Tools for Macroeconomists

The Essentials

Tuesday Assignment

Petr Sedláček

1 Objective

In this assignment you are asked to show how to generate enough employment volatility in a simple matching model by adjusting wage stickiness and the share of entrepreneur. Recall that Shimer had pointed out that matching models cannot do this when wages are determined by Nash bargaining and the share of the revenues received by the entrepreneur is substantial.

2 Model

The model assumes that the household is composed of two types of agents, entrepreneurs and consumers. Given that agents of each type are assumed to be identical we can concentrate on a representative entrepreneur and consumer. The entrepreneur runs a rm that posts vacancies, hires workers, and produces output. For simplicity, producing output only requires labor. The consumer only works and consumes (there is no investment-saving decision). The model equations are as follows:

Consumer's budget constraint:

$$c_t + \psi v_t = w_t n_{t-1} + Q_t, \tag{1}$$

where c_t is consumption, v_t vacancies posted by the firm, w_t is the wage rate, and Q_t are total firm's profits. Parameter ψ is the cost of vacancies.

The number of employed workers, n_t, evolves according to the following law of motion:

$$n_t = (1 - \rho_x) n_{t-1} + p_t^f v_t, \tag{2}$$

where n_t is the number of employed workers, ρ_x is exogenous breakup probability, and p_t^f is the probability that the firm will find a worker. The total labor force is normalized to be equal to 1, so n_t is also the fraction of employed agents.

Total output is given by:

$$y_t = \exp(z_t) n_{t-1}, \tag{3}$$

where z_t is the (log of) the productivity shock.

We assume that wages are given by the following equation:

$$w_t = (1 - \omega_e)(\omega \exp(z_t) + (1 - \omega)\exp(0)). \tag{4}$$

If ω is equal to 1, then wages are proportional to the marginal product and if ω is equal to 0, then wages are fully sticky and equal to the steady state value of productivity. By changing ω_e we can control the share the entrepreneur receives on average.

Total firm profits are given by:

$$Q_t = (\exp(z_t) - w_t)n_{t-1}. \tag{5}$$

The matching function determines the number of matches based on the number of unemployed, $(1 - n_{t-1})$, and vacancies. The probability of a firm getting matched with a worker is just the number of matches divided by the number of vacancies.

$$m_t = \phi(1 - n_{t-1})^{\mu} v_t^{1-\mu}, \tag{6}$$

$$p_t^f = \frac{m_t}{v_t}. \tag{7}$$

The exogenous shock evolves according to the following law of motion:

$$z_t = \rho z_{t-1} + \epsilon_t. \tag{8}$$

The entrepreneur (the firm) decides on the number of vacancies to post. The cost of posting a vacancy is ψ and the per period gain of getting an additional worker is equal to the pro t per worker, $\exp(z_t) - w_t$. The firm solves the following problem:

$$\max_{\{n_t, v_t\}_{t=1}^{\infty}} \mathbb{E} \sum_{t=1}^{\infty} \beta^t \frac{c_t^{-\upsilon}}{c_1^{-\upsilon}} \left((\exp(z_t) - w_t)n_{t-1} - \psi v_t \right),$$

$$\text{s.t.}$$

$$n_t = (1 - \rho_x)n_{t-1} + p_t^f v_t,$$

$$n_0 \text{ given}$$

The first-order conditions are given by:

$$\psi = p_t^f g_t, \tag{9}$$

$$g_t = \beta \mathbb{E}_t \left(\frac{c_{t+1}}{c_t} \right)^{-\upsilon} \left[\exp(z_{t+1}) - w_{t+1} + (1 - \rho_x) g_{t+1} \right], \tag{10}$$

where g_t is the Lagrange multiplier corresponding to the constraint on the changes in n_t. Note that it can be interpreted as the value to the firm of getting one extra worker.

These two equations have a nice interpretation. The first one says that, at the optimum, the cost of posting a vacancy has to be equal to the probability that the firm finds a worker, p_t^f, times the value of this additional worker to the firm, g_t (which is just the Lagrange multiplier on the law of motion for labour). The second equation says that the value of an additional worker to the firm is equal to the discounted profits this worker will bring in the next period, plus the discounted value of next period's continuation value.

The model consists of the 10 numbered equations and the following 10 unknowns:

$$c_t, n_t, y_t, w_t, Q_t, m_t, p_t^f, z_t, v_t, g_t.$$

Two files have been written for you. The file `SimpleMatching.mod` is the Dynare file with the model described above. The file `MatchingMain.m` is a Matlab file that runs the Dynare file and computes some of the labour market statistics for different values of the entrepreneur share using a "for loop".

3 Assignment

- Examine both files (more info below). Make sure that you know what `MatchingMain.m` does with the output of `SimpleMatching.mod`.
- Fill in the model part in `SimpleMatching.mod`
- At the start of the for loop complete the following:
  1. save the ith value of the entrepreneur share to an external file
  2. give the command to run `SimpleMatching.mod` with Dynare

Next you have to simulate the economy using the policy functions computed in Dynare for a given time path for the realizations of the productivity levels. Let's focus only on three variables, namely employment, aggregate productivity, and output.

You have to do the following:

1. Specify sensible initial values for these variables (hint: the program has stored some sensible initial values somewhere, so you don't have to do any calculations).
2. Program the simulation part of the economy given (i) the policy functions and (ii) the time path for the innovations. That is, write up the "for loop" to generate output, employment, and aggregate productivity.

   Now that you have simulated the economy, you can also compute some business cycle statistics. The key statistic in this debate is the ratio of standard deviations of employment and output.

   You have to do the following:

   (a) Compute the cyclical components of output and employment using the `hpfilter2.m` function. This function takes as arguments the given variable and the smoothing coefficient. Set this coefficient to 1600. The output of the `hpfilter.m` function is the trend of the given series. The Matlab command is

   ```
   hp_trend_x=hpfilter2(x,1600)
   ```

   (b) Calculate the standard deviation of log employment relative to the standard deviation of log output using their cyclical components you calculated above.

   (c) Also (still within the loop) plot in one figure say 100 values of the cyclical component of log employment and that of log output.

What to look for? At the end of each iteration is a pause statement so you can look at the graph and the calculated ratio. Do you understand why the simulated series change the way they do?

Additional exercises

1. Try changing the wage rigidity parameter in `SimpleMatching.mod` and look at the relative standard deviations again.

2. Another way to analyze models is to look at the impulse response functions (IRFs). Computing IRFs is essentially the same as simulating the economy, but with a specific draw for the shocks. Namely, only the first value of the shock series is positive (typically the value of the shocks standard deviation), while the other values are zero. Do the following:

   (a) Define the shock series you will use for the IRFs. That is, set the first element of the shock to the standard deviation of the shock and set zeros everywhere else (the length of the series is given by $T_i = 24$).

   (b) Specify the initial values for the variables of interest and program the recursion that calculates the IRFs (hint: that is almost identical to the simulation part).

   (c) Plot the IRFs as % deviations from steady states.

Structure of `matchingmain.m`

1. sets ω_e
2. writes ω_e to a file
3. runs `SimpleMatching.mod` (within the program) to get policy functions
4. creates policy function matrix
5. generates time paths
6. calculates volatilities
7. considerse next value of ω_e

Tricks to calculate steady state

- Steady state is a nontrivial nonlinear system of 10 equations in 10 unknowns.
- Moreover, correct value of ψ not known
- We want the steady state value for pf to be sensible
- Solution to last two problems: instead of taking ψ as given and p^f as unknown, take p^f ss as given and solve for ψ