u_y, \tag{2}$$

$$\ddot{z} = -\frac{(1 - \pi_2)\, z}{\sigma^3} - \frac{\pi_2\, z}{\psi^3} + u_z. \tag{3}$$

Define the states X and input u of the systems to be

$$X = [\, x,\ \dot{x},\ y,\ \dot{y},\ z,\ \dot{z}\,]^\top,$$
$$u = [\, u_x,\ u_y,\ u_z\,]^\top.$$

we can transfer the equations of motion to the state-space form

$$\begin{aligned}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= -\frac{1-\pi_2}{\sigma^3}(X_1+\pi_2) - \frac{\pi_2}{\psi^3}(X_1 - 1 + \pi_2) + 2X_4 + X_1 + u_x \\
\dot{X}_3 &= X_4 \\
\dot{X}_4 &= -\frac{1-\pi_2}{\sigma^3}X_3 - \frac{\pi_2}{\psi^3}X_3 - 2X_2 + X_3 + u_y \\
\dot{X}_5 &= X_6 \\
\dot{X}_6 &= -\frac{1-\pi_2}{\sigma^3}X_5 - \frac{\pi_2}{\psi^3}X_5 + u_z
\end{aligned} \tag{4}$$

These equations describe the motion of JWST in the uniformly rotating frame, accounting for the combined effects of gravitational attraction from each other bodies, and Coriolis acceleration. The control terms (u_x, u_y, u_z) are included for later use for modeling the Orbital maneuver using feedforward and feedback control. We can now use the numerical integrator to solve (4), given an initial condition $X(0)$, and input $u(t)$.

At this stage, use do not include any input, so $u(t) = 0$. The initial condition use is achieved from the JPL Horizon system in non-dimensional unit[2]:

$$X(0) = \begin{bmatrix} 1.006\,201\,041\,659\,247\,6 \\ -9.807\,436\,982\,071\,242\,3 \times 10^{-16} \\ -2.152\,307\,851\,825\,963\,0 \times 10^{-23} \\ -1.325\,347\,792\,466\,051\,1 \times 10^{-2} \\ 1.238\,031\,120\,134\,930\,3 \times 10^{-2} \\ -1.095\,660\,345\,906\,113\,3 \times 10^{-14} \end{bmatrix}$$

As shown in Fig. 1, this configuration places JWST slightly beyond L_2 along the Sun–Earth axis, with a small out-of-plane component that induces the characteristic three-dimensional halo motion observed in the simulation.

3. Numerical Implementation and Validation

The continuous-time dynamics (4) are implemented as a nonlinear state update and simulated using the Python Control Systems Library (`control`, imported as `ct`). We construct a continuous-time nonlinear I/O system with state $X = [x,\ \dot{x},\ y,\ \dot{y},\ z,\ \dot{z}]^\top$ and input $u = [u_x,\ u_y,\ u_z]^\top$, and propagate the state with `ct`'s built-in numerical integrators. For the replication results reported here, the control inputs are set to zero ($u_x = u_y = u_z = 0$). The simulation time span for 20 years. The result is evaluated in both the rotating and the inertial frames.

Two static frames extracted from the simulation animations illustrate the periodic halo-like orbit of JWST around L_2. Figure 1 shows the trajectory in the rotating reference frame, while Figure 2 presents the same motion of JWST in the inertial frame. The trajectory for JWST in Fixed frame is partially visible for clarity

The simulation reproduces a three-dimensional periodic halo orbit around the Sun–Earth L_2 point, validating the non-dimensional equations and the numerical integration approach based on `ct`.

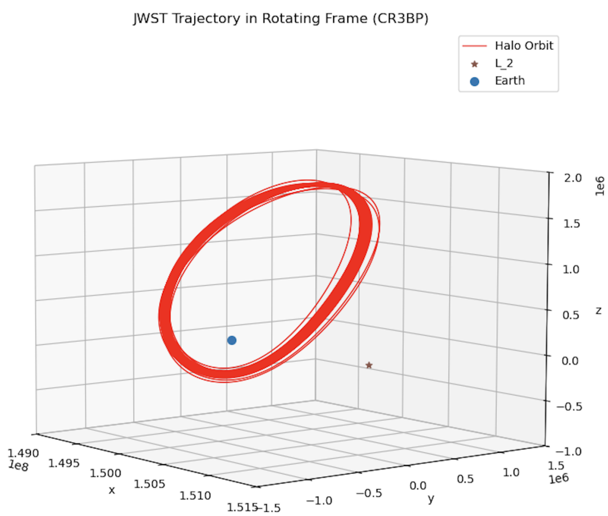

Figure 1: JWST trajectory near L_2 in the rotating (co-rotating) frame, showing the Halo Orbit.

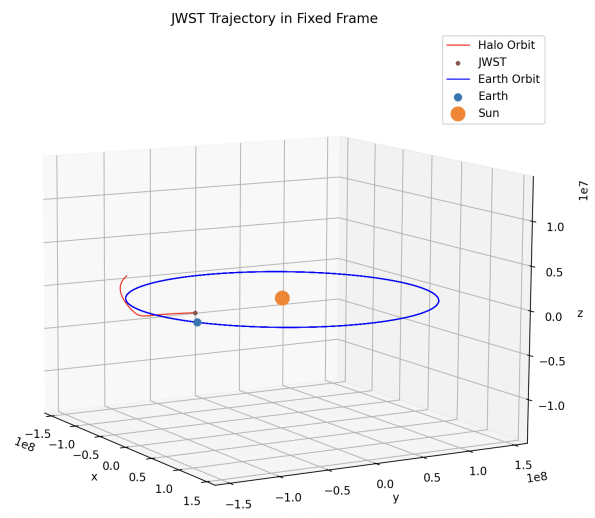

Figure 2: JWST trajectory in the inertial frame, illustrating the Sun–Earth–JWST configuration over several orbital periods.

These results establish a validated baseline for the CR3BP dynamics, providing the foundation for the transfer-trajectory optimization and feedback control to be developed in Milestone 3.

So far, the equation motion of JWST has been successfully implemented and validated. For the next milestone, we want to use the pyvista module for better visualization of the celestials, the JWST, and their orbits in 3D. We also want to add UI that allows user to set some parameters in the simulation. Furthermore, we plan to use the control module to design feedforward and feedback controller to make JWST transfer from LEO to the Halo Orbit. We will design feedback controller to stabilize the JWST motion on the Halo Orbit, which is intrinsically unstable.

References

[1] Ben Weber. Circular restricted three-body problem. https://orbital-mechanics.space/the-n-body-problem/circular-restricted-three-body-problem.html. Accessed: 2025-10-23.

[2] J. D. Giorgini. Three-body periodic orbits. https://ssd.jpl.nasa.gov/tools/periodic_orbits.html#/periodic. Accessed: 2025-10-26.