Modus ponens $\quad \dfrac{A \to B \quad A}{B}$

Prolog konkret:

a

$b \leftarrow a$

Answer sets: $\{a, b\}$ ist die einzige Antwortmenge von

Modelle der zugeh. Theorie $th = \{a, a \to b\}$ in klassischer AL:

$$\{\{a, b\}, \{a, b, c\}, \dots \{a, b, c, d, e, \dots\} \dots\}$$

$\Updownarrow$

$m : \{a, b, c, d \dots\} \to \mathbb{B} \qquad m\, b = \text{True}$

$m\, a = \text{True} \qquad m\, x = \text{False}$

Klassisch: Abzählbare Menge von Aussage-variablen:

$\mathrm{Var} = \{X_0, X_1, X_2, \dots\}$

$\mathrm{Operatoren} = \{\wedge, \vee, \neg\}$

$(\mathbb{IP} =)$ Interpretation $\doteq \mathrm{Var} \to \mathbb{B}$

$\wedge_{\mathbb{B}} : \mathbb{B} \to \mathbb{B} \to \mathbb{B}$

$\vee_{\mathbb{B}} : \mathbb{B} \to \mathbb{B} \to \mathbb{B}$

$\neg_{\mathbb{B}} : \mathbb{B} \to \mathbb{B}$

Formeln:

data F where

$V : \mathrm{Var} \to F$

$\wedge : F \to F \to F$

$\vee : F \to F \to F$

$\neg : F \to F$

$\mathrm{eval} : \mathbb{IP} \to F \to \mathbb{B}$

$\mathrm{eval}\ f\ (V\,x) = f\,x$

$\mathrm{eval}\ f\ (A \wedge B) =$
$(\mathrm{eval}\ f\ A) \wedge_{\mathbb{B}} (\mathrm{eval}\ f\ B)$

$\mathrm{eval}\ f\ (A \vee B) =$
$(\mathrm{eval}\ f\ A) \vee_{\mathbb{B}} (\mathrm{eval}\ f\ B)$

$\mathrm{eval}\ f\ (\neg A) = \neg_{\mathbb{B}} (\mathrm{eval}\ f\ A)$

klassisch: Abzählbare Menge von Aussagenvariablen:

$Var = \{X_0, X_1, X_2, \dots\}$

Operatoren $= \{\wedge, \vee, \neg\}$

$\wedge_{\mathbb{B}} : \mathbb{B} \to \mathbb{B} \to \mathbb{B}$

$\vee_{\mathbb{B}} : \mathbb{B} \to \mathbb{B} \to \mathbb{B}$

$\neg_{\mathbb{B}} : \mathbb{B} \to \mathbb{B}$

(IP =) Interpretation $= Var \to \mathbb{B}$

data F where

$V : Var \to F$

$\wedge : F \to F \to F$

$\vee : F \to F \to F$

$\neg : F \to F$

$eval : IP \to F \to \mathbb{B}$

$eval\ f\ (V x) = f x$

$eval\ f\ (A \wedge B) = (eval\ f\ A) \wedge_{\mathbb{B}} (eval\ f\ B)$

$eval\ f\ (A \vee B) = (eval\ f\ A) \vee_{\mathbb{B}} (eval\ f\ B)$

$eval\ f\ (\neg A) = \neg_{\mathbb{B}} (eval\ f\ A)$

Theorie:

$Th : Type$

$Th = List\ F$ (eigentlich $\mathcal{P}(F)$)

Modell:

$Mo : Th \to Type$

$$Mo\ th = \sum_{m:IP} \underbrace{\prod_{f:F} f \in th \to (eval\ m\ f =_{\mathbb{B}} True)}_{:Type}$$

$m : IP$ ist ein Modell einer Theorie $th : Th$ $\Leftrightarrow^{def}$

$\forall f : F .\ f \in th \to eval\ m\ f = True$

Alternativ (vielleicht besser?):

Modellrelation:

$MR : Th \to IP \to Type$

$$MR\ th\ m = \prod_{f:F} f \in th \to eval\ m\ f =_{\mathbb{B}} True$$

$\_\models\_ : IP \to Th \to Type$

Unter Verwendung von MR ist dann:

$$Mo\ th = \sum_{m:IP} MR\ th\ m$$

25.05.

wrong question!

In what sense "equivalent"?

Is rule equivalent to formula?

$$y \Leftarrow x \qquad \neg x \vee y$$

Classically: $(x \to y) \leftrightarrow \neg x \vee y$ is a tautology!

$\boxed{\text{Defined semantically, i.e. via (classical) models}}$

$$\vDash (x \to y) \leftrightarrow \neg x \vee y$$

i.e. the type family

$$\mathrm{HR}\big((x \to y) \leftrightarrow (\neg x \vee y)\big) : \mathbb{P} \to \mathrm{Type}$$

has a "section"!

we had

$$m \vDash [f] \quad : \mathrm{Type}$$

$$m \vDash [f] = \prod_{g : [f]} \mathrm{eval}\ m\ g =_{\mathbb{B}} \mathrm{True}$$

with abuse of notation (single formula on the right):

$$m \vDash f = \mathrm{eval}\ m\ f =_{\mathbb{B}} \mathrm{True}$$

another abuse of notation: "f is a tautology"

if every $m : \mathbb{P}$ is a model of f:
$$\vDash f :\equiv \prod_{m : \mathbb{P}} m \vDash f$$

or

$\vdash (x \to y) \Leftrightarrow (\neg x \vee y)$ "Beweisbarkeitsrelation"

$\models f \Leftrightarrow \vdash f$ provability : formula is provable (without any assumption!)

"$\Rightarrow$" Completeness
"$\Leftarrow$" Soundness
$\Big\}$ of classical propositional logic

Locality of formula "packing":

$a \leftarrow b \qquad a \leftarrow c \wedge \neg d \qquad (1)$

$e \vee p \leftarrow b \qquad e \vee p \leftarrow c \wedge \neg d$

is strongly equivalent to

$a \wedge (e \vee p) \leftarrow b \vee (c \wedge \neg d) \qquad (2)$

also with context. e.g. (1) and (2) also equivalent to

$a \wedge (e \vee p) \leftarrow b \qquad a \leftarrow c \wedge \neg d \qquad e \vee p \leftarrow c \wedge \neg d$

Let A, B be sets of (logic program) rules
(or, better, propositional theories)

A is "strongly equivalent" to B

$\overset{def}{\iff}$ $\forall C$ (set of rules) (or, better, propositl. theories)

$A \cup C$ has the same answer sets as

$B \cup C$

$p \leftarrow 0 \leq \{q = 1, r = -1\}$ is an "aggregation",

it is an abbreviation for

$p \leftarrow \neg q \wedge \neg r$ $\quad$ $p \leftarrow q \wedge \neg r$ $\quad$ $p \leftarrow q \wedge r$

$\cancel{p \leftarrow \neg q \wedge r}$

$$p \leftarrow \neg(\neg q \wedge r)$$

$$\text{“}\Leftarrow\text{”}$$

$$p \leftarrow q \vee \neg r$$

$$\text{“}(\Rightarrow)\text{”}$$

$$p \leftarrow (q \leftarrow r)$$

Equivalence of (3) and (4) is at least plausible

1.6.

originally: 90th (Gelfond, Lifschitz) ... verallgemeinert...

$$p \leftarrow (q \leftarrow r)$$

$$p \leftarrow \neg r$$

$$p \leftarrow q$$

$$p \vee \neg q \vee r \leftarrow$$

claim: these are strongly equivalent !?

As $\underline{\text{(classical) propositional formulas:}}$

$$(r \to q) \to p$$

$$\equiv_{PL_0} p \vee \neg(r \to q)$$

$$\equiv_{PL_0} p \vee \neg(q \vee \neg r)$$

$$\equiv_{PL_0} p \vee (\neg q \wedge \neg\neg r)$$

$$\equiv_{PL_0} p \vee (\neg q \wedge r)$$

and also

$(p \leftarrow q) \vee r \qquad p \vee r \leftarrow q$

$\qquad\qquad\qquad\qquad \neg q \vee r \leftarrow \neg p$

maintesalt is proved in two ways

(1) with syntactic rec. transf.

(2) starting from countermodels (in Here-and-There)

in analogy to CNF transform in classical logic:

(1') with syntactic rec. transf.

(2') starting from countermodels (classical)

Interpretation $(X,Y) \mathrel{\hat{=}} f : \Sigma \to \mathbb{B}$ mit (partieller Funktion)

one possibility

$$f\ p = \begin{cases} \text{true} & \text{if } p \in X \\ \text{false} & \text{if } p \notin Y \\ \text{undefined} & \text{otherwise} \end{cases}$$

or $(X,Y) \mathrel{\hat{=}} f : \Sigma \longrightarrow \text{Maybe } \mathbb{B}$

$$f : p = \begin{cases} \text{Just true} & ; \text{if } p \in X \\ \text{Just false} & ; \text{if } p \notin Y \\ \text{Nothing} & ; \text{otherwise} \end{cases}$$

data Maybe : $\mathcal{U} \to \mathcal{U}$ where

Nothing : $\{A : \mathcal{U}\} \longrightarrow \text{Maybe } A$

Just : $\{A : \mathcal{U}\} \to A \to \text{Maybe } A$

$\forall X \subseteq Y \in \Sigma$:

$$\left((X, Y) \models_{HT\text{"true"}} F \right) \iff \left(X \models_{PL} F \right)$$

or intuitionistic?

! check : $X \models \neg\neg a \to a$