Model specification

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Overview

General notation

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Individual n, for $n = 1, ..., N$, where N is the number of individuals

Time period t, for $t = 1, ..., T$, where T is the number of time periods

$\boldsymbol{Y}^{obs}$ is a $N \times T$ matrix representing the observed dependent variable

$\boldsymbol{Y}^{pred}$ is a $N \times T$ matrix representing the predicted dependent variable

$\boldsymbol{p}$ is a row vector of size T representing the time periods, starting at zero

General notation — continued

Latent class (aka, sub-population) c, for $c = 1, ..., C$, where C is the number of classes

$\mathbf{\Pi}$ is a $N \times C$ matrix representing the mixture proportions (e.g., the probability that individual $n = 1$ belongs to class $c = 1$); thus,

$$0 \leq \pi_{n,c} \leq 1 \quad \text{and} \quad \sum_{c=1}^{C} \pi_{n,c} = 1 \tag{1}$$

Model 1

Model 1 — dependent variable

Continuous

Normally distributed

Homoscedasticity

Uncorrelated errors (over individuals and time periods)

Model 1 — constant and trend component

Constant without interindividual differences within-class (i.e., the constant is allowed to vary between classes but not within classes)

Linear (i.e., non-stationary, deterministic) trend component without interindividual differences within-class (i.e., the trend component is allowed to vary between classes but not within classes)

According to Ram and Grimm (2009), latent class growth analysis (or LCGA for short) refers to a subset of GMMs assuming no interindividual differences within-class regarding the constant and trend component

Model 1 — likelihood function

$$y_{n,t}^{obs} \sim \sum_{c=1}^{C} \pi_{n,c} \, Normal(\mu_{c,t}, \sigma_c) \tag{2}$$

where $\boldsymbol{M}$ is a $C \times T$ matrix, and $\boldsymbol{\sigma}$ is a column vector of size C representing the error component.

$$\mu_{c,t} = \beta_{0,c} + \beta_{1,c} \, p_t \tag{3}$$

where $\boldsymbol{\beta}_0$ is a column vector of size C representing the constant, and $\boldsymbol{\beta}_1$ is a column vector of size C representing the trend component.

Model 1 — prior distributions

$$\beta_{0,c} \sim \mathit{Normal}(0, 1) \tag{4}$$

$$\beta_{1,c} \sim \mathit{Normal}(0, 1) \tag{5}$$

$$\sigma_c \sim \mathit{Normal}(0, 1)\, \mathbf{1}(\sigma_c > 0) \tag{6}$$

where the normal distribution is truncated to the left at zero, modeled via the indicator function $\mathbf{1}(\sigma_c \geq 0)$.

$$\pi_{n,c} \sim \begin{cases} \mathit{Uniform}(0, 1) & \text{if } c = 1 \\ \mathit{Uniform}(0, \pi_{n,c-1}) & \text{if } c > 1 \end{cases} \tag{7}$$

where $\pi_{n,c}$ is assigned a hierarchical prior.

THE END