Tools for Macroeconomists

The Essentials

Thursday Assignment

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1 Objective

Solve a DSGE model with financial frictions. Then, using data on real GDP and a measure of the risk premium, estimate the model using Bayesian methods. Decomposing the variation in the model into the contributions of the two shocks, discuss whether or not this is a particularly good model of financial frictions.

2 Model description

The model economy is populated by households and firms. Households consume non-durables and supply labor to firms. Firms need to borrow in order to produce and pay a risk premium that depends negatively on how valuable capital is.

Households. Households solve the following maximization problem:

$$\max_{c_t, l_t} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \ln c_t - \phi_0 \frac{l_t^{1+\phi_1}}{1+\phi_1} \right\}, \qquad \phi_0, \phi_1 > 0,$$

subject to the following budget constraint:

$$c_t = w_t l_t + D_t,$$

where c_t is non-durable consumption, l_t is the amount of labor supplied to firms, w_t is the wage rate, and D_t the profits that the households receive as firm owners. The first-order conditions are:

$$\begin{aligned} \frac{1}{c_t} &= \lambda_t \\ w_t \lambda_t &= \phi_0 l_t^{\phi_1}, \end{aligned}$$

where λ_t is the Lagrange multiplier on the budget constraint.

Firms. Firms operate in a perfectly competitive goods market. They are owned by the households and they produce using capital and labor. For simplicity, the aggregate stock of capital is fixed.

Firms are required to pay wages at the beginning of the period, before any revenues from production are received. Firms therefore rely on intraperiod loans. Interest rates consist of two components: (i) a component to compensate for postponing consumption and (ii) a premium to cover risk and financial intermediation costs. For intraperiod loans there is only the second component, which we refer to as x_t. It is assumed to depend negatively on the price of capital, $p_{k,t}$ according the following formula:

$$x_t = \mu_0 z_{x,t} \left(\frac{\overline{p}_k}{p_{k,t}} \right)^{\mu_1}, \quad \mu_0, \mu_1 > 0,$$

where $p_{k,t}$ is the per-unit price of capital, $\overline{p}_k$ is the steady-state price of capital. The idea is that during periods when the value of the capital is low, financial frictions become more severe, as capital serves as collateral for loans. The premium is also affected by an exogenous shock $z_{x,t}$. A high realisation of this shock may be thought of as arising from problems within the financial sector.

Firms maximize the present value of profits, discounted using the stochastic discount factor of the households:

$$\max_{l_t,k_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left\{ y_t - p_{k,t}(k_t - k_{t-1}) - (1 + x_t) w_t l_t \right\},$$

subject to a Cobb-Douglas production function:

$$y_t = z_{a,t} k_t^{\alpha} l_t^{1-\alpha},$$

where y_t is output, k_t is capital, $z_{a,t}$ is an exogenous productivity shock. The first-order conditions are:

$$\begin{aligned} w_t(1 + x_t) =& (1 - \alpha) z_{a,t} \left(\frac{k_t}{l_t} \right)^{\alpha}, \\ p_{k,t} =& \alpha z_{a,t} \left(\frac{k_t}{l_t} \right)^{\alpha - 1} + \mathbb{E}_t \beta \frac{c_t}{c_{t+1}} p_{k,t+1}. \end{aligned}$$

Market clearing. In the aggregate, all resources are allocated to non-durable consumption and financial intermediation costs:

$$y_t = c_t + x_t w_t l_t.$$

The stock of capital is fixed in the aggregate at a level $\overline{k}$. Note that by symmetry $k_t = \overline{k}$.

Shock processes. The exogenous productivity and risk premium shocks obey the following laws of motion:

$$\begin{aligned} \log(z_{a,t}) =& \rho_a \log(z_{a,t-1}) + \epsilon_{a,t}, \\ \log(z_{x,t}) =& \rho_x \log(z_{x,t-1}) + \epsilon_{x,t}, \end{aligned}$$

where $\epsilon_{a,t}$ and $\epsilon_{x,t}$ are normally distributed iid shocks with mean zero and standard deviation σ_a and σ_x, respectively.

Supply of capital. For simplicity we assume that the supply of capital is fixed. This fixed value is denoted by $\overline{k}$.

State variables. Given that capital is fixed, the only two state variables are $z_{a,t}$ and $z_{x,t}$. The policy functions are thus functions of these two variables.

Equilibrium. Since capital supply is fixed, we know that the capital demand of the representative firm has to satisfy

$$k_t = \overline{k} \quad \forall t.$$

We will impose this condition below.

A competitive equilibrium is defined by four policy functions $c_t = c(z_{a,t}; z_{x,t})$, $w_t = w(z_{a,t}; z_{x,t})$, $p_{k,t} = p_k(z_{a,t}; z_{x,t})$, and $l_t = l(z_{a,t}; z_{x,t})$, that satisfy the following dynamic system of equations:

$$\frac{w_t}{c_t} = \phi_0 l_t^{\phi_1}, \tag{1}$$

$$w_t \left(1 + \mu_0 z_{x,t} \left(\frac{\overline{p}_k}{p_{k,t}}\right)^{\mu_1}\right) = (1-\alpha) z_{a,t} \left(\frac{\overline{k}}{l_t}\right)^{\alpha}, \tag{2}$$

$$p_{k,t} = \alpha z_{a,t} \left(\frac{\overline{k}}{l_t}\right)^{\alpha-1} + \mathbb{E}_t \beta \frac{c_t}{c_{t+1}} p_{k,t+1}, \tag{3}$$

$$z_{a,t} \overline{k}^{\alpha} l_t^{1-\alpha} = c_t + \mu_0 z_{x,t} \left(\frac{\overline{p}_k}{p_{k,t}}\right)^{\mu_1} w_t l_t, \tag{4}$$

with

$$\log(z_{a,t}) = \rho_a \log(z_{a,t-1}) + \epsilon_{a,t}, \tag{5}$$

$$\log(z_{x,t}) = \rho_x \log(z_{x,t-1}) + \epsilon_{x,t}. \tag{6}$$

Fixed parameter values. We normalize $\overline{k} = 1$ and we set $\alpha = 0.33$, $\beta = 0.99$, $\mu_0 = 0.02$, and $\phi_1 = 1$. The value of ϕ_0 is discussed below.

Parameters to be estimated. The following parameters are estimated: μ_1, ρ_x, ρ_a, σ_x and σ_a.

Data. We will use two data series to estimate the model: (i) log real GDP (detrended and normalized to zero), and (ii) log bond spreads (10 year corporate bond rate minus the federal funds rate). The data run from 1970Q1 to 2010Q1 and are contained in the file `data_gdp_bondspread.m`.

3 Assignment

1. Suppose we would like to choose ϕ_0 such that the steady state value of employment is equal to 1. Show that $\phi_0 = \frac{1-\alpha}{1+\alpha\mu_0}$ ensures that $l = 1$. What is the steady state price of capital, p_k?

2. Use the Dynare program `model_solve.mod` to analyze the IRFs of the two shocks.

3. What is the interpretation of the parameter μ_1? Specify a prior distribution for this parameter that you think is reasonable.

4. Write a program to estimate the model. For ρ_x, ρ_a, σ_x and σ_a use uniform distributions with supports $[0, 1]$. For μ_1, use your own prior.

    - You will find the names of the data at the end of the data file.

5. Estimate the model using $5,000$ replications of the MH algorithm (or more if you have time), and try to obtain an acceptance rate of around 0.25.

6. Using the `shock_decomposition` command, analyze the contribution of the two shocks to the realizations of output and bond spreads during the recent recession. How would you interpret the results?