ADJUSTMENT THEORY I

Exercise 8: Adjustment Calculation - part III

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```
clc; clear all; close all;
warning('off')
```

Task 2 - Copper:

Given Values:

```
a = 11.60; m = 15.15; a_s = 0.05; m_s = 0.05; p = 8.93/1000;
table([a,m]',[a_s,m_s]','VariableNames',...
    {'Li' 'sLi' },'RowNames',{'a [mm]' 'm [g]'})
```

ans = 2×2 table

	Li	sLi
1 a [mm]	11.6	0.05
2 m [g]	15.15	0.05

Functional model:

$V = a^3 \;\Rightarrow a = \sqrt[3]{V} \qquad \rightarrow \varphi_1$ V - unknown

$m = V \cdot p \;\Rightarrow m = V \cdot p \qquad \rightarrow \varphi_2$ a and m measurements

Discription of the data:

Our observations Parameter are the side lenght of a cube called 'a' and the mass of the same object 'm'.

We also have an error-free observation of the density of the cude 'p'.

The unknow parameter we want the calculate is the volume 'V'.

Linear or non-linear?

To determine if the problem is a linear or non-linear adjustment problem we have to look at the unkowns we want to calculate. Only they show if we need to solve the problem linear or non-linear. The other variables dosen't matter for this classification, because they are not the unkowns we want to determine.

In this case we see that our unknow V dosen't scale linear in the first equation $a = \sqrt[3]{V}$. That indicates that we need a non-linear solution to solve the problem. In other words, it need to be linearized.

Vector of observations:

```
L = [ a; m ];
```

Number of observations:

```
no_n = length (L);
```

Initial values for the unknowns:

```
V = a^3; % cm^3
```

Test for same units of measurement:

```
if (abs((V * p) - L(2))<L(2))
disp('Measurements seems to have the same units of measurement')
else
error("Measurements DON'T seems to have the same units of measurement")
end
```

```
Measurements seems to have the same units of measurement
```

Number of unknowns:

```
no_u = length(V);
```

Redundancy:

```
r = no_n - no_u
```

```
r =
     1
```

Stochastic model:

VC Matrix of the observations:

```
S_LL = diag([a_s,m_s])
```

```
S_LL = 2×2
    0.0500         0
         0    0.0500
```

Theoretical standard deviation:

```
sigma_0 = 1;    %a priori
```

Cofactor matrix of the observations:

```
Q_LL = (1/(sigma_0^2)) * S_LL;
```

Weight matrix:

```
P = inv(Q_LL);
```

Non- Linear adjustment problem:

```
%break-off conditions
epsilon = 10^-5;
delta = 10^-12;
max_x_hat = inf;
Check2 = inf;

%Number of iterations
iteration = 0;

while (max_x_hat > epsilon) || (Check2 > delta)

     % Observations as functions of the approximations for the unknowns
     L_0 (1) = nthroot(V,3); % a
     L_0 (2) = V * p; %m

     % Vector of reduced observations
     l = L - L_0';

     % Design matrix with the elements from the Jacobian matrix J
     A(1,1) = 1 / (3*V^(2/3));%a
     A(2,1) = p; %m

     % Normal matrix
     N = A'*P*A;

     % Vector of right hand side of normal equations
     n = A'*P*l;

     % Inversion of normal matrix / Cofactor matrix of the unknowns
     Q_xx = inv(N);

     % Solution of the normal equations
     x_hat = Q_xx*n;

     % Update
     V = V+x_hat;

     % Check 1
     max_x_hat = max(abs(x_hat));

     % Vector of residuals
     res = A*x_hat-l;

     % Vector of adjusted observations
     L_hat = L+res;

     % Objective function
     vTPv = res' * P * res;

     % Functional relationships without observations
     phi_X_hat(1) = nthroot(V,3);
```

```
    phi_X_hat(2) = V * p;

    % Check 2
    Check2 = max(abs(L_hat-phi_X_hat'));

    % Update number of iterations
    iteration = iteration+1;

end

if Check2<=delta
    disp ('Everything is fine!')
else
    disp ('Something is wrong.')
end
```

```
Everything is fine!
```

```
% Empirical reference standard deviation
s_0 = sqrt (vTPv/r);

% VC matrix of adjusted unknowns
S_XX_hat = s_0^2 * Q_xx;

% Standard deviation of the adjusted unknowns
s_X = sqrt(diag(S_XX_hat));

% Cofactor matrix of adjusted observations
Q_LL_hat = A*Q_xx*A';

% VC matrix of adjusted observations
S_LL_hat = s_0^2 * Q_LL_hat;

% Standard deviation of the adjusted observations
s_L_hat = sqrt (diag (S_LL_hat));

% Cofactor matrix of the residuals
Q_vv = Q_LL-Q_LL_hat;

%VC matrix of residuals
S_vv = s_0^2*Q_vv;

%Standard deviation of the residuals
s_v = sqrt (diag(S_vv));
```

Adjusted observations, their standart deviations and there Risduals:

```
format default
table(L,L_hat,s_L_hat,res,s_v,'VariableNames',...
{'L'...
'L_hat',...
's_L_hat',...
```

```
'v',...
's_v'}, ...
'RowNames',{'a [mm]' 'm [g]'})
```

ans = 2×5 table

	L	L_hat	s_L_hat	v	s_v
1 a [mm]	11.6000	11.9055	0.0805	0.3055	0.3055
2 m [g]	15.1500	15.0695	0.3055	-0.0805	0.0805

Adjusted unknow V, and his standart deviation:

```
format shortG
table(11.60^3,V,s_X,'VariableNames',...
{'X',...
'X_hat',...
's_X_hat'},...
'RowNames',{'V [Volume]'})
```

ans = 1×3 table

	X	X_hat	s_X_hat
1 V [Volume]	1560.9	1687.5	34.216