Formalizing reachability, viability and avoidability in the context of sequential decision problems

Nicola Botta

Outline

- Why formalizing what?
- Minimal goals
- Sequential decision problems
- Reachability, viability and avoidability
- Decision procedures
- Wrap-up

Why formalizing what?

- *International emissions trading: Good or bad?*, Holtsmark & Sommervoll, 2012: "[. . .] we find that an agreement with international emissions trading leads to increased emissions and reduced efficiency."
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- *Confronting Climate Change: Avoiding the Unmanageable and Managing the Unavoidable*, P. Raven, R. Bierbaum, J. Holdren, UN-Sigma Xi Climate Change Report, 2007.

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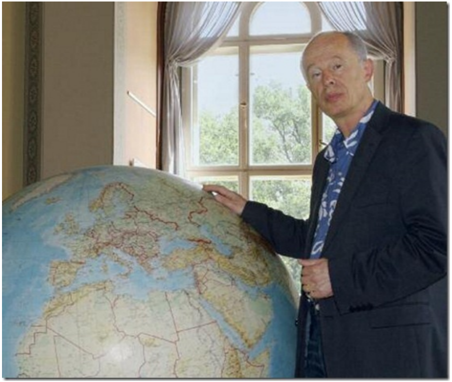
But what does it mean (for atmospheric GHG concentrations) to be *avoidable*?

Why formalizing what?



"Die Rolle der Klimaforschung bleibt weiterhin, die Problemfakten auf den Tisch zu knallen und Optionen für geeignete Lösungswege zu identifizieren."

H.-J. Schellnhuber in *Frankfurter Allgemeine* from 2012-06-19

Why formalizing what?

But how can we produce "hard facts" if the notions used to phrase specific, concrete problems are ambiguous, devoid of precise, well established, meanings?

Minimal goals

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Further questions, goals

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Further questions, goals

- Can one exploit decidability to derive useful avoidability (levity?) measures?

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- Explain under which conditions the question of whether future states are avoidable [reachable, viable, ...] can be decided

Further questions, goals

- Can one exploit decidability to derive useful avoidability (levity?) measures?
- Can one refine decidable notions of viability, avoidability to derive decidable notions (measures?) of sustainability, adaptability, resilience?

Sequential decision problems (intuition)

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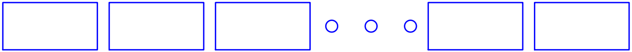
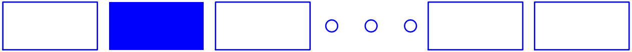


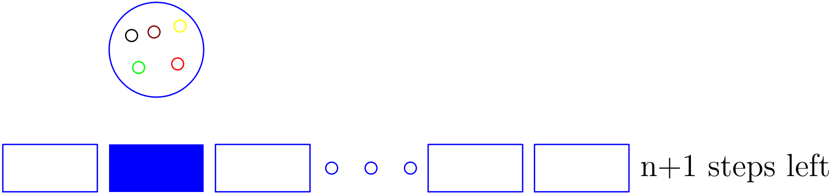
n+1 steps left

You are here...

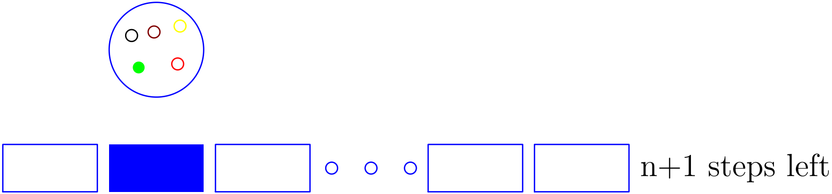


n+1 steps left

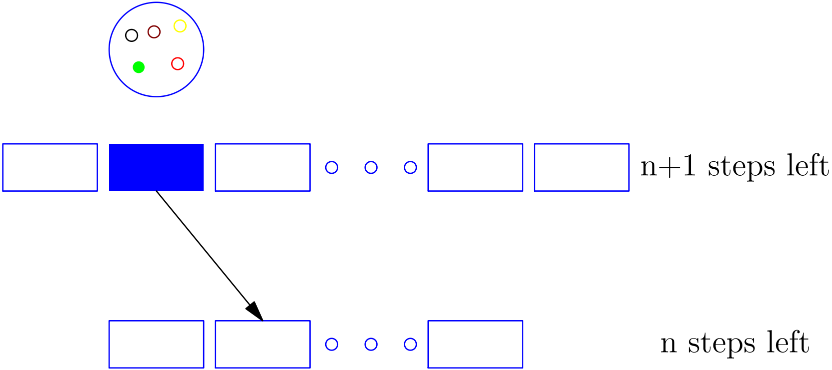
These are your options. . .



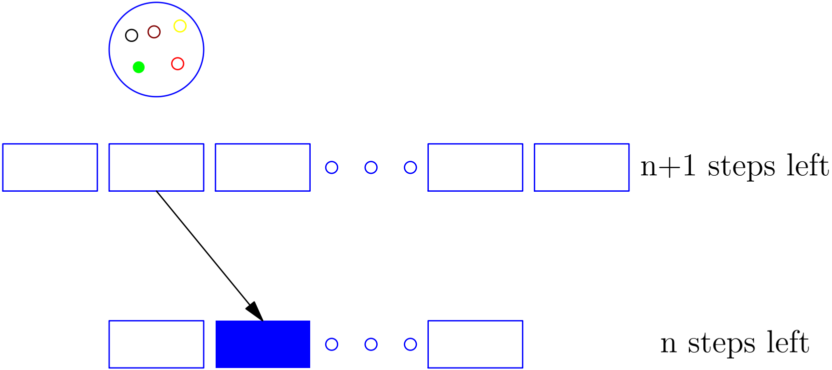
Pick one!



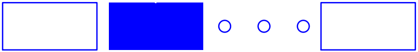
Advance one step. . .



...collect rewards ...



. . . and go!



n steps left

General sequential decision problems (intuition)

This intuition is a bit too simplistic ...

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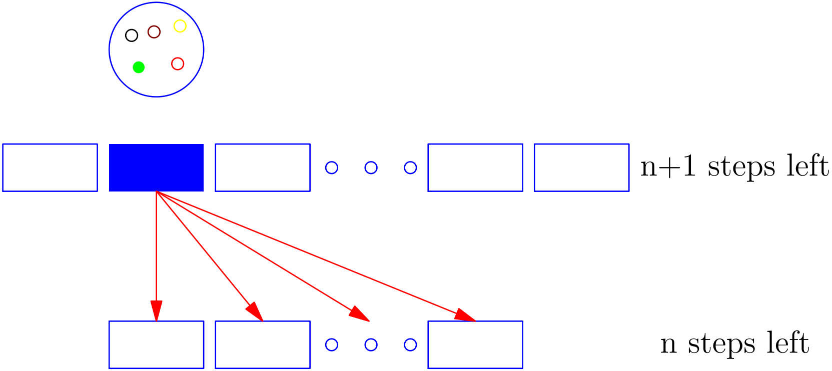
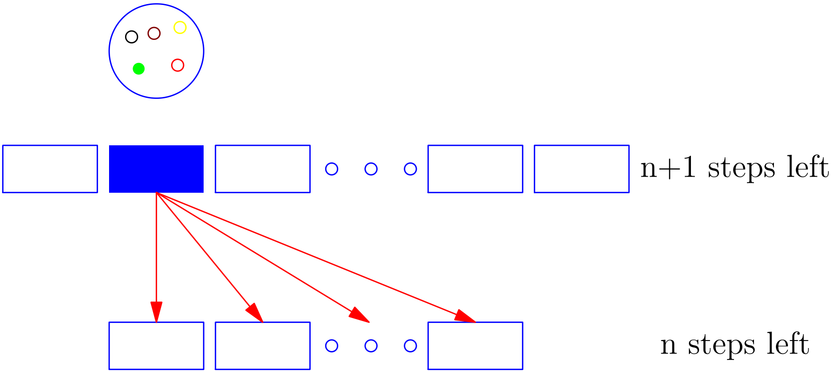


Notation (language)

Idris	set theory, logic
$A : Type$	A is a set
$a : A$	$a \in A$
$f : A \to B$	$f : A \to B$
$b = f\ a$	$b = f(a)$
$(a + b) * c$	$(a + b) * c$
$P : Type$	P is a predicate
$p : P$	p is a proof of P
$p : P \to Void$	p is a proof of $\neg P$
$P \to Q$	P implies Q
$P : A \to Type$	P is a predicate on A
$pa : P\ a$	pa is a proof of $P(a)$
$(a : A ** P\ a)$	there exists an $a \in A$ such that $P(a)$ holds
$(a : A) \to P\ a$	forall $a \in A$, $P(a)$ holds

Figure: Curry-Howard correspondence relating Idris and set theory, logic.

Sequential decision problems (basic notions)



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At each decision step, a set of possible states:

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$$Y \; : \; (t \; : \; \mathbb{N}) \; \rightarrow \; (x \; : \; X \; t) \; \rightarrow \; \mathit{Type}$$

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A transition function

$$\mathit{step} \,:\, (t \,:\, \mathbb{N}) \,\rightarrow\, (x \,:\, X\; t) \,\rightarrow\, (y \,:\, Y\; t\; x) \,\rightarrow\, M\,(X\,(S\; t))$$

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What are M and S?

Sequential decision problems (uncertainties)

$S\ t$ is just the successor of t:

$$
\begin{aligned}
&\textbf{data}\ \mathbb{N}\ :\ \mathit{Type}\ \textbf{where} \\
&\quad Z\ :\ \mathbb{N} \\
&\quad S\ :\ \mathbb{N} \rightarrow \mathbb{N}
\end{aligned}
$$

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$M\ :\ \mathit{Type}\ \rightarrow\ \mathit{Type}$ represents the uncertainties of the problem:

- deterministic problems: $M = \mathit{Id}$
- non-deterministic problems: $M = \mathit{List}$
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$$
\begin{array}{l}
\textbf{data}\ \mathit{Prob}\ :\ \mathit{Type}\ \rightarrow\ \mathit{Type}\ \textbf{where} \\
\quad \mathit{mkProb}\ :\ (\mathit{as}\ :\ \mathit{Vect}\ n\ a)\ \rightarrow\ (\mathit{ps}\ :\ \mathit{Vect}\ n\ \mathit{Float})\ \rightarrow \\
\qquad\qquad\quad \mathit{sum}\ \mathit{ps} = 1.0\ \rightarrow\ \mathit{Prob}\ a
\end{array}
$$

Sequential decision problems (container monad)

Formally, M is a container monad, that is M is a monad:

$$
\begin{aligned}
&fmap &&: (a \rightarrow b) \rightarrow M\ a \rightarrow M\ b \\
&ret &&: a \rightarrow M\ a \\
&bind &&: M\ a \rightarrow (a \rightarrow M\ b) \rightarrow M\ b \\
&join &&: M\ (M\ a) \rightarrow M\ a
\end{aligned}
$$

$$
\begin{aligned}
&functorSpec1 &&: fmap \circ id = id \\
&functorSpec2 &&: fmap\ (f \circ g) = (fmap\ f) \circ (fmap\ g) \\
&monadSpec1 &&: (fmap\ f) \circ ret = ret \circ f \\
&monadSpec2 &&: bind\ (ret\ a)\ f = f\ a \\
&monadSpec3 &&: bind\ ma\ ret = ma \\
&monadSpec4 &&: \{f : a \rightarrow M\ b\} \rightarrow \{g : b \rightarrow M\ c\} \rightarrow \\
& &&\quad bind\ (bind\ ma\ f)\ g = bind\ ma\ (\lambda x \Rightarrow bind\ (f\ x)\ g) \\
&monadSpec5 &&: join\ mma = bind\ mma\ id
\end{aligned}
$$

Sequential decision problems (container monad)

and M is a container:

$$
\begin{aligned}
&\mathit{Elem} &&: a \to M\ a \to \mathit{Type} \\
&\mathit{All} &&: (a \to \mathit{Type}) \to M\ a \to \mathit{Type}
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{containerSpec1} &&: a\ \text{`}\mathit{Elem}\text{`}\ (\mathit{ret}\ a) \\
&\mathit{containerSpec2} &&: a\ \text{`}\mathit{Elem}\text{`}\ \mathit{ma} \to \mathit{ma}\ \text{`}\mathit{Elem}\text{`}\ \mathit{mma} \to a\ \text{`}\mathit{Elem}\text{`}\ (\mathit{join}\ \mathit{mma}) \\
&\mathit{containerSpec3} &&: \mathit{All}\ p\ \mathit{ma} \to a\ \text{`}\mathit{Elem}\text{`}\ \mathit{ma} \to p\ a
\end{aligned}
$$

Sequential decision problems (basic notions)

Thus, a concrete sequential decision problem is defined (up to the rewards) in terms of 4 entities: X, Y, M and *step*

$$X \; : \; (t \; : \; \mathbb{N}) \; \rightarrow \; Type$$

$$Y \; : \; (t \; : \; \mathbb{N}) \; \rightarrow \; (x \; : \; X \; t) \; \rightarrow \; Type$$

$$M \; : \; Type \; \rightarrow \; Type$$

$$step \; : \; (t \; : \; \mathbb{N}) \; \rightarrow \; (x \; : \; X \; t) \; \rightarrow \; (y \; : \; Y \; t \; x) \; \rightarrow \; M \, (X \, (S \; t))$$

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We formalize reachability, viability and avoidability in terms of these notions

Reachability and viability (intuition)

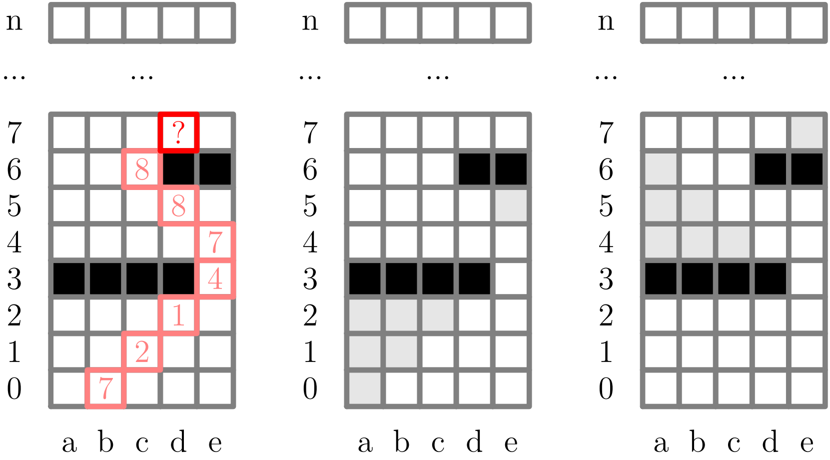

Figure: Possible evolution starting from b (left), states with limited viability (middle) and unreachable states (right).

Predecessor relation, reachability and viability

The (possible) predecessor relation:

$$\begin{aligned}&\mathit{Pred}\ :\ X\ t\ \rightarrow\ X\ (S\ t)\ \rightarrow\ \mathit{Type}\\&\mathit{Pred}\ \{t\}\ x\ x' = (y\ :\ Y\ t\ x\ **\ x'\ `\mathit{Elem}`\ (\mathit{step}\ t\ x\ y))\end{aligned}$$

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\end{aligned}
$$

reachability

$$
\begin{aligned}
&\mathit{Reachable} \;:\; X\ t' \;\to\; \mathit{Type} \\
&\mathit{Reachable}\ \{t' = Z\}\ x' \quad = \mathit{Unit} \\
&\mathit{Reachable}\ \{t' = S\ t\}\ x' = (x \;:\; X\ t\ \ast\ast\ (\mathit{Reachable}\ x, x\ `\mathit{Pred}`\ x'))
\end{aligned}
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\end{aligned}
$$

and viability

$$
\begin{aligned}
&\mathit{Viable} \ : (n \ : \ \mathbb{N}) \ \to \ X\ t \ \to \ \mathit{Type} \\
&\mathit{Viable}\ \{t\}\ Z\ x \qquad = \mathit{Unit} \\
&\mathit{Viable}\ \{t\}\ (S\ m)\ x = (y \ : \ Y\ t\ x \ ** \ \mathit{All}\ (\mathit{Viable}\ m)\ (\mathit{step}\ t\ x\ y))
\end{aligned}
$$

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- We are interested in the avoidability of "possible" future states. Specifically, we are interested in the avoidability of states which are reachable from some given state.
- The notion of avoidability entails the notion of an alternative.

Avoidability

We are interested in the avoidability of states which are reachable from some given state:

$$
\begin{array}{l}
\mathit{ReachableFrom} \; : \; X \; t'' \;\to\; X \; t \;\to\; \mathit{Type} \\
\mathit{ReachableFrom} \, \{ t'' = Z \} \, \{ t \} \; x'' \; x \quad = (t = Z, x = x'') \\
\mathit{ReachableFrom} \, \{ t'' = S \; t' \} \, \{ t \} \; x'' \; x = \\
\quad \mathit{Either} \; (t = S \; t', x = x'') \\
\qquad\qquad (x' \; : \; X \; t' \; ** \, (x' \, `\mathit{ReachableFrom}` \; x, x' \, `\mathit{Pred}` \; x''))
\end{array}
$$

where

$$
\textbf{data} \; \mathit{Either} \; a \; b = \mathit{Left} \; a \mid \mathit{Right} \; b
$$

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&\mathit{ReachableFrom}\ \{t'' = S\ t'\}\ \{t\}\ x''\ x = \\
&\quad \mathit{Either}\ (t = S\ t', x = x'') \\
&\qquad\qquad (x' \;:\; X\ t' \;**\; (x'\ `\mathit{ReachableFrom}`\ x, x'\ `\mathit{Pred}`\ x''))
\end{aligned}
$$

where

$$\mathbf{data}\ \mathit{Either}\ a\ b = \mathit{Left}\ a \mid \mathit{Right}\ b$$

Proof of concept: show that

$$
\begin{aligned}
\mathit{reachableFromLemma} \;:\; &(x'' \;:\; X\ t'') \;\rightarrow\; (x \;:\; X\ t) \;\rightarrow \\
&x''\ `\mathit{ReachableFrom}`\ x \;\rightarrow\; t''\ `\mathit{GTE}`\ t
\end{aligned}
$$

Avoidability

The notion of avoidability entails the notion of an alternative state x''. This has to fulfill three conditions:

$$
\begin{aligned}
&\mathit{AvoidableFrom} \;:\; (x' \;:\; X\ t') \;\rightarrow\; (x \;:\; X\ t) \;\rightarrow \\
&\qquad\qquad\qquad\quad x'\ \text{`}\mathit{ReachableFrom}\text{`}\ x \;\rightarrow\; (m \;:\; \mathbb{N}) \;\rightarrow\; \mathit{Type} \\
&\mathit{AvoidableFrom}\ \{\,t'\,\}\ x'\ x\ r\ m = \\
&\quad (x'' \;:\; X\ t' \ **\ (x''\ \text{`}\mathit{ReachableFrom}\text{`}\ x\ ,\ (\mathit{Viable}\ m\ x''\ ,\ \mathit{Not}\ (x'' = x'))))
\end{aligned}
$$

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$$
\begin{aligned}
&\mathit{AvoidableFrom} \;:\; (x' : X\; t') \to (x : X\; t) \to \\
&\qquad\qquad\qquad x' \,`\mathit{ReachableFrom}`\, x \to (m : \mathbb{N}) \to \mathit{Type} \\
&\mathit{AvoidableFrom}\, \{t'\}\, x'\; x\; r\; m = \\
&\quad (x'' : X\; t' \;{*}{*}\, (x'' \,`\mathit{ReachableFrom}`\, x \;,\; (\mathit{Viable}\; m\; x'' \;,\; \mathit{Not}\; (x'' = x'))))
\end{aligned}
$$

Back to the minimal goals: under which conditions are reachability, viability and avoidability decidable?

Decision procedures

For every type (predicate) $P : Type$, $Not\ P$ is just a synonym for $P \rightarrow Void$:

$$
\begin{aligned}
&Not : Type \rightarrow Type \\
&Not\ P = P \rightarrow Void
\end{aligned}
$$

Decision procedures

For every type (predicate) $P : Type$, $Not\ P$ is just a synonym for $P \rightarrow Void$:

$$Not : Type \rightarrow Type$$
$$Not\ P = P \rightarrow Void$$

A predicate $P : Type$ is *decidable* if one can compute either a value $p : P$ or a value of type $Not\ P$:

$$Decidable : Type \rightarrow Type$$
$$Decidable\ P = Either\ P\ (Not\ P)$$

Decision procedures

Thus, the question is under which conditions one can implement

$$decReachable : (x : X\ t) \rightarrow Decidable\ (Reachable\ x)$$

$$decViable : (n : \mathbb{N}) \rightarrow (x : X\ t) \rightarrow Decidable\ (Viable\ n\ x)$$

$$\begin{aligned} decAvoidableFrom : & \{t' : \mathbb{N}\} \rightarrow \{t : \mathbb{N}\} \rightarrow \\ & (x' : X\ t') \rightarrow (x : X\ t) \rightarrow \\ & (r : x'\ \text{`}ReachableFrom\text{`}\ x) \rightarrow (n : \mathbb{N}) \rightarrow \\ & Decidable\ (AvoidableFrom\ \{t'\}\ \{t\}\ x'\ x\ r\ n) \end{aligned}$$

Decision procedures

As one would expect, the conditions

$$fX : (t : \mathbb{N}) \rightarrow Finite\ (X\ t)$$

$$fY : (t : \mathbb{N}) \rightarrow (x : X\ t) \rightarrow Finite\ (Y\ t\ x)$$

$$\begin{aligned} decElem : \{t : \mathbb{N}\} \rightarrow (x : X\ t) \rightarrow (mx : M\ (X\ t)) \rightarrow \\ Decidable\ (x\ `Elem`\ mx) \end{aligned}$$

$$\begin{aligned} decAll : \{t : \mathbb{N}\} \rightarrow (P : X\ t \rightarrow Type) \rightarrow ((x : X\ t) \rightarrow \\ Decidable\ (P\ x)) \rightarrow (mx : M\ (X\ t)) \rightarrow \\ Decidable\ (All\ P\ mx) \end{aligned}$$

are sufficient for decidability.

Decision procedures

The key lemma for implementing decision procedures for *Reachable*, *Viable* and *AvoidableFrom* is intuitively obvious

$$
\begin{aligned}
\mathit{finiteDecidableLemma} \; : \; & \{A \; : \; \mathit{Type}\} \; \rightarrow \\
& \{P \; : \; A \; \rightarrow \; \mathit{Type}\} \; \rightarrow \\
& \mathit{Finite} \; A \; \rightarrow \\
& ((a \; : \; A) \; \rightarrow \; \mathit{Decidable} \; (P \; a)) \; \rightarrow \\
& \mathit{Decidable} \; (a \; : \; A \; ** \; P \; a)
\end{aligned}
$$

Decision procedures

The key lemma for implementing decision procedures for *Reachable*, *Viable* and *AvoidableFrom* is intuitively obvious

$$\begin{aligned} \mathit{finiteDecidableLemma} \; : \; & \{A \; : \; \mathit{Type}\} \; \to \\ & \{P \; : \; A \; \to \; \mathit{Type}\} \; \to \\ & \mathit{Finite} \; A \; \to \\ & ((a \; : \; A) \; \to \; \mathit{Decidable} \; (P \; a)) \; \to \\ & \mathit{Decidable} \; (a \; : \; A \; {*}{*} \; P \; a) \end{aligned}$$

but implementing *finiteDecidableLemma* is not trivial!

Decision procedures

With *finiteDecidableLemma*, *fY* and decidability of *Elem* one immediately has decidability of *Pred*

$$
\begin{aligned}
&decPred : \{t : \mathbb{N}\} \rightarrow (x : X\ t) \rightarrow (x' : X\ (S\ t)) \rightarrow \\
&\qquad\qquad Decidable\ (x\ `Pred`\ x') \\
&decPred\ \{t\}\ x\ x' = finiteDecidableLemma\ (fY\ \ t\ x)\ prf\ \textbf{where} \\
&\quad prf\ \ : (y\ :\ Y\ t\ x)\ \rightarrow\ Decidable\ (x'\ `Elem`\ (step\ t\ x\ y)) \\
&\quad prf\ \ y = decElem\ x'\ (step\ t\ x\ y)
\end{aligned}
$$

Decision procedures

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$$\begin{aligned}
&\mathit{decPred} : \{t : \mathbb{N}\} \to (x : X\ t) \to (x' : X\ (S\ t)) \to \\
&\qquad\qquad \mathit{Decidable}\ (x\ `\mathit{Pred}`\ x') \\
&\mathit{decPred}\ \{t\}\ x\ x' = \mathit{finiteDecidableLemma}\ (\mathit{fY}\ t\ x)\ \mathit{prf}\ \textbf{where} \\
&\quad \mathit{prf}\ : (y : Y\ t\ x) \to \mathit{Decidable}\ (x'\ `\mathit{Elem}`\ (\mathit{step}\ t\ x\ y)) \\
&\quad \mathit{prf}\ y = \mathit{decElem}\ x'\ (\mathit{step}\ t\ x\ y)
\end{aligned}$$

Remember

$$\mathit{Pred}\ \{t\}\ x\ x' = (y : Y\ t\ x\ {*}{*}\ x'\ `\mathit{Elem}`\ (\mathit{step}\ t\ x\ y))$$

Decision procedures

and, with

$$decPair \;:\; Decidable\; p \;\rightarrow\; Decidable\; q \;\rightarrow\; Decidable\,(p, q)$$

decidability of *Reachable*:

$$
\begin{array}{l}
decReachable \;:\; \{\, t' \;:\; \mathbb{N} \,\} \;\rightarrow\; (x' \;:\; X\; t') \;\rightarrow\; Decidable\,(Reachable\; x') \\
decReachable\,\{\, t' = Z \,\}\; x' = Left\;() \\
decReachable\,\{\, t' = S\;\; t \,\}\; x' = s1\; \textbf{where} \\
\quad s1 \;:\; Decidable\,(x \;:\; X\;\; t \;**\, (Reachable\; x, x\,`Pred`\,x')) \\
\quad s1 = finiteDecidableLemma \\
\qquad\quad (fX\;\; t) \\
\qquad\quad (\lambda x \Rightarrow decPair\;(decReachable\; x)\;(decPred\; x\; x'))
\end{array}
$$

Decision procedures

Similarly, one can implement (prove) decidability of *Viable*:

$$
\begin{aligned}
&\mathit{decViable} \;:\; \{t : \mathbb{N}\} \to (n : \mathbb{N}) \to (x : X\ t) \to \\
&\qquad\qquad\quad \mathit{Decidable}\,(\mathit{Viable}\ n\ x)\\
&\mathit{decViable}\,\{t\}\ Z\ x = \mathit{Left}\ ()\\
&\mathit{decViable}\,\{t\}\ (S\ m)\ x = \mathit{s3}\ \textbf{where}\\
&\quad \mathit{s1} \quad : (y : Y\ t\ x) \to \mathit{Decidable}\,(\mathit{All}\,(\mathit{Viable}\ m)\,(\mathit{step}\ t\ x\ y))\\
&\quad \mathit{s1}\ y = \mathit{decAll}\,(\mathit{Viable}\ m)\,(\mathit{decViable}\ m)\,(\mathit{step}\ t\ x\ y)\\
&\quad \mathit{s2} \quad : \mathit{Decidable}\,(y : Y\ t\ x \ast\ast \mathit{All}\,(\mathit{Viable}\ m)\,(\mathit{step}\ t\ x\ y))\\
&\quad \mathit{s2} \quad = \mathit{finiteDecidableLemma}\,(\mathit{fY}\ t\ x)\ \mathit{s1}\\
&\quad \mathit{s3} \quad : \mathit{Decidable}\,(\mathit{Viable}\,(S\ m)\ x)\\
&\quad \mathit{s3} \quad = \mathit{s2}
\end{aligned}
$$

Decision procedures

Similarly, one can implement (prove) decidability of *Viable*:

$$
\begin{aligned}
&decViable \;:\; \{t : \mathbb{N}\} \to (n : \mathbb{N}) \to (x : X\ t) \to \\
&\qquad\qquad\quad Decidable\,(Viable\ n\ x) \\
&decViable\,\{t\}\ Z\ x = Left\ () \\
&decViable\,\{t\}\ (S\ m)\ x = s3\ \textbf{where} \\
&\quad s1 \quad : (y : Y\ t\ x) \to Decidable\,(All\,(Viable\ m)\,(step\ t\ x\ y)) \\
&\quad s1\ y = decAll\,(Viable\ m)\,(decViable\ m)\,(step\ t\ x\ y) \\
&\quad s2 \quad : Decidable\,(y : Y\ t\ x \ast\ast\ All\,(Viable\ m)\,(step\ t\ x\ y)) \\
&\quad s2 \quad = finiteDecidableLemma\,(fY\ t\ x)\ s1 \\
&\quad s3 \quad : Decidable\,(Viable\,(S\ m)\ x) \\
&\quad s3 \quad = s2
\end{aligned}
$$

Implementing a decision procedure for *AvoidableFrom* is a bit more complicated but conceptually equivalent.

Wrap-up

- It is not difficult to formalize viability, reachability and avoidability for a large class of decision problems.

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- It is not difficult to formalize viability, reachability and avoidability for a large class of decision problems.
- The specific aspects of a concrete decision problem are captured by four abstractions: X, Y, M and *step*.
- In particular, different kinds of uncertainty (for instance, of the models underlying *step*) are covered by M, a monadic container.
- For finite X and Y decision procedures for viability, reachability and avoidability can be derived rigorously.
- Decidable generic [viability, reachability, avoidability] notions are hopefully a good starting point for deriving decidable domain specific notions: sustainability, adaptability, resilience, ...

Acknowledgments

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These slides:
`https://github.com/nicolabotta/SeqDecProbs/tree/master/talks/2015.06.rd4_seminar`

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The code shown in these slides:
`https://github.com/nicolabotta/SeqDecProbs/blob/master/talks/2015.06.rd4_seminar/code/Theory.lidr`