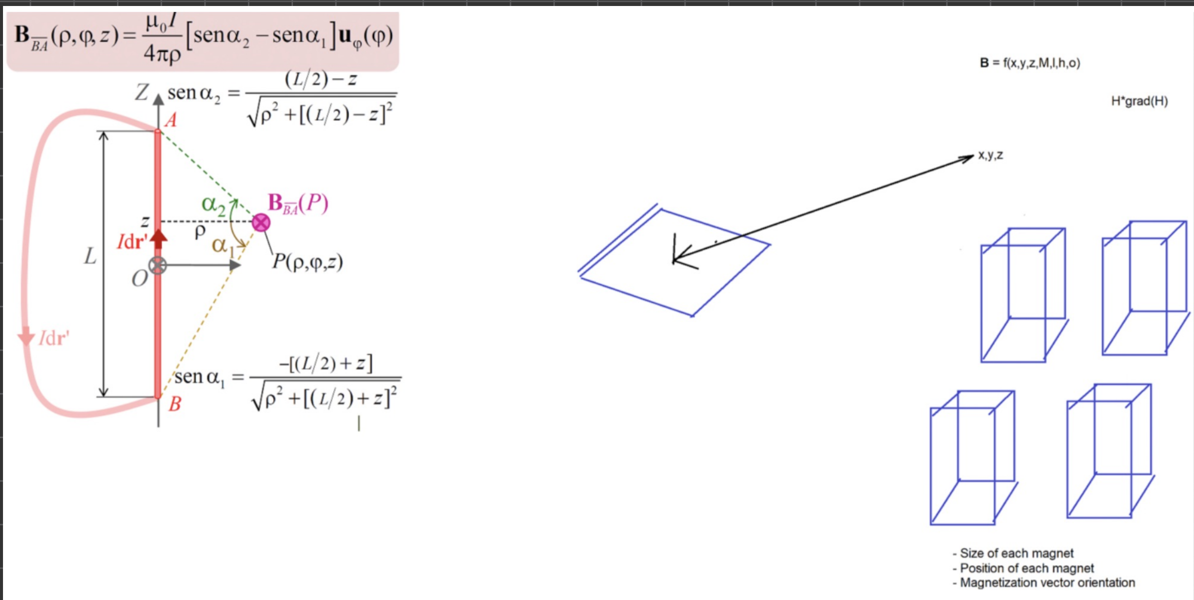
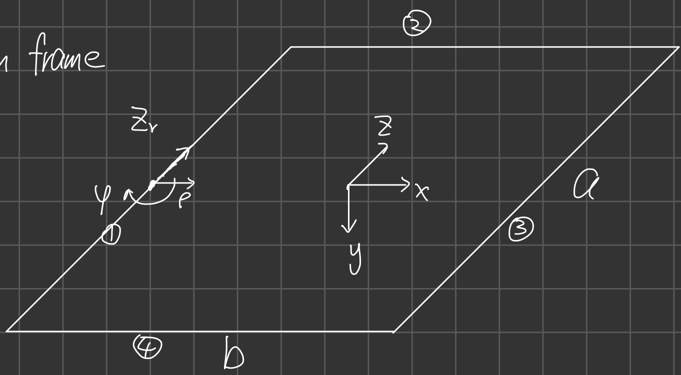
Compute B field produced by an array of cubic magnets located next to each other and with variable size, magnetization, magnetization direction, and location.



Change coordinates from cylindrical frame to Cartesian frame



$$x = \rho\cos\varphi \qquad y = \rho\sin\varphi \qquad z = z_r$$

$$\begin{cases} \rho^2 = x^2+y^2 \\ \tan\varphi = \frac{y}{x} \\ z_r = z \end{cases}$$

$$\sin\alpha_2 = \frac{\frac{L}{2}-z}{\sqrt{x^2+y^2+(\frac{L}{2}-z)^2}}$$

$$\sin\alpha_1 = \frac{-(\frac{L}{2}+z)}{\sqrt{x^2+y^2+(\frac{L}{2}+z)^2}}$$

Transfer $B_{\overline{BA}}(\rho,\varphi,z_r)$ to $B_{\overline{BA}}(x,y,z)$,

$$B_{\overline{BA}}(x,y,z) = \frac{\mu_0 I}{4\pi\sqrt{x^2+y^2}}(\sin\alpha_2 - \sin\alpha_1)\,\hat{\varphi}$$

$$\hat{\varphi} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$$

$$\hat{\varphi} = -\sin(\tan^{-1}(\tfrac{y}{x}))\,\hat{x} + \cos(\tan^{-1}(\tfrac{y}{x}))\,\hat{y}$$

$$\Rightarrow B_{\overline{BA}}(x,y,z) = \frac{\mu_0 I}{4\pi\sqrt{x^2+y^2}}(\sin\alpha_2 - \sin\alpha_1)(-\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y})$$

Shift the origin $\frac{b}{2}$ along x-direction. Replace all x by $x - x_0$, where x_0 is how much the original origin shifted along the x-direction

Generalized Solution of $\bar{B}$ around a segment in Cartesian Coordinate system with shifted origin along x-direction:

$$\begin{cases} B_{BA}(x,y,z) = \dfrac{\mu_0 I}{4\pi\sqrt{(x-x_0)^2+y^2}} (\sin\alpha_2 - \sin\alpha_1)\left[-\sin\left(\tan^{-1}\dfrac{y}{x-x_0}\right)\hat{x} + \cos\left(\tan^{-1}\dfrac{y}{x-x_0}\right)\hat{y}\right] \\ \sin\alpha_2 = \dfrac{\frac{L}{2}-z}{\sqrt{(x-x_0)^2+y^2+(\frac{L}{2}-z)^2}} \qquad \sin\alpha_1 = \dfrac{-(\frac{L}{2}+z)}{\sqrt{(x-x_0)^2+y^2+(\frac{L}{2}+z)^2}} \end{cases}$$

x_0 : how much the origin shifted along the x-direction

L : Segment length.

For Segment ①, $x_0 = \frac{b}{2}$, $L = a$

① $$\begin{cases} \bar{B}_{①}(x,y,z) = \dfrac{\mu_0 I}{4\pi\sqrt{(x-\frac{b}{2})^2+y^2}} (\sin\alpha_{①2} - \sin\alpha_{①1})\left[-\sin\left(\tan^{-1}\left(\dfrac{y}{x-\frac{b}{2}}\right)\right)\hat{x} + \cos\left(\tan^{-1}\left(\dfrac{y}{x-\frac{b}{2}}\right)\right)\hat{y}\right] \\ \text{where} \quad \sin\alpha_{①2} = \dfrac{\frac{a}{2}-z}{\sqrt{(x-\frac{b}{2})^2+y^2+(\frac{a}{2}-z)^2}}, \quad \sin\alpha_{①1} = \dfrac{-(\frac{a}{2}+z)}{\sqrt{(x-\frac{b}{2})^2+y^2+(\frac{a}{2}+z)^2}} \end{cases}$$

For Segment ②, $x_0 = \frac{a}{2}$, $L = b$; Replace x by $-z$, y by y, z by x
$\hat{x}$ by $-\hat{z}$, $\hat{y}$ by $\hat{y}$, $\hat{z}$ by $\hat{x}$

② $$\begin{cases} \bar{B}_{②}(x,y,z) = \dfrac{\mu_0 I}{4\pi\sqrt{(-z-\frac{a}{2})^2+y^2}} (\sin\alpha_{②2} - \sin\alpha_{②1})\left[-\sin\left(\tan^{-1}\left(\dfrac{y}{-z-\frac{a}{2}}\right)\right)(-\hat{z}) + \cos\left(\tan^{-1}\left(\dfrac{y}{-z-\frac{a}{2}}\right)\right)\hat{y}\right] \\ \text{where} \quad \sin\alpha_{②2} = \dfrac{\frac{b}{2}-x}{\sqrt{(-z-\frac{a}{2})^2+y^2+(\frac{b}{2}-x)^2}}, \quad \sin\alpha_{②1} = \dfrac{-(\frac{b}{2}+x)}{\sqrt{(-z-\frac{a}{2})^2+y^2+(\frac{b}{2}+x)^2}} \end{cases}$$

For segment ③, $x_0 = \frac{b}{2}$, $L = a$, Replace x by $-x$, y by y, z by $-z$
$\hat{x}$ by $-\hat{x}$, $\hat{y}$ by $\hat{y}$, $\hat{z}$ by $-\hat{z}$

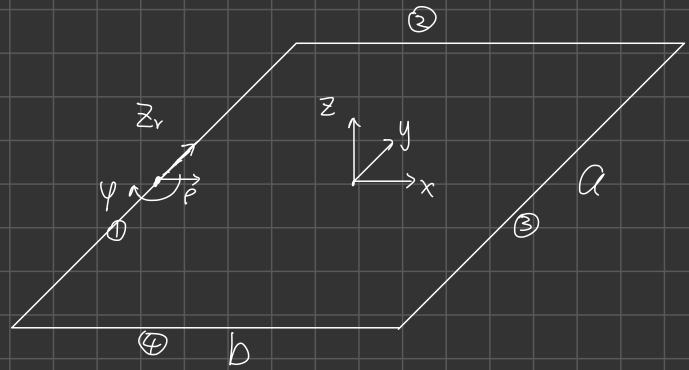
③ $$\begin{cases} \bar{B}_{③}(x,y,z) = \dfrac{\mu_0 I}{4\pi\sqrt{(-x-\frac{b}{2})^2+y^2}} (\sin\alpha_{③2} - \sin\alpha_{③1})\left[-\sin\left(\tan^{-1}\dfrac{y}{-x-\frac{b}{2}}\right)(-\hat{x}) + \cos\left(\tan^{-1}\dfrac{y}{-x-\frac{b}{2}}\right)\hat{y}\right] \\ \text{where} \quad \sin\alpha_{③2} = \dfrac{\frac{a}{2}+z}{\sqrt{(-x-\frac{b}{2})^2+y^2+(\frac{a}{2}+z)^2}}, \quad \sin\alpha_{③1} = \dfrac{-(\frac{a}{2}-z)}{\sqrt{(-x-\frac{b}{2})^2+y^2+(\frac{a}{2}-z)^2}} \end{cases}$$

For segment ④, $x_0 = \frac{a}{2}$, $L = b$. Replace x by z, y by y, z by $-x$; $\hat{x}$ by $\hat{z}$, $\hat{y}$ by $\hat{y}$, $\hat{z}$ by $-\hat{x}$

④
$$\bar{B}_{\textcircled{4}}(x,y,z) = \frac{\mu_0 I}{4\pi\sqrt{(z-\frac{a}{2})^2+y^2}}\left(\sin\alpha_{\textcircled{4}2} - \sin\alpha_{\textcircled{4}1}\right)\left[-\sin\left(\tan^{-1}\left(\frac{y}{z-\frac{a}{2}}\right)\right)\hat{z} + \cos\left(\tan^{-1}\left(\frac{y}{z-\frac{a}{2}}\right)\right)\hat{y}\right]$$

where
$$\sin\alpha_{\textcircled{4}2} = \frac{\frac{b}{2}+x}{\sqrt{(z-\frac{a}{2})^2+y^2+(\frac{b}{2}+x)^2}}, \quad \sin\alpha_{\textcircled{4}1} = \frac{-(\frac{b}{2}-x)}{\sqrt{(z-\frac{a}{2})^2+y^2+(\frac{b}{2}-x)^2}}$$

$$\bar{B}_{rect}(x,y,z) = \sum_{i=1}^{4} \bar{B}_{\textcircled{i}}(x,y,z) \qquad ⑤$$



Now, Replace all z by y, y by $-z$; $\hat{z}$ by $\hat{y}$, $\hat{y}$ by $-\hat{z}$

in ⑤ to meet convention

$$\Rightarrow \quad \bar{B}_{rect}(x,y,z) = \bar{B}_{\textcircled{1}}(x,y,z) + \bar{B}_{\textcircled{2}}(x,y,z) + \bar{B}_{\textcircled{3}}(x,y,z) + \bar{B}_{\textcircled{4}}(x,y,z)$$

$$= \frac{\mu_0 I}{4\pi\sqrt{(x-\frac{b}{2})^2+(-z)^2}}\left(\sin\alpha_{\textcircled{1}2} - \sin\alpha_{\textcircled{1}1}\right)\left[-\sin\left(\tan^{-1}\left(\frac{-z}{x-\frac{b}{2}}\right)\right)\hat{x} + \cos\left(\tan^{-1}\left(\frac{-z}{x-\frac{b}{2}}\right)\right)(-\hat{z})\right]$$

$$+ \frac{\mu_0 I}{4\pi\sqrt{(-y-\frac{a}{2})^2+(-z)^2}}\left(\sin\alpha_{\textcircled{2}2} - \sin\alpha_{\textcircled{2}1}\right)\left[-\sin\left(\tan^{-1}\left(\frac{-z}{-y-\frac{a}{2}}\right)\right)(-\hat{y}) + \cos\left(\tan^{-1}\left(\frac{-z}{-y-\frac{a}{2}}\right)\right)(-\hat{z})\right]$$

$$+ \frac{\mu_0 I}{4\pi\sqrt{(-x-\frac{b}{2})^2+(-z)^2}}\left(\sin\alpha_{\textcircled{3}2} - \sin\alpha_{\textcircled{3}1}\right)\left[-\sin\left(\tan^{-1}\frac{-z}{-x-\frac{b}{2}}\right)(-\hat{x}) + \cos\left(\tan^{-1}\frac{-z}{-x-\frac{b}{2}}\right)(-\hat{z})\right]$$

$$+ \frac{\mu_0 I}{4\pi\sqrt{(y-\frac{a}{2})^2+(-z)^2}}\left(\sin\alpha_{\textcircled{4}2} - \sin\alpha_{\textcircled{4}1}\right)\left[-\sin\left(\tan^{-1}\left(\frac{-z}{y-\frac{a}{2}}\right)\right)\hat{y} + \cos\left(\tan^{-1}\left(\frac{-z}{y-\frac{a}{2}}\right)\right)(-\hat{z})\right]$$

where
$$\sin\alpha_{\textcircled{1}2} = \frac{\frac{a}{2}-y}{\sqrt{(x-\frac{b}{2})^2+(-z)^2+(\frac{a}{2}-y)^2}}, \quad \sin\alpha_{\textcircled{1}1} = \frac{-(\frac{a}{2}+y)}{\sqrt{(x-\frac{b}{2})^2+(-z)^2+(\frac{a}{2}+y)^2}}$$

$$\sin\alpha_{\textcircled{2}2} = \frac{\frac{b}{2}-x}{\sqrt{(-y-\frac{a}{2})^2+(-z)^2+(\frac{b}{2}-x)^2}}, \quad \sin\alpha_{\textcircled{2}1} = \frac{-(\frac{b}{2}+x)}{\sqrt{(-y-\frac{a}{2})^2+(-z)^2+(\frac{b}{2}+x)^2}}$$

$$\sin\alpha_{\textcircled{3}2} = \frac{\frac{a}{2}+y}{\sqrt{(-x-\frac{b}{2})^2+(-z)^2+(\frac{a}{2}+y)^2}}, \quad \sin\alpha_{\textcircled{3}1} = \frac{-(\frac{a}{2}-y)}{\sqrt{(-x-\frac{b}{2})^2+(-z)^2+(\frac{a}{2}-y)^2}}$$

$$\sin\alpha_{\textcircled{4}2} = \frac{\frac{b}{2}+x}{\sqrt{(y-\frac{a}{2})^2+(-z)^2+(\frac{b}{2}+x)^2}}, \quad \sin\alpha_{\textcircled{4}1} = \frac{-(\frac{b}{2}-x)}{\sqrt{(y-\frac{a}{2})^2+(-z)^2+(\frac{b}{2}-x)^2}}$$