

7.1)

a) $|M| = 5$

$$b) \text{Informantik} = I, \text{macht} = m, \text{Spaß} = s, \text{ist} = i, \text{super} = s$$

$$\text{Pot}(M) = 2^M = \{\emptyset, \{I\}, \{m\}, \{s\}, \{i\}, \{s\}, \{I, m\}, \{I, s\}, \{I, i\}, \{I, s\}, \{m, s\}, \{m, i\}, \{m, s\}, \{s, i\}, \{s, s\}, \{i, s\}, \{I, m, s\}, \{I, m, i\}, \{I, m, s\}, \{I, s, i\}, \{I, s, s\}, \{I, i, s\}, \{m, s, i\}, \{m, s, s\}, \{m, i, s\}, \{s, i, s\}, \{I, m, s, i\}, \{I, m, s, s\}, \{I, m, i, s\}, \{I, s, i, s\}, \{m, s, i, s\}, \{I, m, s, i, s\}\}$$

c) $|2^M| = 2^{|M|} = 32$

d) $(2^M)^N = \{\{\emptyset, \{I\}\}, \{\{I\}, \{m\}\}, \dots\}$

e) $M = \{1, 2, 3\}$

$$N \times M = \{(i, j) \mid i \in N, j \in M\}$$

7.2) (a) nein, da $(x, x) \in M \notin R$

(b) nein

(c) $R = \{(1, s), (s, 4), (2, 1), (3, 1), (s, 3)\}$

$$R^2 = \{(1, 4), (2, s), (3, s), (1, 3), (s, 1)\}$$

$$R^3 = \{(1, 1), (2, 4), (2, 3), (3, 3), (3, 4), (s, s)\}$$

$$R^4 = \{(1, s), (s, 4), (2, 1), (3, 1), (s, 3)\}$$

$$\rightarrow R^+ = \bigcup_{k \in \mathbb{N}} R^k = R \cup R^2 \cup R^3 \cup \dots$$

$$= R \cup R^2 \cup R^3$$

$$= \{(1, 1), (1, 3), (1, 4), (1, s), (2, 1), (2, 3), (2, 4), (2, s), (3, 1), (3, 3), (3, 4), (3, s), (s, 1), (s, 3), (s, 4), (s, s)\}$$

(d) $S = \{(2, s), (3, 2), (1, 3), (4, s)\}$

$$S^2 = \{(1, 2), (3, s)\}$$

$$S^3 = \{(1, s)\}$$

$$S^* = \bigcup_{k \in \mathbb{N}} S^k = S^0 \cup S^1 \cup S^2 \cup S^3 \cup \dots$$

$$= S^0 \cup S^1 \cup S^2 \cup S^3$$

$$= \{(1, 1), (1, 2), (1, 3), (1, s), (2, 2), (2, s), (3, 2), (3, 3), (3, s), (4, 4), (4, s), (s, s)\}$$

(e) $SR = \{(1, 1), (2, 3), (2, 4), (3, 1), (4, 3), (4, 4)\}$