Assignment

Oszillation of a Fluidsphere

Name TN 1: Tim Peinkofer

tim.peinkofer@student.uni-tuebingen.de

Name TN 2: Fabian Kostow

fabian.kostow@student.uni-tuebingen.de

Tutor: Jose Carlos Olvera Meneses

Date: February 4, 2025

Contents

1 Theoretical derivation

Lets assume the following:

$$\frac{d\theta}{d\xi} = \frac{\phi}{\xi^2} \tag{1}$$

If we plug this in the Lane-Emden equation we get:

$$\frac{d\phi}{d\xi} = -\theta^n \cdot \xi^2 \tag{2}$$

We can now use this results to calculate the fluidsphere oszillation of our star.

2 Code

To get the solution of the Lane-Emden equation we use the following code with Runke-Kutta 4th order:

```
import numpy as np
import matplotlib.pyplot as plt

# Lane-Emden equation
def Lane_Emden_system(y, xi, n):
    theta, dtheta_dxi = y
    if xi == 0:
        return np.array([dtheta_dxi, 0])  # If we divide by
                                            zero
    return np.array([dtheta_dxi, - (2 / xi) * dtheta_dxi -
                                    theta ** n])

# Runge-Kutta 4 (RK4)
def rk4(System, y0, xi_max, h, n):
    xi_values = np.arange(1E-4, xi_max, h)
    y = np.zeros((len(xi_values), len(y0)))
    y[0] = y0

    for i in range(1, len(xi_values)):
        xi = xi_values[i - 1]
        k1 = System(y[i - 1], xi, n)
        k2 = System(y[i - 1] + h * k1 / 2, xi + h / 2, n)
        k3 = System(y[i - 1] + h * k2 / 2, xi + h / 2, n)
        k4 = System(y[i - 1] + h * k3, xi + h, n)
        y[i] = y[i - 1] + h * (k1 + 2 * k2 + 2 * k3 + k4) / 6

    return xi_values, y[:, 0], y[:, 1]

# BC
y0 = np.array([1, 0])
xi_max = 10
h = 0.001

for n in [0, 1, 2, 3, 4]:
    xi_vals, theta_vals, dtheta_vals = rk4(Lane_Emden_system,
                                        y0, xi_max, h, n)
```

```
    # Plot
    plt.figure(figsize=(6, 4))
    plt.plot(xi_vals, theta_vals, label=f'Lane-Emden Solution
                                        (n={n})')
    plt.xlabel("Radius")
    plt.ylabel("Density")
    plt.legend()
    plt.title(f"Lane-Emden equation with RK4 (n={n})")
    plt.grid()


    plt.savefig(f"Lane_Emden_n{n}.png", dpi=300)
    plt.close()
```

3 Solutions for different polytropic indexes

We solved the equation for different indexes. As a result we get:

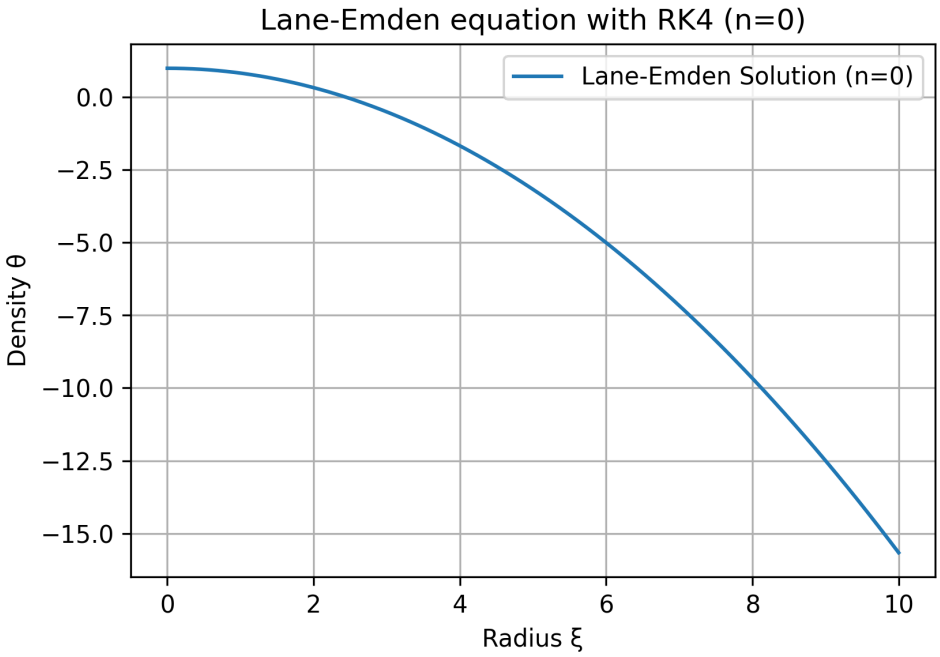

Figure 1: $n = 0$

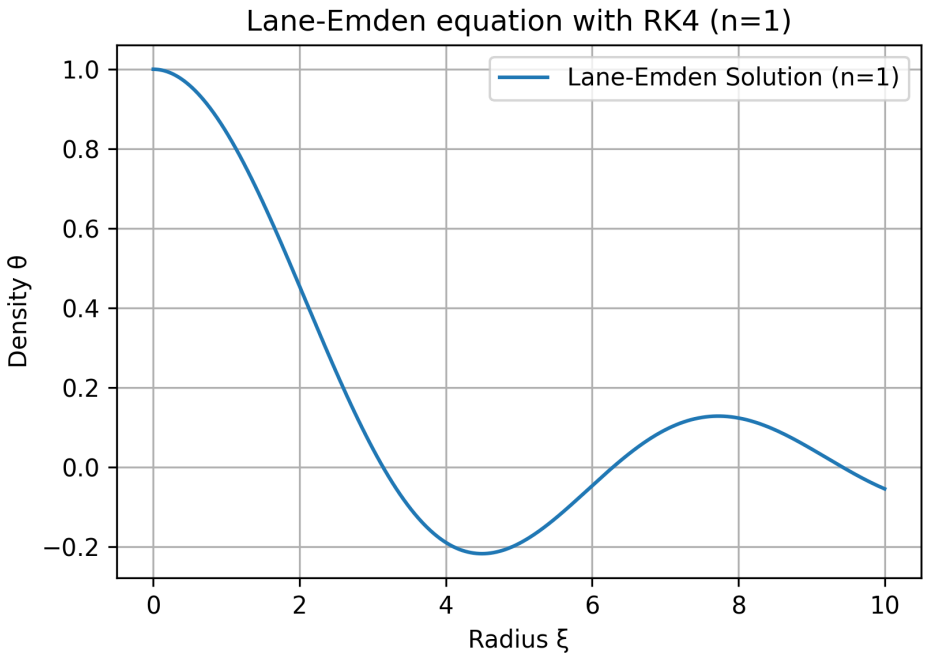
Lane-Emden equation with RK4 (n=1)
Lane-Emden Solution (n=1)
Density θ
Radius ξ
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
2
4
6
8
10

Figure 2: $n = 1$

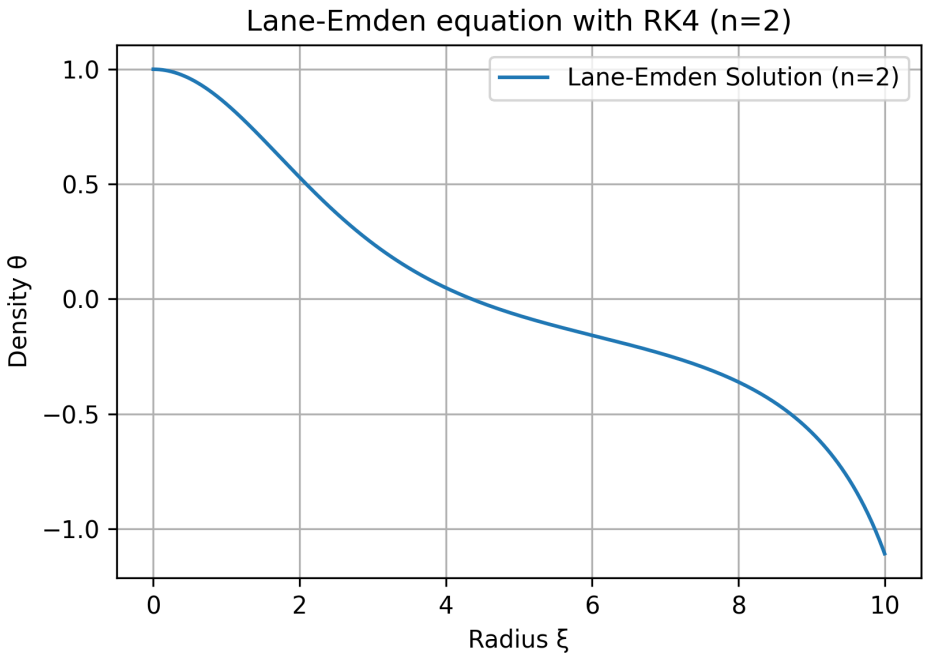
Lane-Emden equation with RK4 (n=2)
Lane-Emden Solution (n=2)
Density θ
Radius ξ
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10

Figure 3: $n = 2$

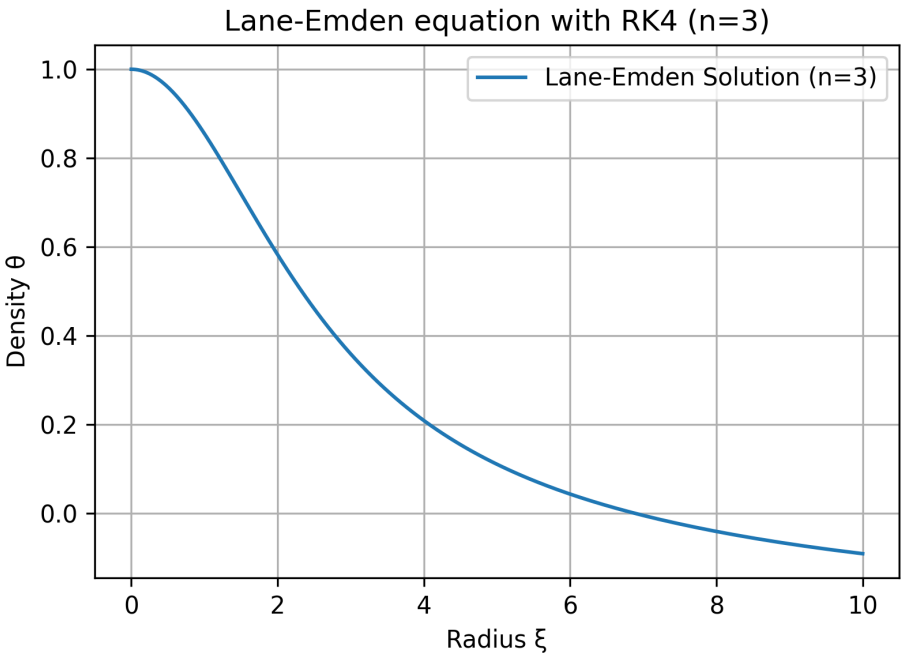
Lane-Emden equation with RK4 (n=3)
Lane-Emden Solution (n=3)
Density θ
Radius ξ
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10

Figure 4: $n = 3$

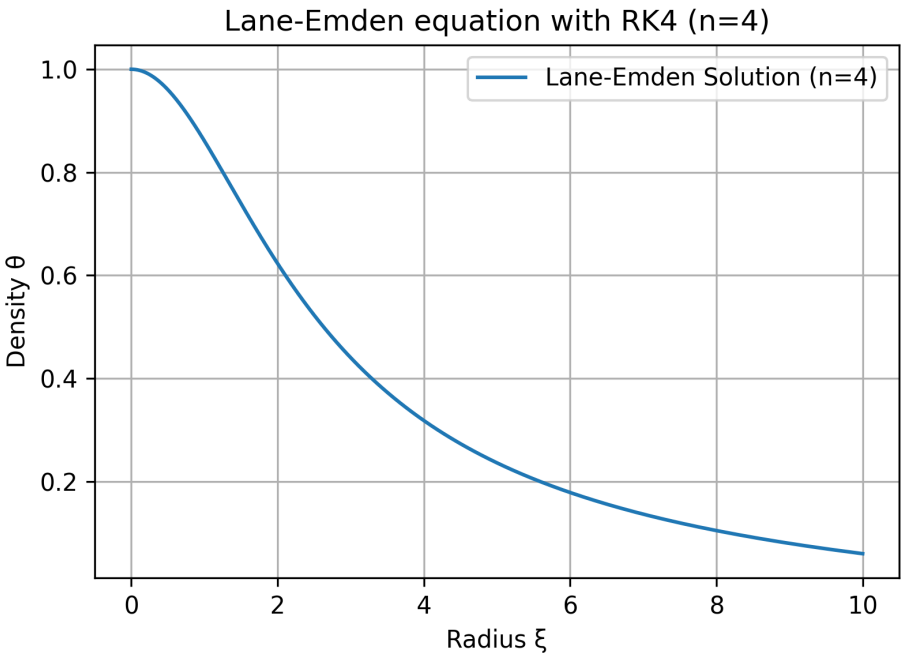
Lane-Emden equation with RK4 (n=4)
Lane-Emden Solution (n=4)
Density θ
Radius ξ
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10

Figure 5: $n = 4$

4 Sources

Wikipedia Lame-Emden equation: https://de.wikipedia.org/wiki/Lane-Emden-Gleichung