

Formalizing reachability, viability and avoidability in the context of sequential decision problems

Nicola Botta

Outline

- ▶ Why formalizing what?
- ▶ Minimal goals
- ▶ Sequential decision problems
- ▶ Reachability, viability and avoidability
- ▶ Decision procedures
- ▶ Conclusions

Why formalizing what?

- ▶ *International emissions trading: Good or bad?*, Holtsmark & Sommervoll, 2012: “[...] we find that an agreement with international emissions trading leads to increased emissions and reduced efficiency.”
- ▶ *The case for international emission trade in the absence of cooperative climate policy*, Carbone et al., 2009: “[...] we find that emission trade agreements can be effective [...]”

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- ▶ *Confronting Climate Change: Avoiding the Unmanageable and Managing the Unavoidable*, P. Raven, R. Bierbaum, J. Holdren, UN-Sigma Xi Climate Change Report, 2007.

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Some notion of avoidability is implicit in the WG3_IPCC_AR5_2014 definitions of

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But what does it mean (for atmospheric GHG concentrations) to be *avoidable*?

Why formalizing what?



“Die Rolle der Klimaforschung bleibt weiterhin, die Problemfakten auf den Tisch zu knallen und Optionen für geeignete Lösungswege zu identifizieren.”

H.-J. Schellnhuber in *Frankfurter Allgemeine* from 2012-06-19

Why formalizing what?

But how can we produce “hard facts” if the notions used to phrase specific, concrete problems are ambiguous, devoid of precise, well established, meanings?

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Further questions, goals

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Further questions, goals

- ▶ Can one exploit decidability to derive useful avoidability (levity?) measures?

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Further questions, goals

- ▶ Can one exploit decidability to derive useful avoidability (levity?) measures?
- ▶ Can one refine decidable notions of viability, avoidability to derive decidable notions (measures?) of sustainability, adaptability, resilience?

Sequential decision problems (intuition)

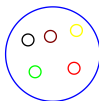
Sequential decision problems (intuition)



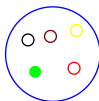
You are here. . .



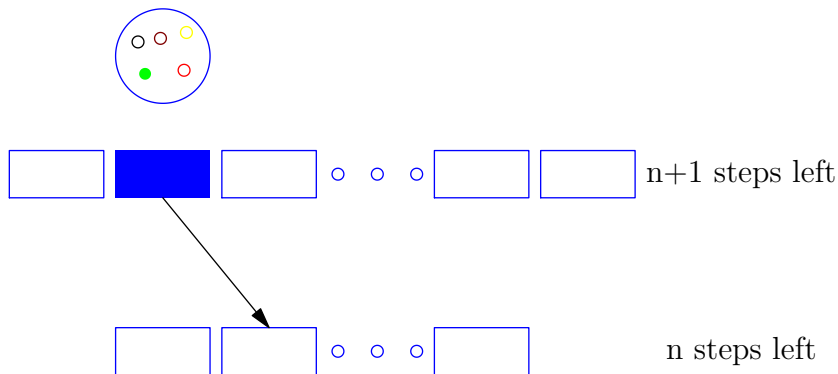
These are your options. . .



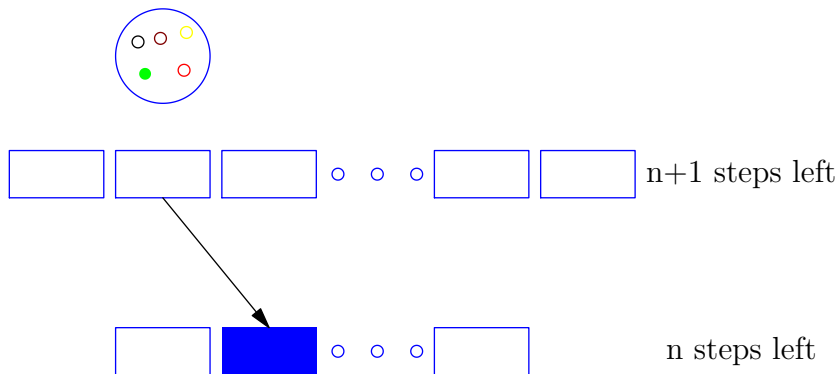
Pick one!



Advance one step...



... collect rewards ...



... and go!



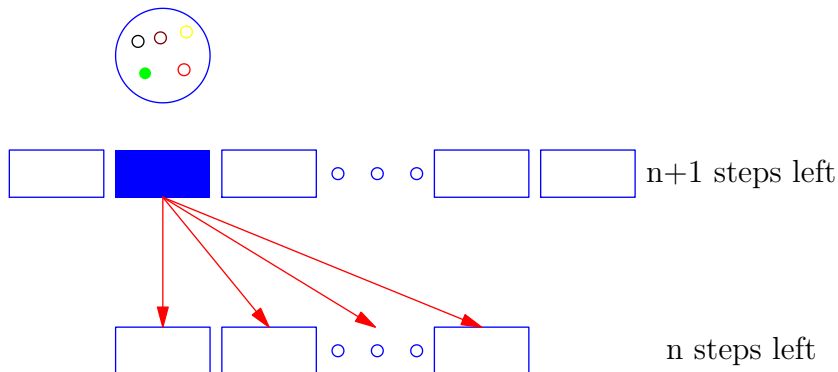
n steps left

General sequential decision problems (intuition)

This intuition is a bit too simplistic ...

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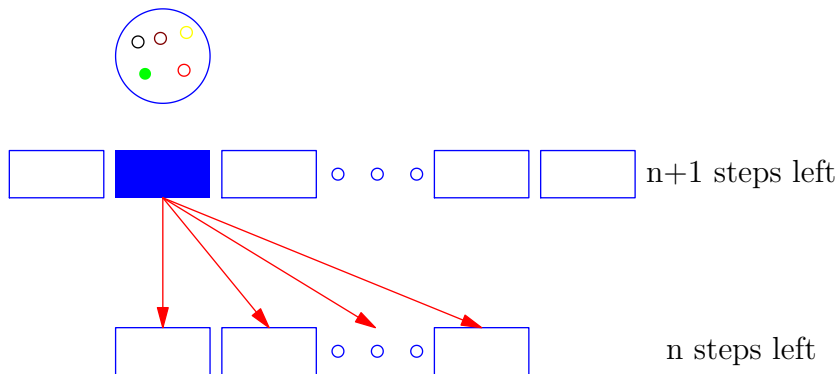


Notation (language)

Idris	set theory, logic
$A : \text{Type}$	A is a set
$a : A$	$a \in A$
$f : A \rightarrow B$	$f : A \rightarrow B$
$b = f\ a$	$b = f(a)$
$(a + b) * c$	$(a + b) * c$
$P : \text{Type}$	P is a predicate
$p : P$	p is a proof of P
$p : P \rightarrow \text{Void}$	p is a proof of $\neg P$
$P \rightarrow Q$	P implies Q
$P : A \rightarrow \text{Type}$	P is a predicate on A
$pa : P\ a$	pa is a proof of $P(a)$
$(a : A ** P\ a)$	there exists an $a \in A$ such that $P(a)$ holds
$(a : A) \rightarrow P\ a$	forall $a \in A$, $P(a)$ holds

Figure: Curry-Howard correspondence relating Idris and set theory, logic.

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A transition function

$$step : (t : \mathbb{N}) \rightarrow (x : X\ t) \rightarrow (y : Y\ t\ x) \rightarrow M\ (X\ (S\ t))$$

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What are M and S ?

Sequential decision problems (uncertainties)

$S\ t$ is just the successor of t :

```
data  $\mathbb{N}$  : Type where
```

```
   $Z$  :  $\mathbb{N}$ 
```

```
   $S$  :  $\mathbb{N} \rightarrow \mathbb{N}$ 
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- ▶ deterministic problems: $M = Id$
- ▶ non-deterministic problems: $M = List$
- ▶ stochastic problems: $M = Prob$

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```
data Prob : Type  $\rightarrow$  Type where
  mkProb : (as : Vect  $n$  a)  $\rightarrow$  (ps : Vect  $n$  Float)  $\rightarrow$ 
    sum ps = 1.0  $\rightarrow$  Prob a
```

Sequential decision problems (container monad)

Formally, M is a container monad, that is M is a monad:

$$fmap : (a \rightarrow b) \rightarrow M a \rightarrow M b$$

$$ret : a \rightarrow M a$$

$$bind : M a \rightarrow (a \rightarrow M b) \rightarrow M b$$

$$join : M (M a) \rightarrow M a$$

$$functorSpec1 : fmap \circ id = id$$

$$functorSpec2 : fmap (f \circ g) = (fmap f) \circ (fmap g)$$

$$monadSpec1 : (fmap f) \circ ret = ret \circ f$$

$$monadSpec2 : bind (ret a) f = f a$$

$$monadSpec3 : bind ma ret = ma$$

$$monadSpec4 : \{f : a \rightarrow M b\} \rightarrow \{g : b \rightarrow M c\} \rightarrow \\ bind (bind ma f) g = bind ma (\lambda x \Rightarrow bind (f x) g)$$

$$monadSpec5 : join mma = bind mma id$$

Sequential decision problems (container monad)

and M is a container:

$$Elem : a \rightarrow M\ a \rightarrow Type$$
$$All : (a \rightarrow Type) \rightarrow M\ a \rightarrow Type$$
$$containerSpec1 : a \text{ 'Elem' } (ret\ a)$$
$$containerSpec2 : a \text{ 'Elem' } ma \rightarrow ma \text{ 'Elem' } mma \rightarrow a \text{ 'Elem' } (join\ mma)$$
$$containerSpec3 : All\ p\ ma \rightarrow a \text{ 'Elem' } ma \rightarrow p\ a$$

Sequential decision problems (basic notions)

Thus, a concrete sequential decision problem is defined (up to the rewards) in terms of 4 entities: X , Y , M and $step$

$$X : (t : \mathbb{N}) \rightarrow Type$$

$$Y : (t : \mathbb{N}) \rightarrow (x : X\ t) \rightarrow Type$$

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We formalize reachability, viability and avoidability in terms of these notions

Reachability and viability (intuition)

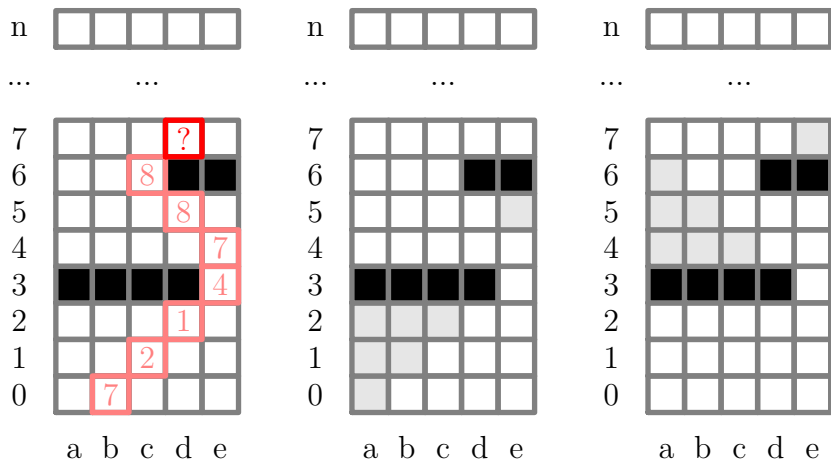


Figure: Possible evolution starting from b (left), states with limited viability (middle) and unreachable states (right).

Predecessor relation, reachability and viability

The (possible) predecessor relation:

$$Pred : X \times t \rightarrow X \times (S \times t) \rightarrow Type$$

$$Pred \{ t \} \times x \times x' = (y : Y \times t \times x \times x' \times 'Elem' (step \ t \times x \ y))$$

Predecessor relation, reachability and viability

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reachability

$$Reachable : X \ t' \rightarrow Type$$

$$Reachable \ \{t' = Z\} \ x' = Unit$$

$$Reachable \ \{t' = S \ t\} \ x' = (x : X \ t \ ** \ (Reachable \ x, x \ 'Pred' \ x'))$$

Predecessor relation, reachability and viability

The (possible) predecessor relation:

$$\begin{aligned} \text{Pred} &: X\ t \rightarrow X\ (S\ t) \rightarrow \text{Type} \\ \text{Pred}\ \{t\}\ x\ x' &= (y : Y\ t\ x\ **\ x'\ \text{'Elem'}\ (\text{step}\ t\ x\ y)) \end{aligned}$$

reachability

$$\begin{aligned} \text{Reachable} &: X\ t' \rightarrow \text{Type} \\ \text{Reachable}\ \{t' = Z\}\ x' &= \text{Unit} \\ \text{Reachable}\ \{t' = S\ t\}\ x' &= (x : X\ t\ **\ (\text{Reachable}\ x, x\ \text{'Pred'}\ x')) \end{aligned}$$

and viability

$$\begin{aligned} \text{Viable} &: (n : \mathbb{N}) \rightarrow X\ t \rightarrow \text{Type} \\ \text{Viable}\ \{t\}\ Z\ x &= \text{Unit} \\ \text{Viable}\ \{t\}\ (S\ m)\ x &= (y : Y\ t\ x\ **\ \text{All}\ (\text{Viable}\ m)\ (\text{step}\ t\ x\ y)) \end{aligned}$$

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- ▶ We are interested in the avoidability of “possible” future states. Specifically, we are interested in the avoidability of states which are reachable from some given state.
- ▶ The notion of avoidability entails the notion of an alternative.

Avoidability

We are interested in the avoidability of states which are reachable from some given state:

$$\begin{aligned}
 & \textit{ReachableFrom} : X \ t'' \rightarrow X \ t \rightarrow \textit{Type} \\
 & \textit{ReachableFrom} \ \{t'' = Z\} \ \{t\} \ x'' \ x = (t = Z, x = x'') \\
 & \textit{ReachableFrom} \ \{t'' = S \ t'\} \ \{t\} \ x'' \ x = \\
 & \quad \textit{Either} \ (t = S \ t', x = x'') \\
 & \quad (x' : X \ t' \ ** \ (x' \ \textit{'ReachableFrom'} \ x, x' \ \textit{'Pred'} \ x''))
 \end{aligned}$$

where

data *Either* *a b* = *Left* *a* | *Right* *b*

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 \end{aligned}$$

where

data $\textit{Either} \ a \ b = \textit{Left} \ a \mid \textit{Right} \ b$

Proof of concept: show that

$$\begin{aligned}
 \textit{reachableFromLemma} : (x'' : X \ t'') \rightarrow (x : X \ t) \rightarrow \\
 x'' \ \textit{'ReachableFrom'} \ x \rightarrow t'' \ \textit{'GTE'} \ t
 \end{aligned}$$

Avoidability

The notion of avoidability entails the notion of an alternative state x'' . This has to fulfill three conditions:

$$\text{AvoidableFrom} : (x' : X \ t') \rightarrow (x : X \ t) \rightarrow \\ x' \text{ 'ReachableFrom' } x \rightarrow (m : \mathbb{N}) \rightarrow \text{Type}$$

$$\text{AvoidableFrom} \{t'\} x' x \ r \ m = \\ (x'' : X \ t' \ ** (x'' \text{ 'ReachableFrom' } x , (Viable \ m \ x'' , \text{Not } (x'' = x')))))$$

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Back to the minimal goals: under which conditions are reachability, viability and avoidability decidable?

Decision procedures

For every type (predicate) $P : \text{Type}$, $\text{Not } P$ is just a synonym for $P \rightarrow \text{Void}$:

$$\text{Not} : \text{Type} \rightarrow \text{Type}$$
$$\text{Not } P = P \rightarrow \text{Void}$$

Decision procedures

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$$\text{Not } P = P \rightarrow \text{Void}$$

A predicate $P : \text{Type}$ is *decidable* if one can compute either a value $p : P$ or a value of type $\text{Not } P$:

$$\text{Decidable} : \text{Type} \rightarrow \text{Type}$$
$$\text{Decidable } P = \text{Either } P (\text{Not } P)$$

Decision procedures

Thus, the question is under which conditions one can implement

$$decReachable : (x : X \ t) \rightarrow Decidable (Reachable \ x)$$

$$decViable : (n : \mathbb{N}) \rightarrow (x : X \ t) \rightarrow Decidable (Viable \ n \ x)$$

$$\begin{aligned} decAvoidableFrom : \{t' : \mathbb{N}\} \rightarrow \{t : \mathbb{N}\} \rightarrow \\ (x' : X \ t') \rightarrow (x : X \ t) \rightarrow \\ (r : x' \text{ 'ReachableFrom' } x) \rightarrow (n : \mathbb{N}) \rightarrow \\ Decidable (AvoidableFrom \ \{t'\} \ \{t\} \ x' \ x \ r \ n) \end{aligned}$$

Decision procedures

As one would expect, the conditions

$$fX : (t : \mathbb{N}) \rightarrow \text{Finite } (X \ t)$$

$$fY : (t : \mathbb{N}) \rightarrow (x : X \ t) \rightarrow \text{Finite } (Y \ t \ x)$$

$$\text{decElem} : \{t : \mathbb{N}\} \rightarrow (x : X \ t) \rightarrow (mx : M \ (X \ t)) \rightarrow \\ \text{Decidable } (x \text{ 'Elem' } mx)$$

$$\text{decAll} : \{t : \mathbb{N}\} \rightarrow (P : X \ t \rightarrow \text{Type}) \rightarrow ((x : X \ t) \rightarrow \\ \text{Decidable } (P \ x)) \rightarrow (mx : M \ (X \ t)) \rightarrow \\ \text{Decidable } (\text{All } P \ mx)$$

are sufficient for decidability.

Decision procedures

The key lemma for implementing decision procedures for *Reachable*, *Viable* and *AvoidableFrom* is intuitively obvious

$$\begin{aligned} \text{finiteDecidableLemma} : & \{A : \text{Type}\} \rightarrow \\ & \{P : A \rightarrow \text{Type}\} \rightarrow \\ & \text{Finite } A \rightarrow \\ & ((a : A) \rightarrow \text{Decidable } (P \ a)) \rightarrow \\ & \text{Decidable } (a : A ** P \ a) \end{aligned}$$

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but implementing *finiteDecidableLemma* is not trivial!

Decision procedures

With *finiteDecidableLemma*, *fY* and decidability of *Elem* one immediately has decidability of *Pred*

$$\text{decPred} : \{t : \mathbb{N}\} \rightarrow (x : X \ t) \rightarrow (x' : X \ (S \ t)) \rightarrow \\ \text{Decidable } (x \text{ 'Pred' } x')$$

$$\text{decPred } \{t\} \ x \ x' = \text{finiteDecidableLemma } (fY \ t \ x) \ \text{prf} \ \mathbf{where} \\ \text{prf} : (y : Y \ t \ x) \rightarrow \text{Decidable } (x' \text{ 'Elem' } (\text{step } t \ x \ y)) \\ \text{prf } y = \text{decElem } x' \ (\text{step } t \ x \ y)$$

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Remember

$$\text{Pred } \{t\} \ x \ x' = (y : Y \ t \ x \ ** \ x' \text{ 'Elem' } (\text{step } t \ x \ y))$$

Decision procedures

and, with

$$\text{decPair} : \text{Decidable } p \rightarrow \text{Decidable } q \rightarrow \text{Decidable } (p, q)$$

decidability of *Reachable*:

$$\text{decReachable} : \{t' : \mathbb{N}\} \rightarrow (x' : X \ t') \rightarrow \text{Decidable } (\text{Reachable } x')$$

$$\text{decReachable } \{t' = Z\} \ x' = \text{Left } ()$$

$$\text{decReachable } \{t' = S \ t\} \ x' = s1 \ \mathbf{where}$$

$$s1 : \text{Decidable } (x : X \ t \ ** (\text{Reachable } x, x' \text{ 'Pred' } x'))$$

$$s1 = \text{finiteDecidableLemma}$$

$$(fX \ t)$$

$$(\lambda x \Rightarrow \text{decPair } (\text{decReachable } x) \ (\text{decPred } x \ x'))$$

Decision procedures

Similarly, one implement (prove) decidability of *Viable*:

$$\text{decViable} : \{t : \mathbb{N}\} \rightarrow (n : \mathbb{N}) \rightarrow (x : X \ t) \rightarrow \\ \text{Decidable} \ (\text{Viable} \ n \ x)$$

$$\text{decViable} \ \{t\} \ Z \ x = \text{Left} \ ()$$

$$\text{decViable} \ \{t\} \ (S \ m) \ x = s3 \ \mathbf{where}$$

$$s1 : (y : Y \ t \ x) \rightarrow \text{Decidable} \ (\text{All} \ (\text{Viable} \ m) \ (\text{step} \ t \ x \ y))$$

$$s1 \ y = \text{decAll} \ (\text{Viable} \ m) \ (\text{decViable} \ m) \ (\text{step} \ t \ x \ y)$$

$$s2 : \text{Decidable} \ (y : Y \ t \ x \ ** \ \text{All} \ (\text{Viable} \ m) \ (\text{step} \ t \ x \ y))$$

$$s2 = \text{finiteDecidableLemma} \ (fY \ t \ x) \ s1$$

$$s3 : \text{Decidable} \ (\text{Viable} \ (S \ m) \ x)$$

$$s3 = s2$$

Decision procedures

Similarly, one implement (prove) decidability of *Viable*:

```

decViable : { t : ℕ } → ( n : ℕ ) → ( x : X t ) →
              Decidable ( Viable n x )

decViable { t } Z x = Left ()

decViable { t } ( S m ) x = s3 where
  s1   : ( y : Y t x ) → Decidable ( All ( Viable m ) ( step t x y ) )
  s1 y = decAll ( Viable m ) ( decViable m ) ( step t x y )
  s2   : Decidable ( y : Y t x ** All ( Viable m ) ( step t x y ) )
  s2   = finiteDecidableLemma ( fY t x ) s1
  s3   : Decidable ( Viable ( S m ) x )
  s3   = s2

```

Implementing a decision procedure for *AvoidableFrom* is a bit more complicated but conceptually equivalent.

Conclusions

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- ▶ For finite X and Y decision procedures for viability, reachability and avoidability can be derived rigorously.
- ▶ Decidable generic [viability, reachability, avoidability] notions are hopefully a good starting point for deriving decidable domain specific notions: sustainability, adaptability, resilience, ...

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These slides:

https://github.com/nicolabotta/SeqDecProbs/tree/master/talks/2015.06.rd4_seminar

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The code shown in these slides:

https://github.com/nicolabotta/SeqDecProbs/blob/master/talks/2015.06.rd4_seminar/code/Theory.lidr