

Bayesian growth mixture models

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General notation

General notation

Individual n , for $n = 1, \dots, N$, where N is the number of individuals

Time period t , for $t = 1, \dots, T$, where T is the number of time periods

\mathbf{Y}^{obs} is a $N \times T$ matrix representing the observed dependent variable

\mathbf{Y}^{pred} is a $N \times T$ matrix representing the predicted dependent variable

\mathbf{p} is a row vector of size T representing the time periods, starting at zero

Latent class (aka, sub-population) c , for $c = 1, \dots, C$, where C is the number of classes

Π is a $N \times C$ matrix representing the mixture proportions (e.g., the probability that individual $n = 1$ belongs to class $c = 1$); thus,

$$0 \leq \pi_{n,c} \leq 1 \quad \text{and} \quad \sum_{c=1}^C \pi_{n,c} = 1 \quad (1)$$

Model 1

Model 1 — dependent variable

Continuous

Normally distributed

Homoscedasticity

Uncorrelated errors (over individuals and time periods)

Model 1 — constant and trend component

Constant without interindividual differences within-class (i.e., the constant is allowed to vary between classes but not within classes)

Linear (i.e., non-stationary, deterministic) trend component without interindividual differences within-class (i.e., the trend component is allowed to vary between classes but not within classes)

According to [Ram and Grimm \(2009\)](#), latent class growth analysis (or LCGA for short) refers to a subset of GMMs assuming no interindividual differences within-class regarding the constant and trend component

Model 1 — likelihood function

$$y_{n,t}^{obs} \sim \sum_{c=1}^C \pi_{n,c} \text{Normal}(\mu_{c,t}, \sigma_c) \quad (2)$$

where \mathbf{M} is a $C \times T$ matrix, and $\boldsymbol{\sigma}$ is a column vector of size C representing the error component.

$$\mu_{c,t} = \beta_{0,c} + \beta_{1,c} p_t \quad (3)$$

where β_0 is a column vector of size C representing the constant, and β_1 is a column vector of size C representing the trend component.

Model 1 — prior distributions

$$\beta_{0,c} \sim \text{Normal}(0, 1) \quad (4)$$

$$\beta_{1,c} \sim \text{Normal}(0, 1) \quad (5)$$

$$\sigma_c \sim \text{Normal}(0, 1) \mathbf{1}(\sigma_c > 0) \quad (6)$$

where the normal distribution is truncated to the left at zero, modeled via the indicator function $\mathbf{1}(\sigma_c \geq 0)$.

$$\pi_{n,c} \sim \begin{cases} \text{Uniform}(0, 1) & \text{if } c = 1 \\ \text{Uniform}(0, \pi_{n,c-1}) & \text{if } c > 1 \end{cases} \quad (7)$$

where $\pi_{n,c}$ is assigned a hierarchical prior.

Model evaluation

Model evaluation — NUTS convergence

THE END