Tools for Macroeconomists

Advanced Tools

Wednesday Assignment

Petr Sedláček

1 Objective

The objective of this assignment is to solve a general equilibrium macro model with firm dynamics and aggregate uncertainty using Dynare (similar to Sedláček and Sterk, 2017). Using the model, you can then analyze how the *distribution* of firms responds to aggregate shocks.

2 Firm dynamics model with aggregate uncertainty

Consider a model in which ex-ante heterogeneous firms produce output according to

$$y_{i,a,t} = Z_t \gamma_i n_{i,a,t}^{\alpha}, \tag{1}$$

where i indicates firm type (level of efficiency i), a indicates firm age, γ_i is a firm-specific (permanent) efficiency level, $n_{i,a,t}$ is firm-level employment and α controls returns to scale and Z_t is an aggregate productivity shock evolving according to

$$Z_t = 1 - \rho + \rho Z_{t-1} + \epsilon_t, \tag{2}$$

where $\rho \in (0,1)$ and $\epsilon_t \sim N(0, \sigma^2)$. Firms choose employment in order to maximize firm value

$$V_{i,a,t} = \max_{n_{i,a,t}} y_{i,a,t} - W_t n_{i,a,t} - \frac{\zeta}{2}\left(n_{i,a,t} - n_{i,a-1,t-1}\right)^2 + \beta \frac{C_t}{C_{t+1}}(1-\delta) V_{i,a+1,t+1}, \quad a \geq 1 \tag{3}$$

where W_t is the aggregate wage rate and firms must pay quadratic adjustment costs $\zeta/2(n_{i,a,t} - n_{i,a-1,t-1})^2$ when changing their employment level. Since it is assumed that households own all firms, firms discount the future with a stochastic discount factor $\beta C_t/C_{t+1}$. All firms also shut down exogenously with a probability of δ. Finally, n_0 is assumed to be fixed and identical for all firms.

There is a mass of potential startups which enter the economy as long as the expected benefits are large enough to cover the startup cost χ. In particular

$$\chi = \sum_i \omega_{i,0} V_{i,0,t}, \tag{4}$$

where $\omega_{i,0}$ are *fixed* proportions of firm types among startups. It is assumed that startups enter with n_0 employees. The above free entry condition determines the mass of entrants, E_t. The mass of all other firm age-types is determined by

$$\omega_{i,1,t} = \omega_{i,0} E_t, \tag{5}$$

$$\omega_{i,a,t} = (1-\delta)\omega_{i,a-1,t-1}, \quad a \geq 2 \tag{6}$$

There is a representative household which supplies labor to firms and consumes the produced output good. The household maximizes lifetime utility subject to a budget constraint

$$\max_{\{C_t,N_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \upsilon N_t\right) \quad \text{s.t.}$$

$$C_t = W_t N_t + \Pi_t,$$

where N_t is aggregate employment, C_t is aggregate consumption and Π_t are aggregate profits.

Finally, aggregates are defined as

$$Y_t = \sum_i \sum_a \omega_{i,a,t} y_{i,a,t}, \tag{7}$$

$$N_t = \sum_i \sum_a \omega_{i,a,t} n_{i,a,t}, \tag{8}$$

$$C_t = Y_t - \sum_i \sum_a \omega_{i,a,t} \zeta/2(n_{i,a,t} - n_{i,a-1,t-1})^2 - \sum_i \omega_{i,0} E_t \chi, \tag{9}$$

$$\Omega_t = \sum_i \sum_a \omega_{i,a,t} \tag{10}$$

3 Solving the model

First, derive the optimality conditions associated with the model.

3.1 Part 0 - parameter values

In Part 0 of `FirmModel_Main.m` set the parameters to the following values

- $\alpha = 0.7$, $\beta = 0.97$, $\delta = 0.1$, $\zeta = 1.5$
- $\rho = 0.7$, $\sigma = 0.01$
- $\gamma = (0.21, 0.35, 70)$, $\omega_{i,0} = (0.75, 0.2, 0.05)$

From the above you can see that we will work with three firm types, so set $I = 3$. In order to operationalize the solution of this model, we will "cut off" the age distribution at some point. For today, set the maximum age to $A = 30$.

At this point we will not determine the disutility of labor, υ, but instead we will pin it down later by assuming that steady state wages are equal to $W = 1$. Similarly, we are not setting the entry cost, but instead we will pin it down using the free entry condition such that the price of the final good is equal to 1 (i.e. it is the numeraire).

3.2 Part 1 - steady state

In a way, the toughest part of the solution is obtaining the steady state. This involves computing the *steady state distribution* of firm values and employment choices. This is done in Part 1 of `FirmModel_Main.m`. Specifically, `get_Vn.m` is a function that computes firm values and employment choices for a given set of parameters such that they satisfy (3) and the optimal employment choice you have derived.

In `get_Vn.m` determine firm-level output and then determine the "error functions" (denoted by F and G in the code) associated with firm values and employment choices of startups, older firms and "end-point" firms. The error functions will be zero in steady state.

Now that you've solved for the distribution, go back to `FirmModel_Main.m` and determine the distribution of firm masses $\omega_{i,a}$ and the associated aggregate firm mass Ω. Next, compute the entry cost, χ, implied by all the above and compute the aggregate variables. Finally, you can determine the disutility of labor, υ.

3.3 Part 2 - solving the model with Dynare

In Part 2 of `FirmModel_Main.m` you will solve the model using perturbation. Fortunately, Dynare can do this for you, you just have to tell him what to do. Solving the model before (partly) estimating it is a good idea, so that you know it works.

Open `FirmModel.mod` which contains the entire model, i.e. parameters, all the optimality conditions and starting values. Note, however, that we're dealing with a heterogeneous firm model and you have to write down the optimality equations for *all* the firms (or more precisely firm types). Fortunately, you can use Dynare's "macro language" for this. There should be enough hints for you in the mod-file to figure out how this works. So go ahead and fill in all the optimality conditions and solve the model.[1]

3.4 Part 3 - distributional impulse responses

Now that you've solved the model you can play around with it a little. In particular, you can construct impulse responses of the distribution of firms!

Construct the time path for each point of the distribution (i.e. for each age-type combination). Do this both for firm masses and the associated employment choices. Having done that, try to visualize how the distribution of firms (and employment) changes in response to an aggregate productivity shock. How do you think this compares to fluctuations in the data?

3.5 (Bonus: Part 4 - estimating the model)

If you want to push more, you can try and use the model to estimate aggregate productivity shocks using data on real GDP. The mod-file for this is almost exactly the same as what you've just finished. So, open `FirmModelEstimation.mod` and copy-paste all the equilibrium conditions from `FirmModel.mod` into the model block. In addition, define the variable "Ycyc" which will simply be the cyclical component of aggregate output in percent deviations (the variable is already declared, don't worry). The mod-file has already computed the cyclical component of real GDP in the data for you.

In the estimated parameters block, you need to specify what parameters you want to estimate (and how). We'll do maximum likelihood estimation of ρ and σ. In this case,

[1]Btw, remember that the command `check` shows you errors of all the equilibrium conditions. If you've done everything right (at least in terms of steady states), Dynare should spit out a long list (because we have so many equilibrium conditions) of zeros.

it suffices to set an initial guess, a lower and an upper bound on these two parameters. That's it, now you're ready to go.

The beauty of the estimation is that you have time-paths for *all* the model variables which are consistent with the observable used in the estimation. This means that you can do all kinds of fancy stuff, including with the firm distributions (they're all stored in the structure `oo_.SmoothedVariables`). We'll do something relatively boring - plot the model-predicted time series for firm entry and compare it to that in the data (it's in the excel file). What do you think about the performance of the model in this respect?