DARPA Zenith

Georgia Tech Update: Mid-WFE Analysis

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LGST Lab

Contracting Status

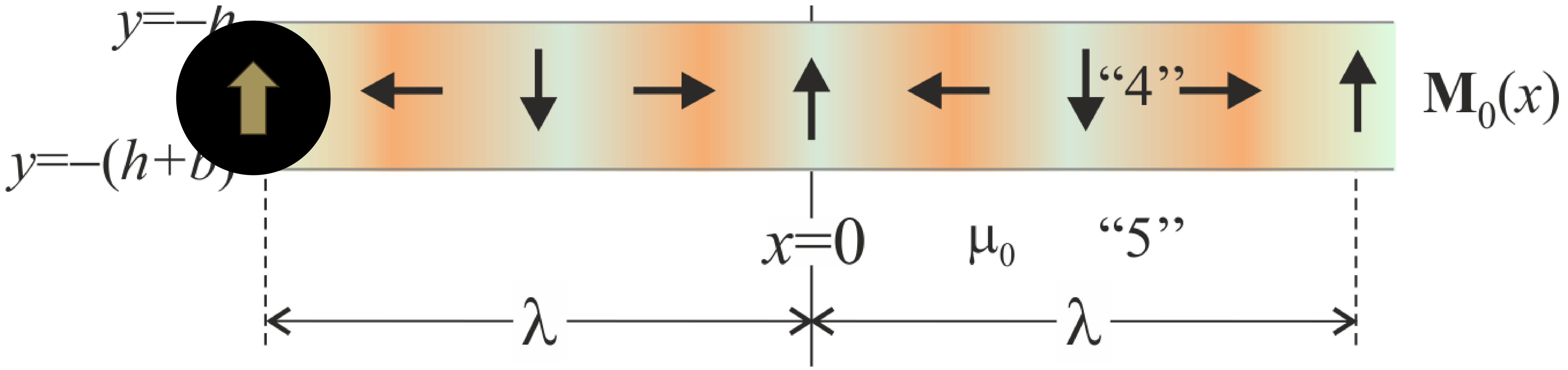
- GT account was released this morning
- Already ordered 3D printer and ferrofluid
- Will soon order ~30 K&J N52 magnets for error characterization
- First 50k only covers our students for 2 months: need to sign full contract ASAP.

Magnetic Fields and Forces on Saturated Ferrofluid Layer

Task 1: Mid-Wavefront-Error Analysis

Mallinson's Magnetic Configuration

Wave number: $k = 2\pi/\lambda$
Magnetization: M_0



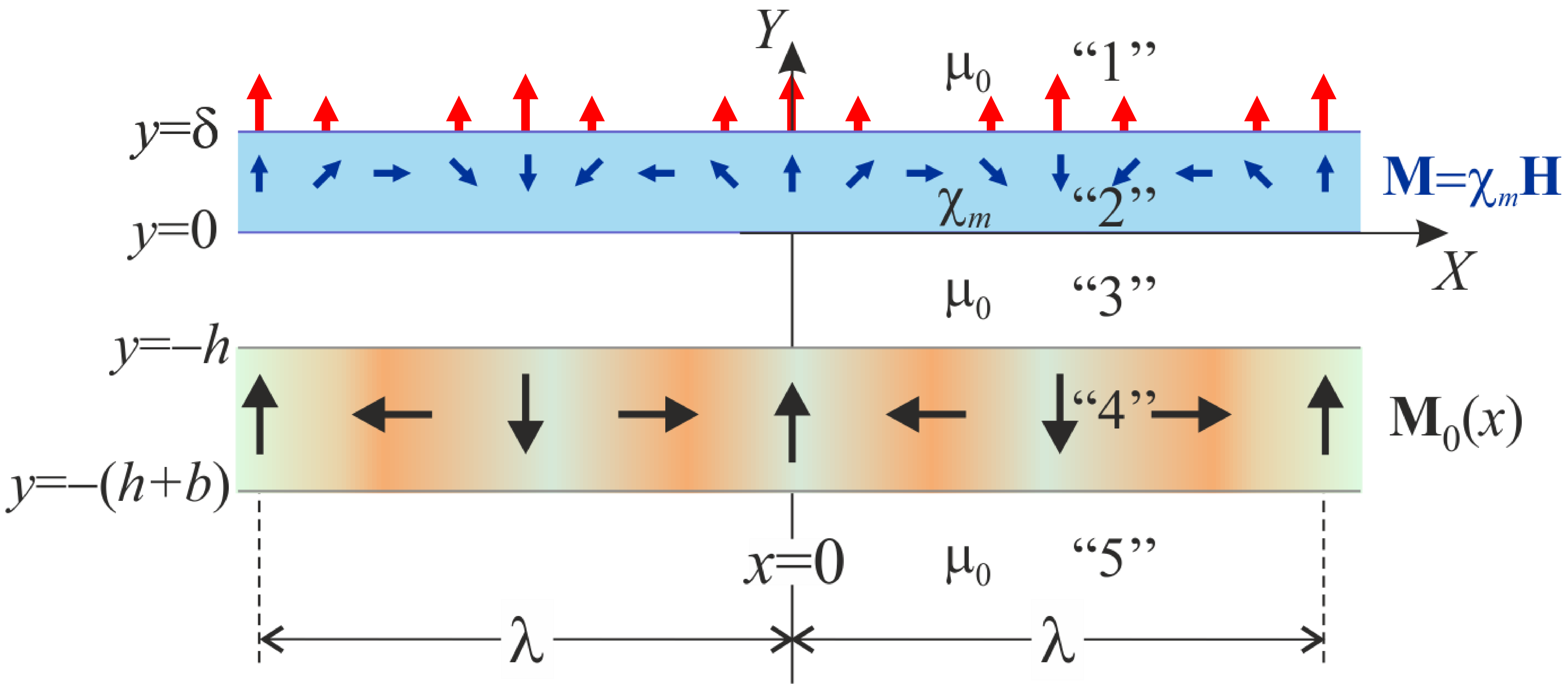
$$\boldsymbol{M}_0 = M_0\left[-\sin(kx)\,\boldsymbol{u}_x + \cos(kx)\,\boldsymbol{u}_y\right]$$

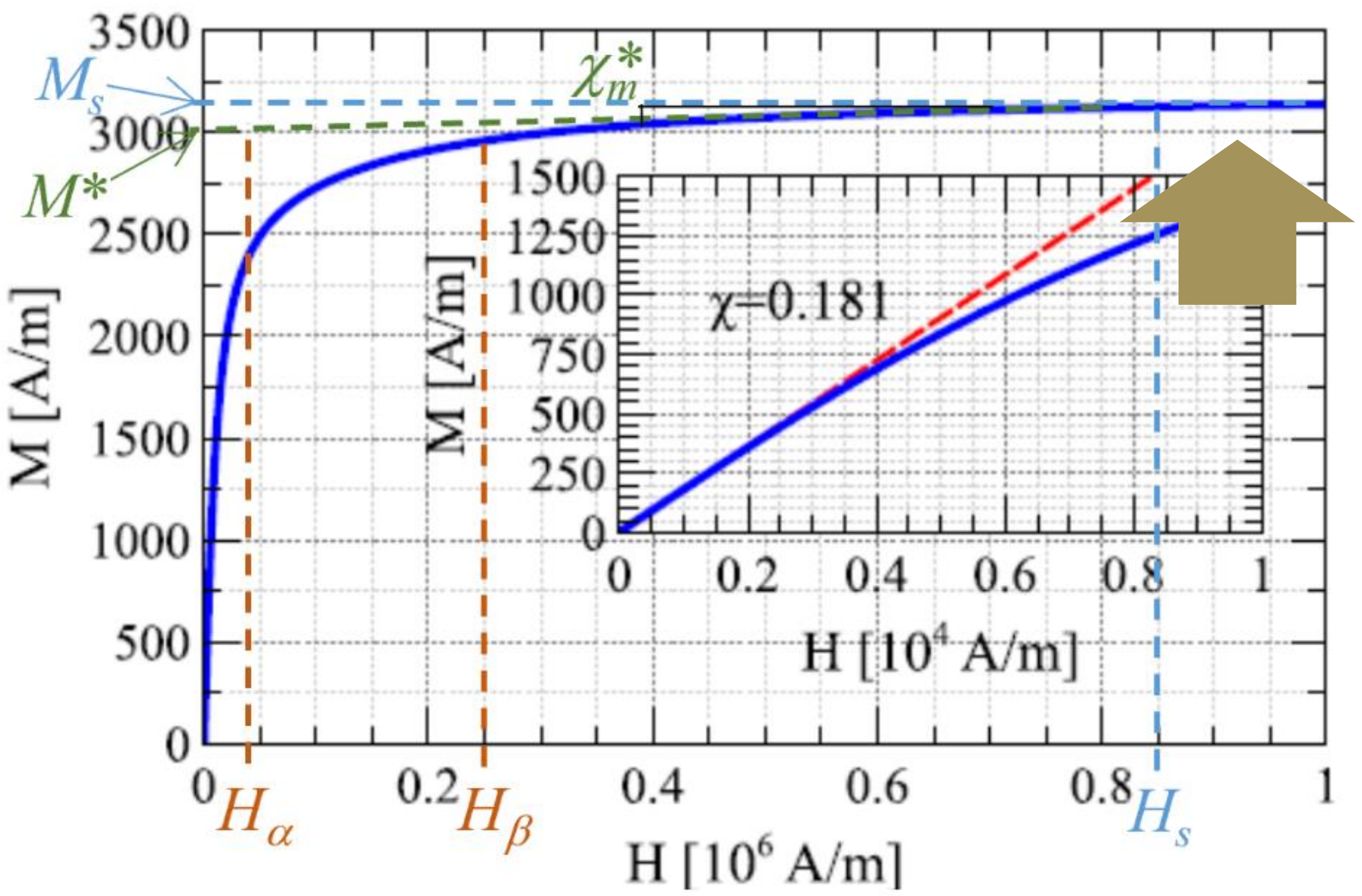
Magnetic Field

$$\boldsymbol{H}_0(x, y > -h) = e^{-k(y+h)} M_0 (1 - e^{-kb}) \left[\sin(kx)\,\boldsymbol{u}_x + \cos(kx)\,\boldsymbol{u}_y\right]$$

$$\|\boldsymbol{H}_0(x,y)\| = H_0 e^{-ky}, \text{ with } H_0 = M_0 e^{-kh}(1 - e^{-kb})$$

Adding a Ferrofluid Layer





Constitutive Equation: $\boldsymbol{M} = \chi(H)\boldsymbol{H}$

Strong force → FF Saturation

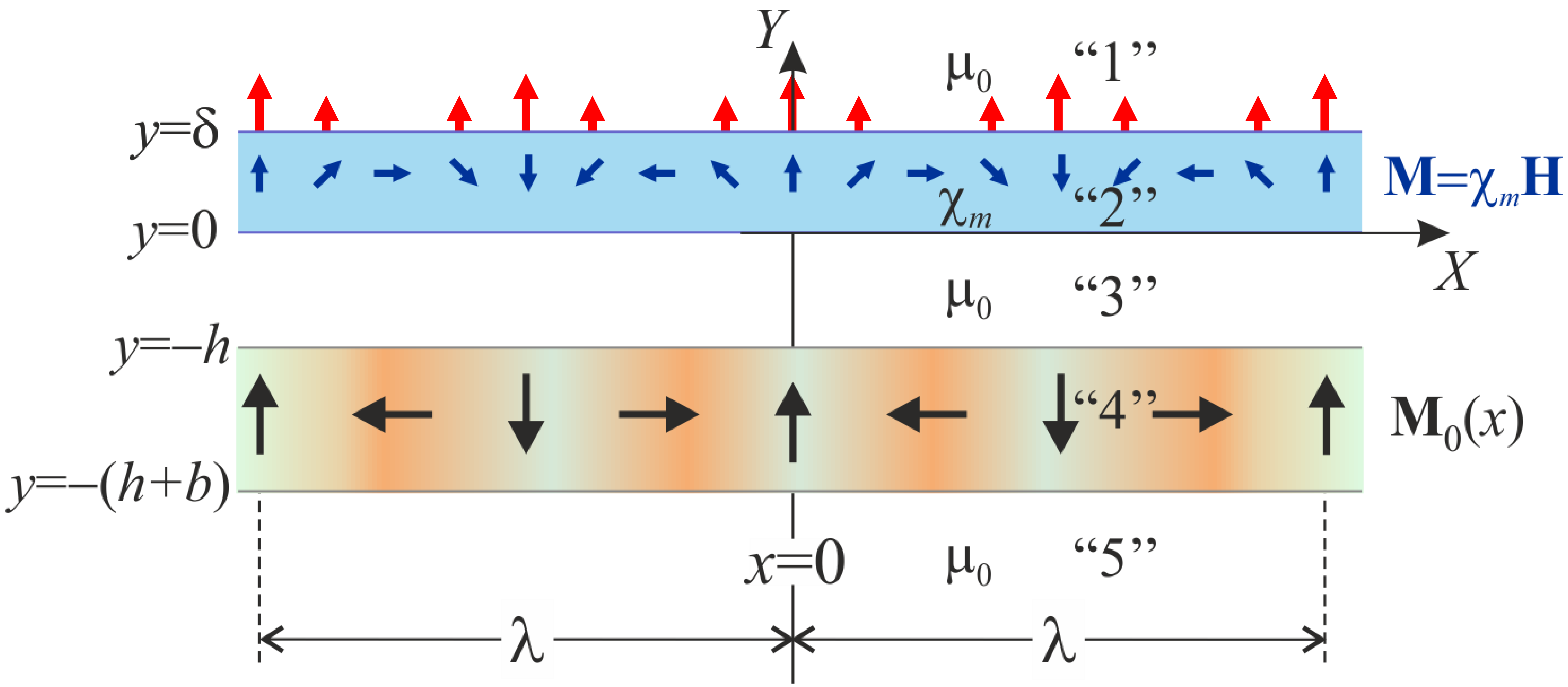
$$\boldsymbol{M}^{sat}(x, 0 < y < \delta) = M_s \frac{\boldsymbol{H}(x,y)}{H(x,y)}$$

Magnetic forces

$$\boldsymbol{f}_m^V = \mu_0 M \nabla H, \qquad \boldsymbol{f}_m^S = \frac{\mu_0}{2}[M_n^2]\boldsymbol{n}$$

Volume term **Surface term**

Magnetic Fields on the Saturated Layer



Total magnetic field

$$\boldsymbol{H}^{sat}(x, 0<y<\delta) = \boldsymbol{H}_0(x,y) + \boldsymbol{H}_d^{sat}(x,y)$$

External magnetic field

$$H^{x\,(\mathrm{sat})}_{\text{"2"}} = \left\{ H_0 e^{-ky} + M_s \left[e^{k(y-\delta)} - 1 \right] \right\} \sin(kx)$$

$$H^{y\,(\mathrm{sat})}_{\text{"2"}} = \left[H_0 e^{-ky} - M_s e^{k(y-\delta)} \right] \cos(kx)$$

Demagnetization field

$$\boldsymbol{H}_d^{sat}(x, 0<y<\delta)$$
$$\approx M_s e^{k(y-\delta)}\left[\sin(kx)\,\boldsymbol{u}_x - \cos(kx)\,\boldsymbol{u}_y\right] - M_s \sin(kx)\,\boldsymbol{u}_x$$

How do they compare?

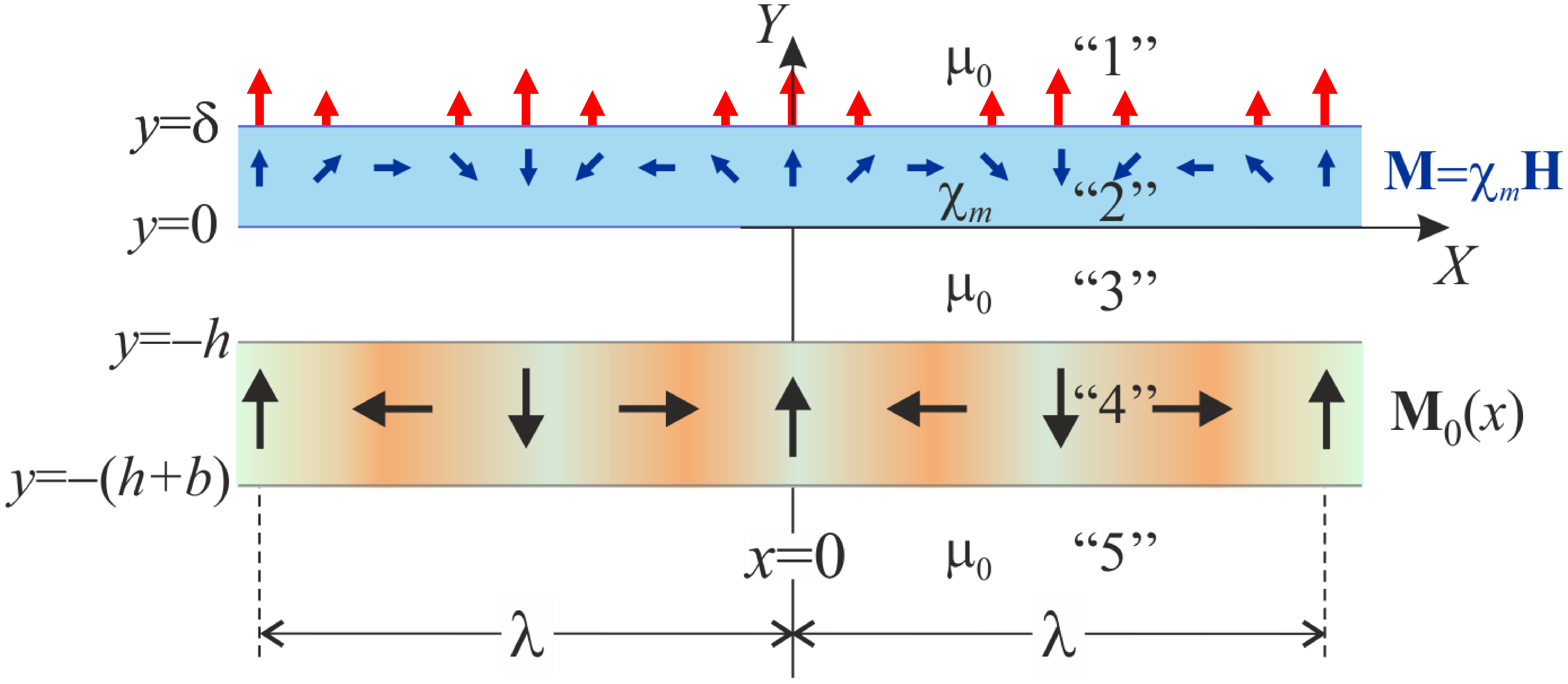
$$\boxed{\frac{|\boldsymbol{H}_d^{(\mathrm{sat})}(x,y)|}{|\boldsymbol{H}_0(x,y)|} < \frac{M_s}{H_0} e^{k\delta} = \frac{M_s}{M_0\,(1-e^{-kb})} e^{k(\delta+h)} = \xi}$$

<<1

Magnetization field:

$$\boldsymbol{M}^{(\mathrm{sat})}(x; 0<y<\delta) \approx M_s \left[\sin(kx)\,\boldsymbol{u}_x + \cos(kx)\,\boldsymbol{u}_y \right] +$$
$$+\xi \frac{M_s}{2} e^{k(y-\delta)} \left[2 e^{k(y-\delta)} - 1 \right] \sin(2kx) \left[\cos(kx)\,\boldsymbol{u}_x - \sin(kx)\,\boldsymbol{u}_y \right]$$

Magnetic Forces on the Saturated Layer



Magnetic forces

$$\boldsymbol{f}_m^V = \mu_0 M \nabla H, \qquad \boldsymbol{f}_m^S = \frac{\mu_0}{2}\left[M_n^2\right]\boldsymbol{n}$$

Volume term **Surface term**

Volume magnetic force

$$\boldsymbol{f}_m^V = \mu_0 M_s \nabla H^{sat} = f_x^{V,sat}\boldsymbol{u}_x + f_y^{V,sat}\boldsymbol{u}_y$$

$$f_x^{V,sat} \approx \mu_0 k M_s^2\left[2e^{k(y-\delta)} - 1\right]\sin(2kx)$$

$$f_y^{V,sat} \approx -\mu_0 k M_s H_0 e^{-ky}\left[1 + \xi e^{2k(y-\delta)}\cos(2kx)\right]$$

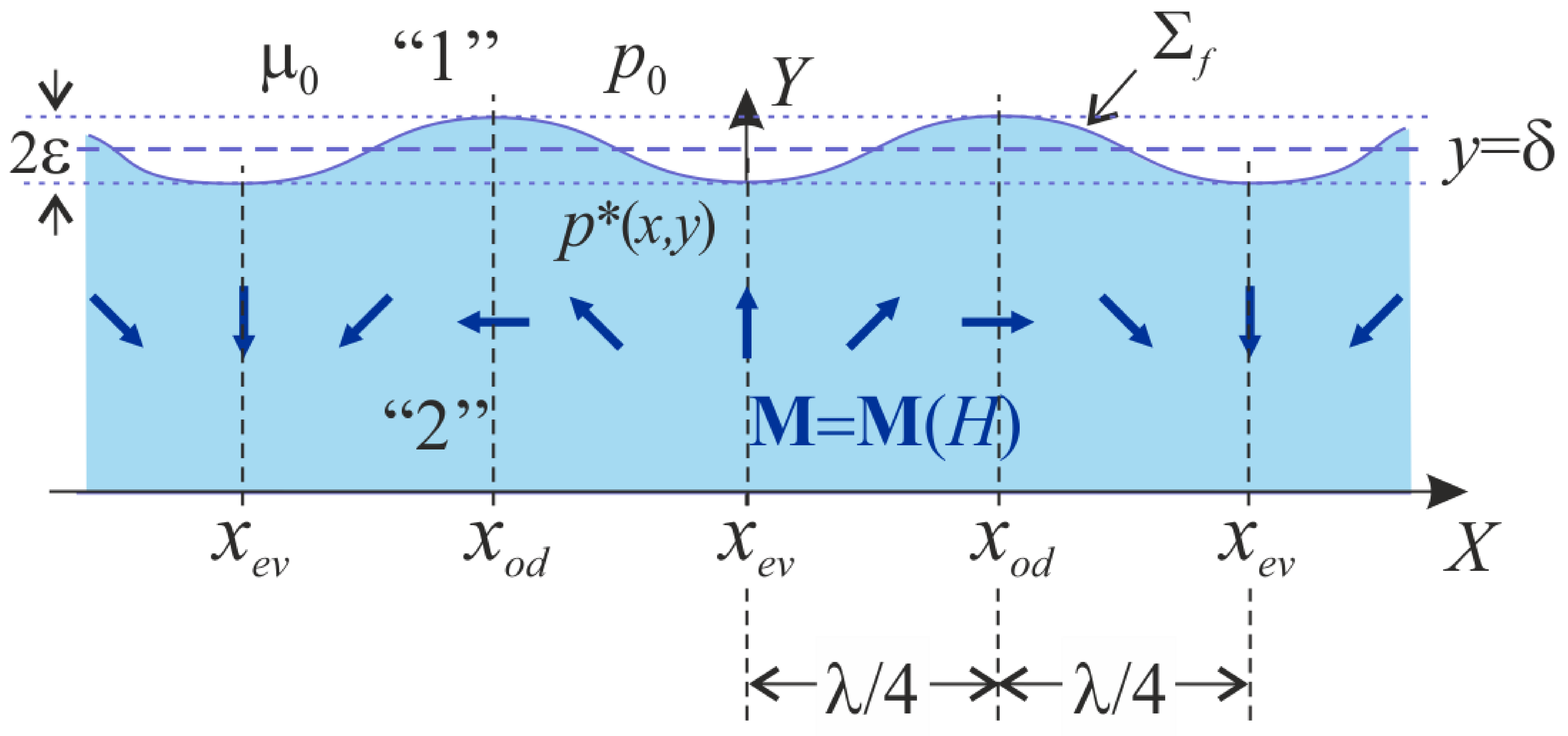
Surface magnetic force

$$\boldsymbol{f}_m^{S,sat}(x, y=\delta) = \frac{\mu_0}{2}\left[\boldsymbol{M}_y^{sat}(y=\delta)\right]^2\boldsymbol{u}_y \approx \frac{\mu_0}{2}M_s^2\cos^2(kx)\left[1 - 2\xi\sin^2(kx)\right]$$

Free Surface Perturbations on Saturated Ferrofluid Layer

Task 1: Mid-Wavefront-Error Analysis

Canceling mid-frequency WFE



Momentum balance: FHD Bernoulli Eq.

$$p^*(x,y) + \rho g y - \Pi_m(x,y) = \Pi$$

Establishes a relation between pressure, layer height, and magnetic forces. **Studying what happens at x_{od} and x_{ev}, we can estimate the surface roughness ϵ**

After multiple pages of math and assuming $\xi = M_s e^{k\delta}/H_0 \ll 1$ (i.e. large force conditions)…

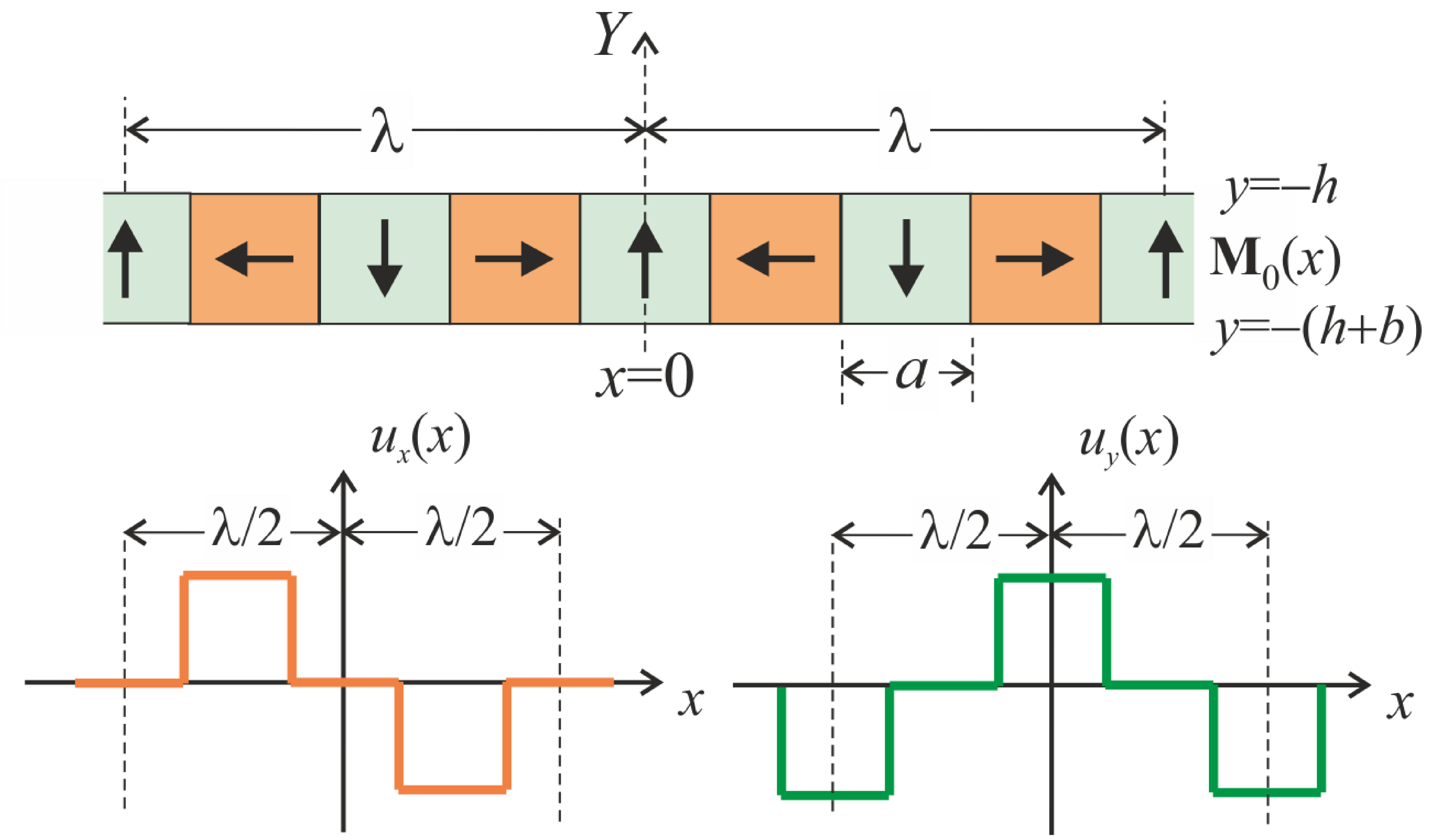
$$k\epsilon \approx \frac{\xi}{4}\left[1 - \frac{4\sigma k + \rho g/k}{\mu_0 M_s H_0 e^{-k\delta}}\right]$$

Roughness at the surface
for the Mallinson Configuration

Halbach Arrays

Task 1: Mid-Wavefront-Error Analysis

The Halbach Discretization



Magnetization pattern

$$\boldsymbol{M}_0 = M_0 \left[f(x)\, \boldsymbol{u}_x + g(x)\, \boldsymbol{u}_y \right]$$

$$\begin{array}{lll} f(x) = 0, & g(x) = 1, & \text{if } \; 0 < x < \lambda/8 \\ f(x) = -1, & g(x) = 0, & \text{if } \; \lambda/8 < x < 3\lambda/8 \\ f(x) = 0, & g(x) = -1, & \text{if } \; 3\lambda/8 < x < \lambda/2 \end{array}$$

Fourier Transform

$$\boldsymbol{M}_0 = \frac{2\sqrt{2}\, M_0}{\pi} \sum_{i=1}^{\infty} \frac{1}{(2i-1)}\, \boldsymbol{m}_{2i-1}(x)$$

$$\boldsymbol{m}_n(x) = \begin{cases} (-1)^{\frac{n+3}{4}} \left[\sin(k_n x)\, \boldsymbol{u}_x - \cos(k_n x)\, \boldsymbol{u}_y \right], & \text{if } \; n = 1, 5, 9, \cdot\cdot \\ (-1)^{\frac{n+5}{4}} \left[\sin(k_n x)\, \boldsymbol{u}_x + \cos(k_n x)\, \boldsymbol{u}_y \right], & \text{if } \; n = 3, 7, 11, \cdot \end{cases}$$

Main concern:

- Cross-harmonics 1-5 and 1-9
- Disturbances are 4 order or magnitude below previous effect
- More updates soon…