Tools for Macroeconomists

The Essentials

Monday Assignment

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1 Objective

In this assignment you are asked to solve a DSGE model in two different ways using linearization techniques: (i) with Dynare and (ii) DIY. At the end, you should be left with exactly the same solution.

2 Model

The model is the standard "neoclassical growth model" with endogenous labor supply. In other words, this is the same model as in the lecture but where, in addition, we allow for a labor supply choice of the household. In particular, we assume

- a representative household maximizing expected life-time utility (derived from consumption and leisure): $\max_{\{c_t,k_{t+1},l_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_t \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l^{1+\eta}}{1+\eta} \right)$
  - where γ is the coefficient of relative risk aversion and η is the Frisch elasticity of labor supply
- the household owns the production technology which uses both capital and labor as input factors and is subject to productivity shocks: $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$
  - where $\alpha \in (0,1)$ and $\ln z_t = \rho \ln z_{t-1} + \epsilon_t$ is aggregate productivity, where $\epsilon_t \sim N(0,\sigma^2)$ and $\rho \in (0,1)$ is a persistence parameter and z_0 is given
- resources are spent on consumption and investment into accumulation of capital and, each period, a fraction (δ) of the capital stock depreciates: $c_t + k_{t+1} = y_t + (1-\delta)k_t$, with k_0 given

3 Assignment

3.1 Derive Optimality Conditions

As a starting point, derive the optimality conditions for capital, labor and consumption. You'll need these to construct the model block in Dynare and to set up the model structure in the DIY linearization.

After you've derived the optimality conditions, have a look through `SolutionMain.m` and `ModelDynare.mod` to make sure you understand the structure of the code. `SolutionMain.m` first sets the model parameters and then proceeds to solve the model using Dynare. The last part of the code is the DIY linearization (and plotting of impulse responses).

3.2 Dynare Solution

In the first part of `SolutionMain.m` you should do the following:

- set the model parameters to the following values: $\alpha = 1/3$, $\beta = 1.03^{-1/4}$, $\gamma = 2$, $\eta = 2$, $\delta = 0.025$, $\rho = 0.9$ and $\sigma = 0.01$.
- define the system of equilibrium conditions (equations as functions of endogenous variables) you derived above. This includes the Euler equation (we'll call it Ee), the resource constraint (we'll call it Rc) and optimal labor supply (we'll call it Ls).
  - to do so, each of the equilibrium conditions should have the following structure, e.g. "$Ee = @(x) \;\; XYZ$", where XYZ is for instance the Euler condition such that it is equal to 0 (i.e. put the left hand side of your condition to the right ;))
  - as inputs, we'll use the vector x which consists of consumption (1st element), capital (2nd element) and labor (3rd element). In other words, Ee, Rc and Ls are *functions* of consumption, capital and labor, which we can then manipulate later
- once we solve for the steady state of consumption, capital and labor, let's also define the steady state of output (yss) and investment (Iss)
- now, move on to `ModelDynare.mod` and fill in the model block and the initial values (steady states). In doing so, specify the model equations in *logarithms* of the variables. This will make the impulse responses that Dynare generates more comparable as they will be in log-deviations from the respective steady states.

3.3 DIY solution

As the final step, let's move on to the DIY solution. As a first step, we define the steady state vector of variables as $x_{ss} = [y_s s, Iss, css, kss, lss, 1]$. What is the "1" for?

Next, the code defines symbolic variables corresponding to x_{t-1}, x_t and x_{t+1} in the slides. The code uses "m" to denote $t-1$ and "p" to denote $t+1$ variables. You should do the following:

- using the above notation, fill in the "system" of equilibrium equations (each row corresponding to an equilibrium condition). This is essentially exactly the same as the model block in dynare. The only difference being our notation and the fact that each line should be written such that it is equal to 0 in equilibrium
- next, write the iteration for solving F. This requires you to provide an update rule for F_{n+1} given an initial value F_n *and* a convergence "metric". Once the metric falls below a certain threshold (e.g. 1e-13) you should stop (i.e. write a "while loop").
- as the final step, we can compute Q, the term in our solution pertaining to the productivity shock, ϵ.

If all goes well, the rest of the code (thanks Pontus!) will generate the same impulse responses as the Dynare code you put together above. Either eyeball the impulse responses or use the outputs to create a new plot that overlays both sets of impulse responses on top of each other – are they the same?!