APG4001S Geoid Computation from spherical harmonic coefficients

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Assignment 1

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1 Introduction

1.1 Subject of Report

This is a report on Assignment 1 for APG4001S Geodesy. The aim of the report was to calculate orthometric heights for each of the 5 stations provided through the use of spherical harmonic coefficients.

1.2 Background

Using the GGM02S Grace derived geopotential model, to degree/order 160, and the coordinates of 5 Trignet stations. As well as an exert from the "Geodesists Handbook" which provides the constants required for the GRS80 ellipsoid.

A program is written to calculate the Geoid-ellipsoid (N) values at each station using the geopotential model provided. Using the N value calculate the orthometric height (H) at each station.

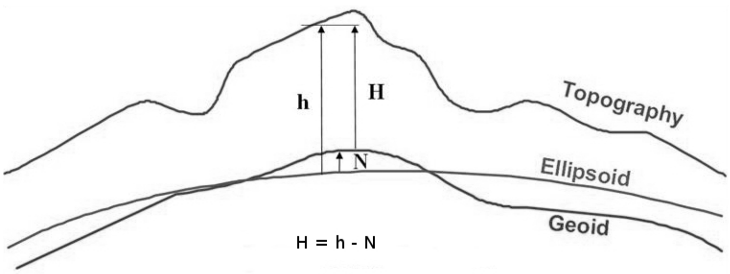

Figure 1: Relationship between ellipsoidal height, Geoidal height and orthometric height

The 5 Trignet stations used are:

Station Name	Latitude	Longitude	Ellipsoidal Height
HNUS	34 25 28.6671 S	19 13 23.0264 E	63.048
PRET	25 43 55.2935 S	28 16 57.4873 E	1387.339
RBAY	28 47 43.9616 S	32 04 42.1896 E	31.752
TDOU	23 04 47.6714 S	30 23 02.4297 E	630.217
ULDI	28 17 35.2196 S	31 25 15.3309 E	607.947

1.3 Objectives

The primary objective of this assignment was to create a program to calculate the geoid-ellipsoid values at each station using the provided global geopotential model, once these values are calculated they are used to calculate the orthometric height at each station.

1.4 Scope and Limitations

This assignment was done using the GSM80 ellipsoid and the GGM02S Grace gopotential model to degree/order 160.

A substantial limitation to the program was that Python 3.4 has a size limit on floating point numbers and the code cannot be run for all 160 coefficients without getting an overflow error, so it is only run for 85.

2 Method

The program has a reasonably simple structure, the GGM02S file is read in and saved using a class structure in Python 3.4. and is accessible be using a simple query of what you want fed into a function. for example if you want S with n = 57 m = 3 you feed in the query string "S 27 16" and it will return the value 0.39484545087973E-08.

Then for each station a Gamma value for the specific longitude is calculated using the formula:

$$\gamma = \gamma_e \frac{n1 + k\sin^2(\Phi)}{\sqrt{1 - e^2\sin^2(\Phi)}} \tag{1}$$

Where γ_e is the normal gravity at the equator

$$N(r, \theta, \lambda) = \frac{GM}{\gamma r} \sum_{n=2}^{\infty} \binom{a}{r} \sum_{m=0}^{n} [(\Delta C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\cos\theta)] \tag{2}$$

Them once the γ has been calculated, N is calculated in parts. by deconstructing equation (2) with the inner sum, outer sum and the outermost dividend calculated almost separately and brought together when returned from the function.

The inner sum which runs inside a loop that runs from 0 to n

$$InnerSum = \sum_{m=0}^{n} [(\Delta C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\cos\theta)] \tag{3}$$

In this equation P_{nm} is equal to:

$$P_n(t) = \frac{1}{2^n n!} . \frac{d^n}{dt^n} (t^n - 1)^n \tag{4}$$

Where the value $t = \cos\theta$

Is then added to the outer sum value and run in a loop from 0 to a specified value, 160.

$$OuterSum = \sum_{n=2}^{\infty} \binom{a}{r} \times InnerSum \tag{5}$$

Them finally we add the values that are outside of the outer sum:

$$N(r, \theta, \lambda) = \frac{GM}{\gamma r} \times OuterSum \tag{6}$$

This gives the final value of N.

Then with this final value of N we calculate the orthometric height with the simple calculation of:

Orthometric Height (H) = Ellipsoidal Height (h) - N

3 Results

The results of calculating N after 85 iterations, n = 85 are as follows:

Station Name	Orthometric Height	Geoidal Height
HNUS	29.638	33.410
PRET	1365.770	21.569
TDOU	611.833	18.384
ULDI	586.555	21.392
RBAY	10.520	21.232

These heights are displayed to 3 decimal places, however the program does calculate to around 10 decimal places so the results are rounded of to a usable and more readable size.

As for only doing 85 iterations the later harmonics are making such a small impact of the heights that when rounding off to 3 decimal places no difference will be noticed.