Introduction to Space Geodesy - Exercise

Assignment 2: Date, time, and time transformations

Exercises

1. Create a function that computes 3D-rotation matrices (equations see attachment). How many input and output arguments are required? Check the correctness of your function by testing its application, for example $R_i(2\pi) = R_i(0) = R_i(\pi)\, R_i(\pi) = I_3$, where I_3 is the (3x3) identity matrix and i equals 1, 2 or 3. Initialize a test vector $\boldsymbol{x} = \boldsymbol{e_1} = (1, 0, 0)^T$ and gradually rotate this vector to match the base vector $\boldsymbol{e_2}$, then $\boldsymbol{e_3}$ and finally $\boldsymbol{e_1}$. From the perspective at the origin, are the rotation matrices defined for clockwise or counterclockwise rotation when using positive angles?
2. Create a function that computes 3D-reflection matrices (equations see attachment). Check the correctness of the function by testing $M_i\, M_i = I_3$, $M_i\, M_j\, M_k = -\, I_3$ with I_3 as in exercise (1), $i, j, k \in \{1,2,3\}$ and $i \neq j \neq k$.
3. In the theory of satellite orbits, a common problem is the KEPLER equation: $M = E - e \sin E$. It relates the given mean anomaly M to the eccentric anomaly E knowing the eccentricity $e \in [0; 1[$ of the satellite orbit. For E, the equation has no closed solution. To compute E from a given M one has to iterate: $E_{i+1} = M + e \sin E_i$ e.g. with starting value $E_i = M$. Write a function file that iteratively computes E from given M and e with the stop criterion $|E_{i+1} - E_i| < 10^{-6}$. Next, extend the number of output arguments so that the function returns the number of iterations as well. Which variable could be introduced to the list of input arguments additionally? For testing the performance, evaluate your function as follows: create a [M;e]-grid with $M \in [0; 2\pi]$ and $e \in [0; 1[$ with resolution 0.01 for M and 0.001 for e. Then compute the difference between the eccentric and mean anomalies $E - M$ and the number of iterations for each grid point. Plot the two quantities over the grid with `mesh` each into an own figure window. Inspect your plots and describe where the anomalies differ at most and at least and explain where the solution needs the most iterations.
4. Create a function that converts a vector of angles with unit decimal degree into vectors of degree, minute, decimal second. (The `fix` command might be useful in this context.) Can you use this function to convert angles given in hour angle decimal to hour, minute and second decimal as well? If so, what would have to be changed? Create another function that does the back conversion from vectors of degree, minute, decimal second to a vector of angles with unit decimal degree. Find a solution for negative angles as well.
5. Write a function that provides the Julian Date and Modified Julian Date when inputting a Gregorian calendar date in year, month, day, hour, minute and second. Second with decimals.
(a) Determine the Earth Rotation Angle (ERA) at 2021-11-22, 01^h:00^m:00^s CET and specify it in degree, minute and second to second precision.

(b) Determine the Greenwich Apparent Sidereal Time (GAST) at 2021-11-22, $01^h{:}00^m{:}00^s$ CET and specify it in hour angle, minute and second to second precision.

6. On a terrestrial baseline, a VLBI group delay $\Delta\tau_{TT} = 20$ ms was observed in terrestrial time TT. Compute the equivalent group delay in TCB, $\Delta\tau_{TCB}$, using the TT to TCG and TCG to TCB coordinate time transformations. How much do the individual gravitational terms contribute to the sum and what solar system bodies are significant for the transformation? Show that the gravitational relativistic term is about twice as large as the special relativistic term. Specify the difference $(\Delta\tau_{TCB} - \Delta\tau_{TT})$ in meter unit.

Attachment

3D - rotation matrices:

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}, \quad R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \quad R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D - reflection matrices:

$$M_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Some Solar system parameters:

body (***A***)	$\boldsymbol{GM_A}$ $(m^3 s^{-2})$	***mean distance*** to Earth $\lvert\vec{x}_e - \vec{x}_A\rvert$ (km)
Sun	$1.32712440018 \cdot 10^{20}$	149,597,870.7 (= 1 AU)
Mercury	$2.2032 \cdot 10^{13}$	91,691,000
Venus	$3.24859 \cdot 10^{14}$	41,400,000
Moon	$4.9048695 \cdot 10^{12}$	385,000.6
Mars	$4.282837 \cdot 10^{13}$	78,340,000
Ceres	$6.26325 \cdot 10^{10}$	414,000,000
Jupiter	$1.26686534 \cdot 10^{17}$	628,730,000
Saturn	$3.7931187 \cdot 10^{16}$	1,275,000,000
Uranus	$5.793939 \cdot 10^{15}$	2,723,950,000
Neptune	$6.836529 \cdot 10^{15}$	4,351,400,000
Pluto	$8.71 \cdot 10^{11}$	5,890,000,000
Eris	$1.108 \cdot 10^{12}$	96.1 AU

The magnitude of the barycentric velocity of Earth be $v_e \approx 30 \cdot 10^3 \text{ m s}^{-1}$.

Due date
December 6, 2021, 12 o'clock (midday)