Formalizing reachability, viability and avoidability in the context of sequential decision problems

Nicola Botta

Outline

- Why formalizing what?
- Minimal goals
- Sequential decision problems
- Reachability, viability and avoidability
- Decision procedures
- Wrap-up

Why formalizing what?

- *International emissions trading: Good or bad?*, Holtsmark & Sommervoll, 2012: "[. . .] we find that an agreement with international emissions trading leads to increased emissions and reduced efficiency."
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- *Confronting Climate Change: Avoiding the Unmanageable and Managing the Unavoidable*, P. Raven, R. Bierbaum, J. Holdren, UN-Sigma Xi Climate Change Report, 2007.

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Some notion of avoidability is implicit in the WG3_IPCC_AR5_2014 definitions of

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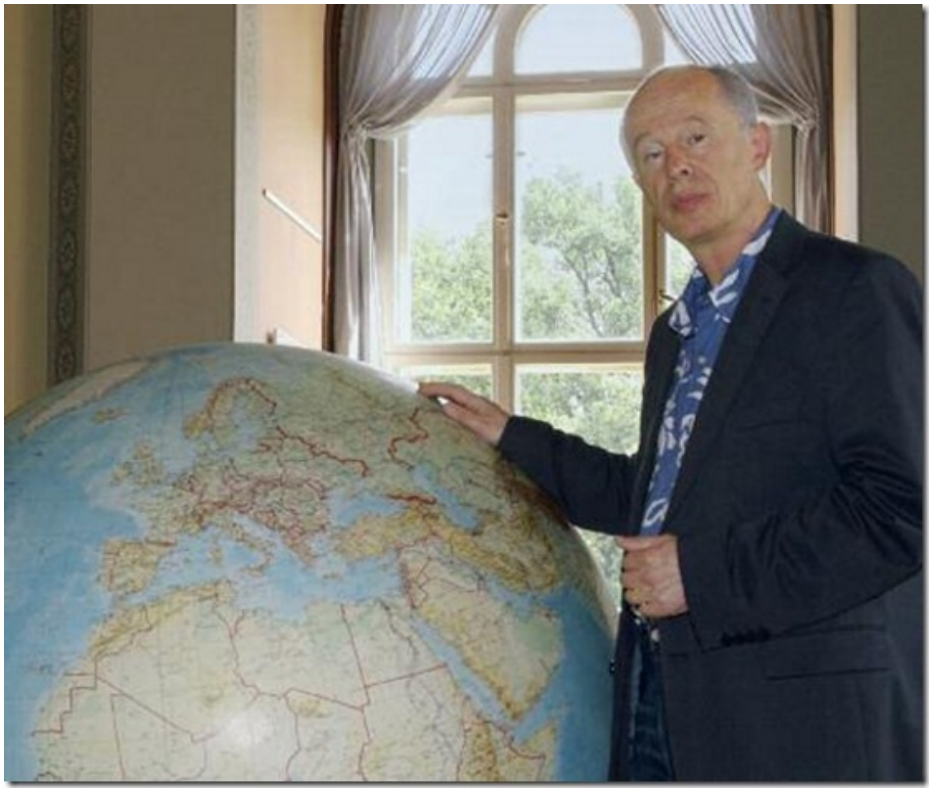
But what does it mean (for atmospheric GHG concentrations) to be *avoidable*?

Why formalizing what?



"Die Rolle der Klimaforschung bleibt weiterhin, die Problemfakten auf den Tisch zu knallen und Optionen für geeignete Lösungswege zu identifizieren."

H.-J. Schellnhuber in *Frankfurter Allgemeine* from 2012-06-19

Why formalizing what?

But how can we produce "hard facts" if the notions used to phrase specific, concrete problems are ambiguous, devoid of precise, well established, meanings?

Minimal goals

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Further questions, goals

- Can one exploit decidability to derive useful avoidability (levity?) measures?

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Further questions, goals

- Can one exploit decidability to derive useful avoidability (levity?) measures?
- Can one refine decidable notions of viability, avoidability to derive decidable notions (measures?) of sustainability, adaptability, resilience?

Sequential decision problems (intuition)

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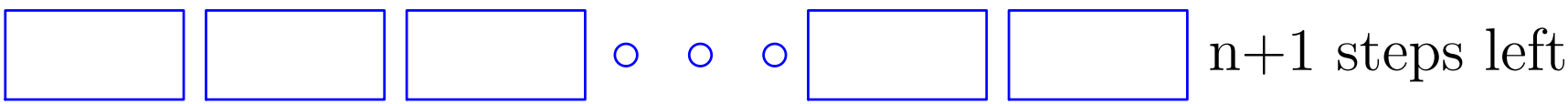
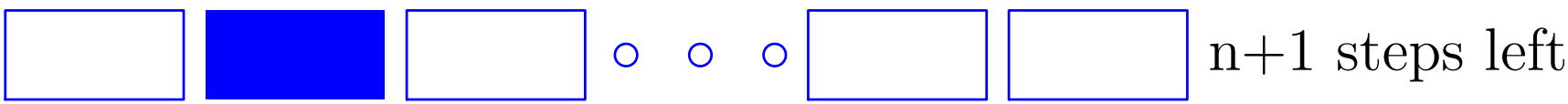
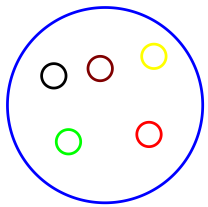
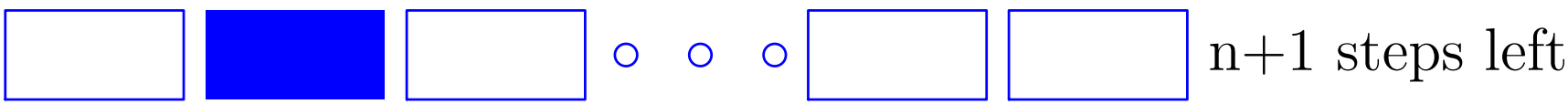


You are here. . .

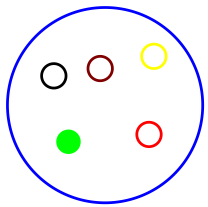


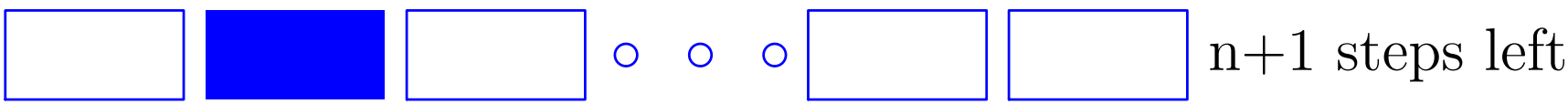
These are your options. . .



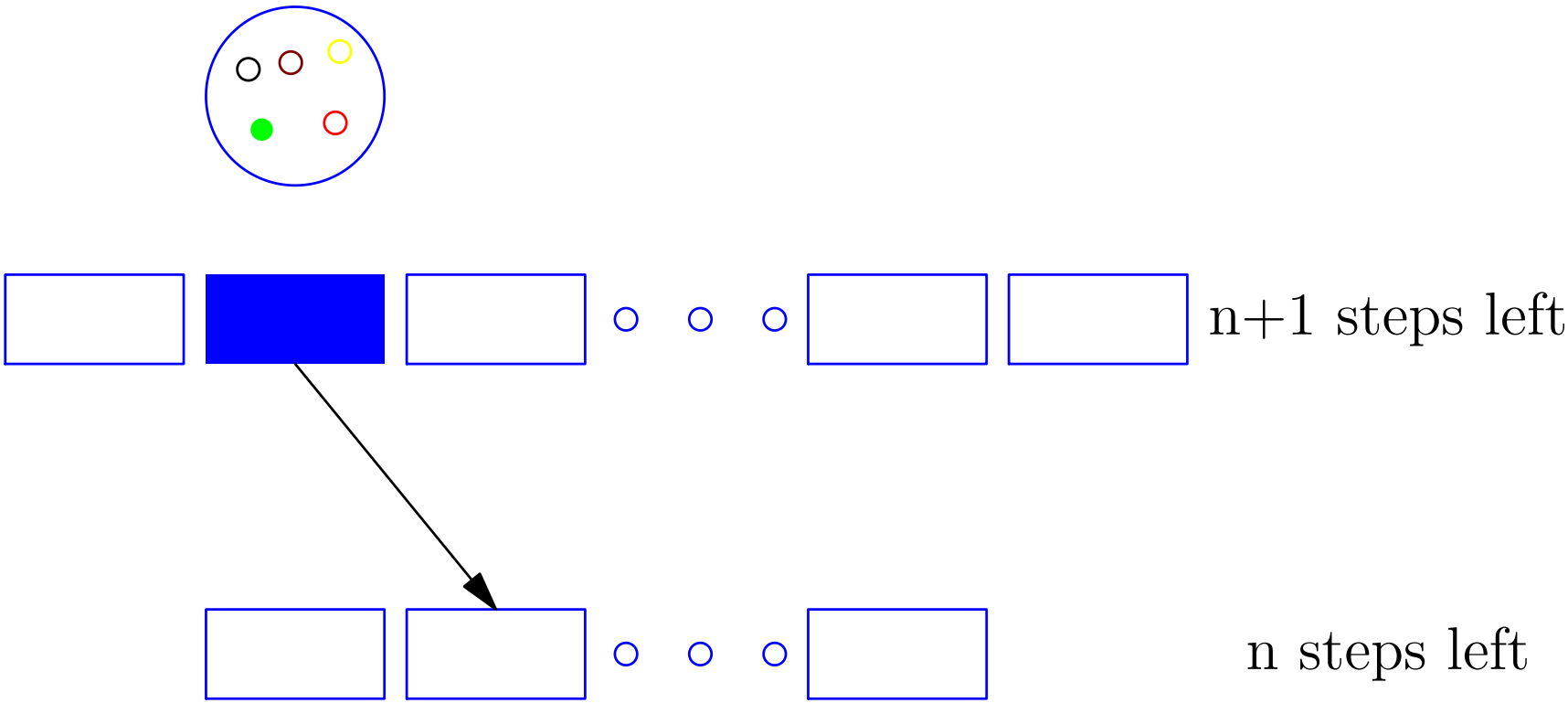


Pick one!

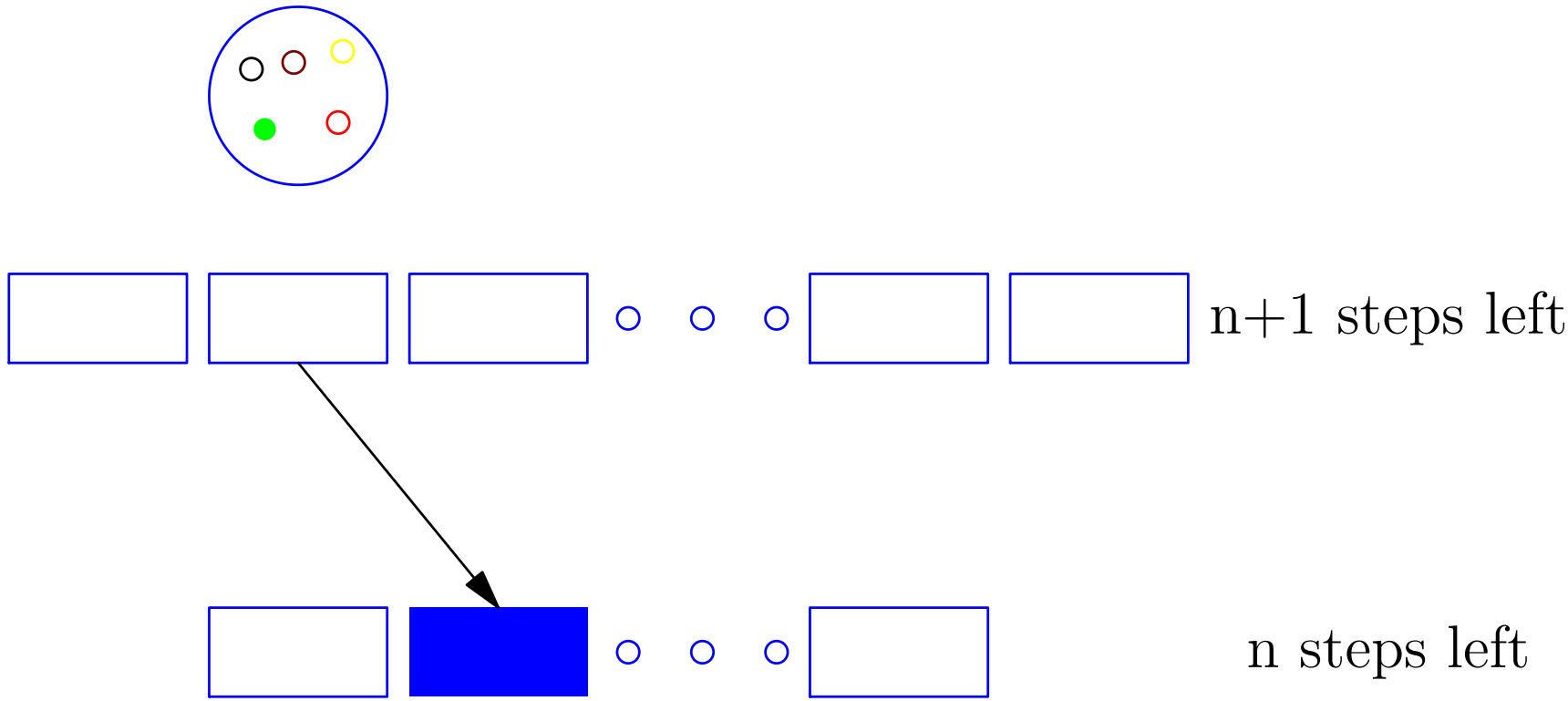




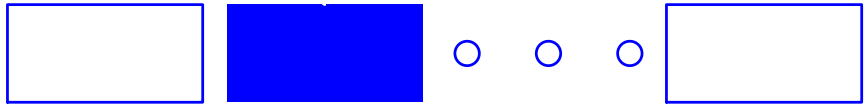
Advance one step. . .



. . . collect rewards . . .



. . . and go!



n steps left

General sequential decision problems (intuition)

This intuition is a bit too simplistic . . .

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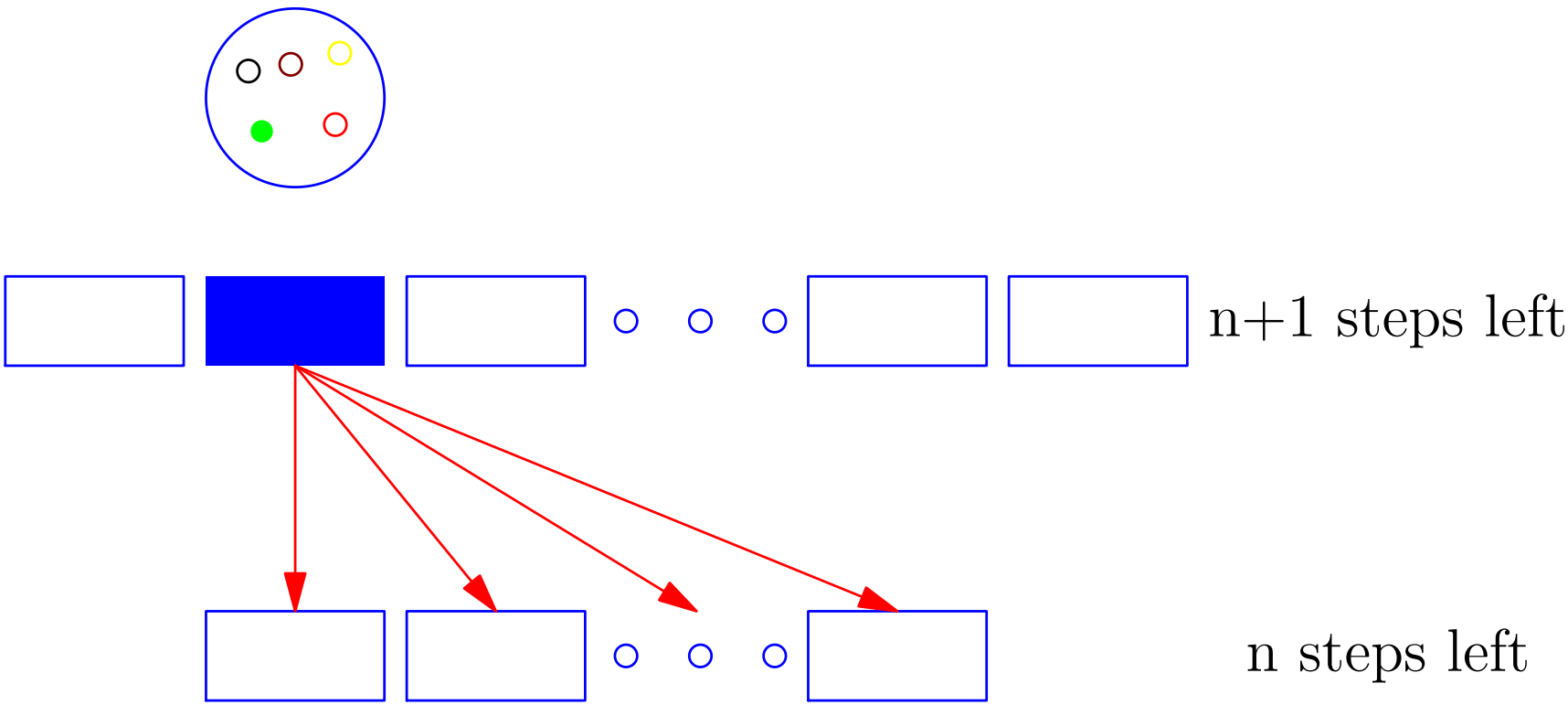
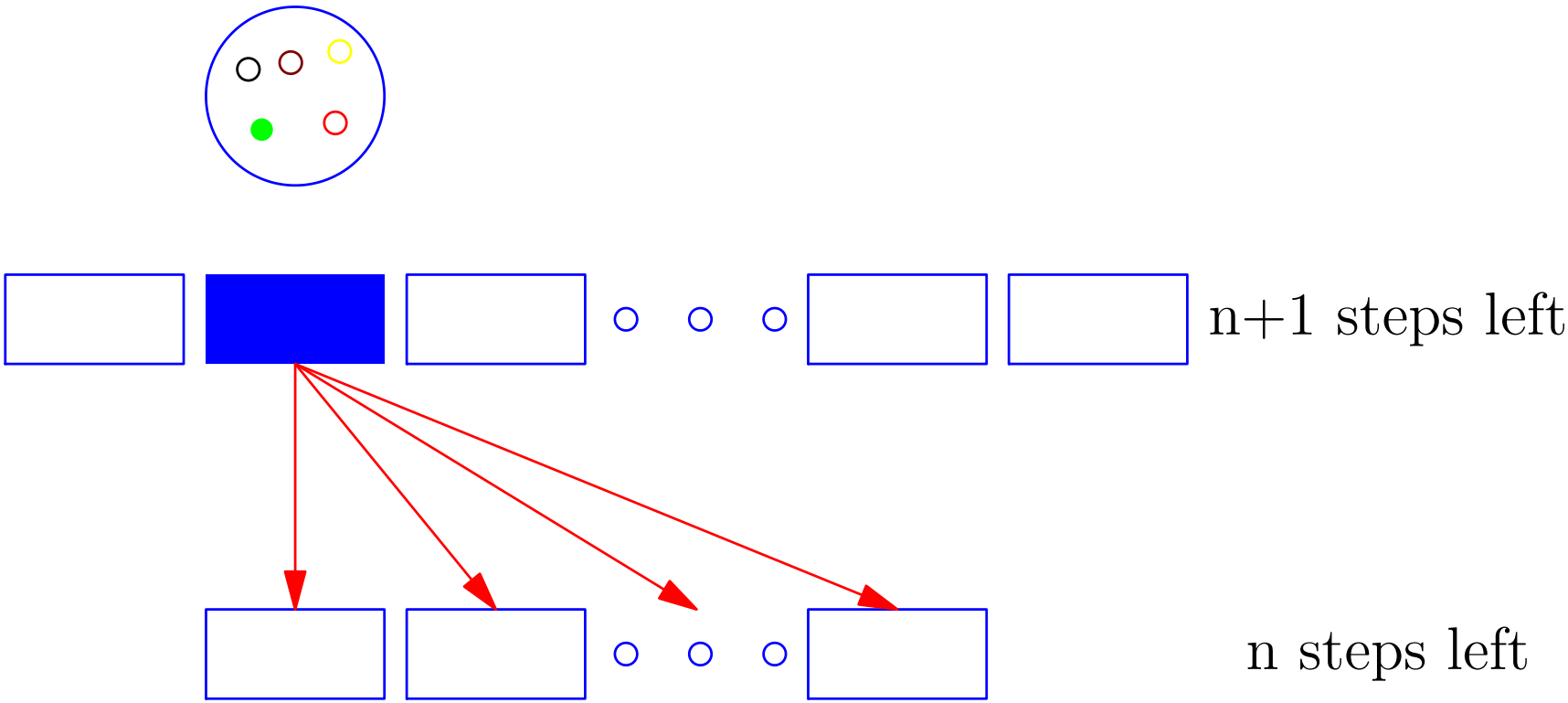


Notation (language)

Idris	set theory, logic
$A : Type$	A is a set
$a : A$	$a \in A$
$f : A \to B$	$f : A \to B$
$b = f\ a$	$b = f(a)$
$(a + b) * c$	$(a + b) * c$
$P : Type$	P is a predicate
$p : P$	p is a proof of P
$p : P \to Void$	p is a proof of $\neg P$
$P \to Q$	P implies Q
$P : A \to Type$	P is a predicate on A
$pa : P\ a$	pa is a proof of $P(a)$
$(a : A ** P\ a)$	there exists an $a \in A$ such that $P(a)$ holds
$(a : A) \to P\ a$	forall $a \in A$, $P(a)$ holds

Figure: Curry-Howard correspondence relating Idris and set theory, logic.

Sequential decision problems (basic notions)



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At each decision step, a set of possible states:

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A transition function

$$\mathit{step} : (t : \mathbb{N}) \to (x : X\ t) \to (y : Y\ t\ x) \to M\ (X\ (S\ t))$$

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What are M and S?

Sequential decision problems (uncertainties)

$S\ t$ is just the successor of t:

```
data ℕ : Type where
  Z : ℕ
  S : ℕ → ℕ
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- deterministic problems: $M = Id$
- non-deterministic problems: $M = List$
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```
data Prob : Type → Type where
  mkProb : (as : Vect n a) → (ps : Vect n Float) →
           sum ps = 1.0 → Prob a
```

Sequential decision problems (container monad)

Formally, M is a container monad, that is M is a monad:

$$
\begin{aligned}
&\mathit{fmap} : (a \to b) \to M\ a \to M\ b \\
&\mathit{ret} : a \to M\ a \\
&\mathit{bind} : M\ a \to (a \to M\ b) \to M\ b \\
&\mathit{join} : M\ (M\ a) \to M\ a
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{functorSpec1} : \mathit{fmap} \circ \mathit{id} = \mathit{id} \\
&\mathit{functorSpec2} : \mathit{fmap}\ (f \circ g) = (\mathit{fmap}\ f) \circ (\mathit{fmap}\ g) \\
&\mathit{monadSpec1} : (\mathit{fmap}\ f) \circ \mathit{ret} = \mathit{ret} \circ f \\
&\mathit{monadSpec2} : \mathit{bind}\ (\mathit{ret}\ a)\ f = f\ a \\
&\mathit{monadSpec3} : \mathit{bind}\ \mathit{ma}\ \mathit{ret} = \mathit{ma} \\
&\mathit{monadSpec4} : \{f : a \to M\ b\} \to \{g : b \to M\ c\} \to \\
&\qquad \mathit{bind}\ (\mathit{bind}\ \mathit{ma}\ f)\ g = \mathit{bind}\ \mathit{ma}\ (\lambda x \Rightarrow \mathit{bind}\ (f\ x)\ g) \\
&\mathit{monadSpec5} : \mathit{join}\ \mathit{mma} = \mathit{bind}\ \mathit{mma}\ \mathit{id}
\end{aligned}
$$

Sequential decision problems (container monad)

and M is a container:

$$\begin{aligned} &\mathit{Elem} : a \to M\ a \to \mathit{Type} \\ &\mathit{All} \quad : (a \to \mathit{Type}) \to M\ a \to \mathit{Type} \end{aligned}$$

$$\begin{aligned} &\mathit{containerSpec1} : a\ \text{`}\mathit{Elem}\text{`}\ (\mathit{ret}\ a) \\ &\mathit{containerSpec2} : a\ \text{`}\mathit{Elem}\text{`}\ ma \to ma\ \text{`}\mathit{Elem}\text{`}\ mma \to a\ \text{`}\mathit{Elem}\text{`}\ (\mathit{join}\ mma) \\ &\mathit{containerSpec3} : \mathit{All}\ p\ ma \to a\ \text{`}\mathit{Elem}\text{`}\ ma \to p\ a \end{aligned}$$

Sequential decision problems (basic notions)

Thus, a concrete sequential decision problem is defined (up to the rewards) in terms of 4 entities: X, Y, M and *step*

$$X : (t : \mathbb{N}) \to Type$$

$$Y : (t : \mathbb{N}) \to (x : X\ t) \to Type$$

$$M : Type \to Type$$

$$step : (t : \mathbb{N}) \to (x : X\ t) \to (y : Y\ t\ x) \to M\ (X\ (S\ t))$$

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We formalize reachability, viability and avoidability in terms of these notions

Reachability and viability (intuition)

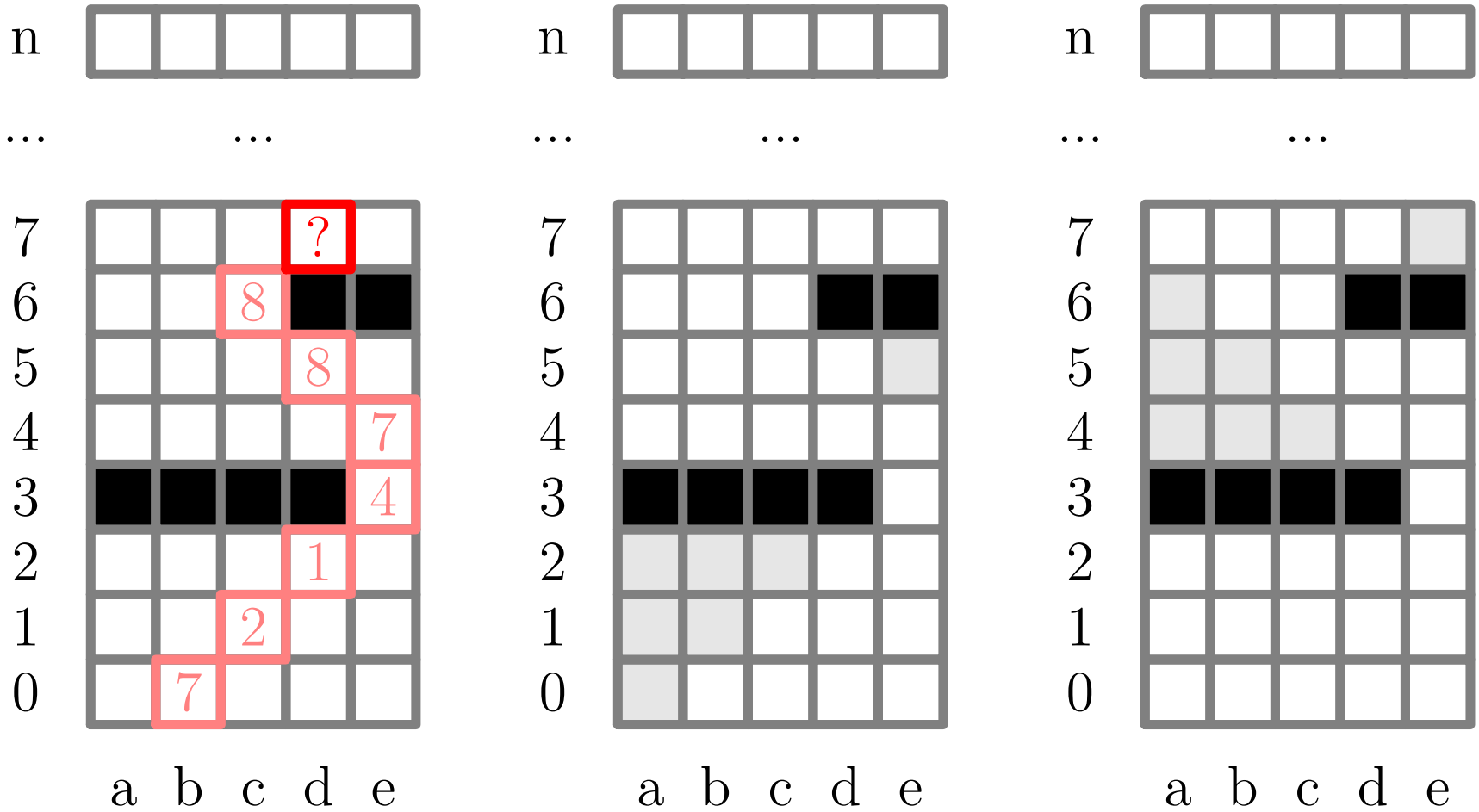

Figure: Possible evolution starting from b (left), states with limited viability (middle) and unreachable states (right).

Predecessor relation, reachability and viability

The (possible) predecessor relation:

$$Pred\ :\ X\ t\ \rightarrow\ X\ (S\ t)\ \rightarrow\ Type$$
$$Pred\ \{t\}\ x\ x' = (y\ :\ Y\ t\ x\ **\ x'\ `Elem`\ (step\ t\ x\ y))$$

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reachability

$$Reachable\ :\ X\ t'\ \rightarrow\ Type$$
$$Reachable\ \{t' = Z\}\ x'\ \ = Unit$$
$$Reachable\ \{t' = S\ t\}\ x' = (x\ :\ X\ t\ **\ (Reachable\ x, x\ `Pred`\ x'))$$

Predecessor relation, reachability and viability

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&\mathit{Pred} : X\ t \to X\ (S\ t) \to \mathit{Type} \\
&\mathit{Pred}\ \{t\}\ x\ x' = (y : Y\ t\ x \ast\ast x'\ `\mathit{Elem}`\ (\mathit{step}\ t\ x\ y))
\end{aligned}
$$

reachability

$$
\begin{aligned}
&\mathit{Reachable} : X\ t' \to \mathit{Type} \\
&\mathit{Reachable}\ \{t' = Z\}\ x' = \mathit{Unit} \\
&\mathit{Reachable}\ \{t' = S\ t\}\ x' = (x : X\ t \ast\ast (\mathit{Reachable}\ x, x\ `\mathit{Pred}`\ x'))
\end{aligned}
$$

and viability

$$
\begin{aligned}
&\mathit{Viable} : (n : \mathbb{N}) \to X\ t \to \mathit{Type} \\
&\mathit{Viable}\ \{t\}\ Z\ x = \mathit{Unit} \\
&\mathit{Viable}\ \{t\}\ (S\ m)\ x = (y : Y\ t\ x \ast\ast \mathit{All}\ (\mathit{Viable}\ m)\ (\mathit{step}\ t\ x\ y))
\end{aligned}
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- We are interested in the avoidability of "possible" future states. Specifically, we are interested in the avoidability of states which are reachable from some given state.
- The notion of avoidability entails the notion of an alternative.

Avoidability

We are interested in the avoidability of states which are reachable from some given state:

```
ReachableFrom : X t'' → X t → Type
ReachableFrom {t'' = Z} {t} x'' x    = (t = Z, x = x'')
ReachableFrom {t'' = S t'} {t} x'' x =
  Either (t = S t', x = x'')
         (x' : X t' ** (x' `ReachableFrom` x, x' `Pred` x''))
```

where

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data Either a b = Left a | Right b
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Proof of concept: show that

```
reachableFromLemma : (x'' : X t'') → (x : X t) →
                     x'' `ReachableFrom` x → t'' `GTE` t
```

Avoidability

The notion of avoidability entails the notion of an alternative state x''. This has to fulfill three conditions:

$$\begin{aligned}
&\mathit{AvoidableFrom} : (x' : X\ t') \to (x : X\ t) \to \\
&\qquad x' \text{ `}\mathit{ReachableFrom}\text{` } x \to (m : \mathbb{N}) \to \mathit{Type} \\
&\mathit{AvoidableFrom}\ \{t'\}\ x'\ x\ r\ m = \\
&\quad (x'' : X\ t' ** (x'' \text{ `}\mathit{ReachableFrom}\text{` } x\ ,\ (\mathit{Viable}\ m\ x''\ ,\ \mathit{Not}\ (x'' = x'))))
\end{aligned}$$

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\end{aligned}$$

Back to the minimal goals: under which conditions are reachability, viability and avoidability decidable?

Decision procedures

For every type (predicate) $P : Type$, $Not\ P$ is just a synonym for $P \to Void$:

$Not : Type \to Type$
$Not\ P = P \to Void$

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For every type (predicate) $P : \mathit{Type}$, $\mathit{Not}\ P$ is just a synonym for $P \to \mathit{Void}$:

$$\mathit{Not} : \mathit{Type} \to \mathit{Type}$$
$$\mathit{Not}\ P = P \to \mathit{Void}$$

A predicate $P : \mathit{Type}$ is *decidable* if one can compute either a value $p : P$ or a value of type $\mathit{Not}\ P$:

$$\mathit{Decidable} : \mathit{Type} \to \mathit{Type}$$
$$\mathit{Decidable}\ P = \mathit{Either}\ P\ (\mathit{Not}\ P)$$

Decision procedures

Thus, the question is under which conditions one can implement

$$\mathit{decReachable} : (x : X\ t) \to \mathit{Decidable}\ (\mathit{Reachable}\ x)$$

$$\mathit{decViable} : (n : \mathbb{N}) \to (x : X\ t) \to \mathit{Decidable}\ (\mathit{Viable}\ n\ x)$$

$$\begin{aligned}\mathit{decAvoidableFrom} : {} & \{t' : \mathbb{N}\} \to \{t : \mathbb{N}\} \to \\ & (x' : X\ t') \to (x : X\ t) \to \\ & (r : x'\ \text{`}\mathit{ReachableFrom}\text{`}\ x) \to (n : \mathbb{N}) \to \\ & \mathit{Decidable}\ (\mathit{AvoidableFrom}\ \{t'\}\ \{t\}\ x'\ x\ r\ n)\end{aligned}$$

Decision procedures

As one would expect, the conditions

$$fX : (t : \mathbb{N}) \to Finite\ (X\ t)$$

$$fY : (t : \mathbb{N}) \to (x : X\ t) \to Finite\ (Y\ t\ x)$$

$$decElem : \{t : \mathbb{N}\} \to (x : X\ t) \to (mx : M\ (X\ t)) \to Decidable\ (x\ `Elem`\ mx)$$

$$decAll : \{t : \mathbb{N}\} \to (P : X\ t \to Type) \to ((x : X\ t) \to Decidable\ (P\ x)) \to (mx : M\ (X\ t)) \to Decidable\ (All\ P\ mx)$$

are sufficient for decidability.

Decision procedures

The key lemma for implementing decision procedures for *Reachable*, *Viable* and *AvoidableFrom* is intuitively obvious

$$
\begin{aligned}
\mathit{finiteDecidableLemma} : {} & \{A : \mathit{Type}\} \to \\
& \{P : A \to \mathit{Type}\} \to \\
& \mathit{Finite}\ A \to \\
& ((a : A) \to \mathit{Decidable}\ (P\ a)) \to \\
& \mathit{Decidable}\ (a : A ** P\ a)
\end{aligned}
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& \mathit{Finite}\ A \to \\
& ((a : A) \to \mathit{Decidable}\ (P\ a)) \to \\
& \mathit{Decidable}\ (a : A ** P\ a)
\end{aligned}
$$

but implementing *finiteDecidableLemma* is not trivial!

Decision procedures

With *finiteDecidableLemma*, *fY* and decidability of *Elem* one immediately has decidability of *Pred*

$$
\begin{aligned}
&\mathit{decPred} : \{t : \mathbb{N}\} \to (x : X\ t) \to (x' : X\ (S\ t)) \to \\
&\qquad \mathit{Decidable}\ (x\ \text{`}\mathit{Pred}\text{`}\ x') \\
&\mathit{decPred}\ \{t\}\ x\ x' = \mathit{finiteDecidableLemma}\ (\mathit{fY}\ t\ x)\ \mathit{prf}\ \textbf{where} \\
&\quad \mathit{prf} : (y : Y\ t\ x) \to \mathit{Decidable}\ (x'\ \text{`}\mathit{Elem}\text{`}\ (\mathit{step}\ t\ x\ y)) \\
&\quad \mathit{prf}\ y = \mathit{decElem}\ x'\ (\mathit{step}\ t\ x\ y)
\end{aligned}
$$

Decision procedures

With *finiteDecidableLemma*, *fY* and decidability of *Elem* one immediately has decidability of *Pred*

```
decPred : {t : ℕ} → (x : X t) → (x' : X (S t)) →
          Decidable (x `Pred` x')
decPred {t} x x' = finiteDecidableLemma (fY t x) prf where
  prf : (y : Y t x) → Decidable (x' `Elem` (step t x y))
  prf y = decElem x' (step t x y)
```

Remember

```
Pred {t} x x' = (y : Y t x ** x' `Elem` (step t x y))
```

Decision procedures

and, with

$$decPair : Decidable\ p \to Decidable\ q \to Decidable\ (p, q)$$

decidability of *Reachable*:

$$decReachable : \{t' : \mathbb{N}\} \to (x' : X\ t') \to Decidable\ (Reachable\ x')$$
$$decReachable\ \{t' = Z\}\ x' = Left\ ()$$
$$decReachable\ \{t' = S\ t\}\ x' = s1\ \textbf{where}$$
$$s1 : Decidable\ (x : X\ t\ **\ (Reachable\ x, x\ `Pred`\ x'))$$
$$s1 = finiteDecidableLemma$$
$$(fX\ t)$$
$$(\lambda x \Rightarrow decPair\ (decReachable\ x)\ (decPred\ x\ x'))$$

Decision procedures

Similarly, one implement (prove) decidability of *Viable*:

```
decViable : {t : ℕ} → (n : ℕ) → (x : X t) →
            Decidable (Viable n x)
decViable {t} Z x = Left ()
decViable {t} (S m) x = s3 where
  s1 : (y : Y t x) → Decidable (All (Viable m) (step t x y))
  s1 y = decAll (Viable m) (decViable m) (step t x y)
  s2 : Decidable (y : Y t x ** All (Viable m) (step t x y))
  s2 = finiteDecidableLemma (fY t x) s1
  s3 : Decidable (Viable (S m) x)
  s3 = s2
```

Decision procedures

Similarly, one implement (prove) decidability of *Viable*:

```
decViable : {t : ℕ} → (n : ℕ) → (x : X t) →
            Decidable (Viable n x)
decViable {t} Z x = Left ()
decViable {t} (S m) x = s3 where
  s1  : (y : Y t x) → Decidable (All (Viable m) (step t x y))
  s1 y = decAll (Viable m) (decViable m) (step t x y)
  s2  : Decidable (y : Y t x ** All (Viable m) (step t x y))
  s2  = finiteDecidableLemma (fY t x) s1
  s3  : Decidable (Viable (S m) x)
  s3  = s2
```

Implementing a decision procedure for *AvoidableFrom* is a bit more complicated but conceptually equivalent.

Wrap-up

- It is not difficult to formalize viability, reachability and avoidability for a large class of decision problems.

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Wrap-up

- It is not difficult to formalize viability, reachability and avoidability for a large class of decision problems.
- The specific aspects of a concrete decision problem are captured by four abstractions: X, Y, M and *step*.
- In particular, different kinds of uncertainty (for instance, of the models underlying *step*) are covered by M, a monadic container.
- For finite X and Y decision procedures for viability, reachability and avoidability can be derived rigorously.
- Decidable generic [viability, reachability, avoidability] notions are hopefully a good starting point for deriving decidable domain specific notions: sustainability, adaptability, resilience, ...

Acknowledgments

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`https://github.com/nicolabotta/SeqDecProbs/tree/master/talks/2015.06.rd4_seminar`

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The code shown in these slides:
`https://github.com/nicolabotta/SeqDecProbs/blob/master/talks/2015.06.rd4_seminar/code/Theory.lidr`