1 Agent based model filament forces

$$
\begin{aligned}
\xi v_{xi} + \eta \sum_{j=1}^{N} O_{ij} \left(\dot{x}_i - \dot{x}_j\right) + \sum_{k=1}^{M} \Theta_{ik} \left(-F_s P_i + \frac{F_s}{V_m}\left(\dot{x}_i - \dot{y}_k\right)\right) \\
+\zeta \sum_{j=A,B} O^a_{ij}\left(\dot{x}_i - \dot{z}_j\right) = 0 \quad \text{for } i = 1, \ldots, N \\
\sum_{i=1}^{N} \Theta_{ik} \left(F_s P_i - \frac{F_s}{V_m}\left(\dot{x}_i - \dot{y}_k\right)\right) = 0 \quad \text{for } k = 1, \ldots, M \\
k(z_B - z_A - L) - \zeta \sum_{i=1}^{N} O^a_{iB}\left(\dot{x}_i - \dot{z}_B\right) = 0 \\
-k(z_B - z_A - L) - \zeta \sum_{i=1}^{N} O^a_{iA}\left(\dot{x}_i - \dot{z}_A\right) = 0
\end{aligned}
$$

2 Converting to continuum model

Agent Based Model		Continuum Model
$\xi \dot{x}_i$	$\Longleftrightarrow$	$\xi \rho^{\pm}(t,x) v^{\pm}(t,x)$
$+\,\eta \sum_{j=1}^{N} O_{ij}\left(\dot{x}_i - \dot{x}_j\right)$	$\Longleftrightarrow$	$+\,\eta \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y)(v^{\pm}(t,x) - v^{n}(t,y))\rho^{\pm}(t,x)\rho^{n}(t,y)dy$
$-\sum_{k=1}^{M} \Theta_{ik} F_s \left(P_i - \frac{\dot{x}_i - \dot{y}_k}{V_m}\right)$	$\Longleftrightarrow$	$-\int_{\mathbb{R}} \Theta^{\pm}(t,x,y) F_s \left(\pm 1 - \frac{v^{\pm}(t,x) - v^{\mp}(t,y)}{2V_m}\right) dy$
$+\,\zeta \sum_{j=A,B} O^a_{ij}\left(\dot{x}_i - \dot{z}_j\right)$	$\Longleftrightarrow$	$+\,\zeta \sum_{j=A,B} O^a(x - z_j)(v^{\pm}(t,x) - V(t,z_j))\rho^{\pm}(t,x)$
$= 0 \quad \text{for } i = 1, \ldots, N$	$\Longleftrightarrow$	$= 0 \quad \forall x \in \mathbb{R}$

3 After non-dimensionalization and scaling

$$\tilde{\xi}\tilde{D}^{\pm} + \tilde{\eta}\tilde{C}^{\pm} - \tilde{M}^{\pm} + \tilde{\zeta}\tilde{A}^{\pm} = 0$$

Where:

$$
\begin{aligned}
\tilde{D}^{\pm} &= \frac{1}{\tilde{l}}\tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{v}^{\pm}(\tilde{t},\tilde{x}) \\
\tilde{C}^{\pm} &= \sum_{n=-1,+1} \int_{\mathbb{R}} \tilde{O}(\tilde{\Delta x}_y)(\tilde{v}^{\pm}(\tilde{t},\tilde{x}) - \tilde{v}^{n}(\tilde{t},\tilde{x} + \tilde{l}\tilde{\Delta x}_y))\tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{\rho}^{n}(\tilde{t},\tilde{x} + \tilde{l}\tilde{\Delta x}_y)\mathrm{d}\tilde{\Delta x}_y \\
\tilde{M}^{\pm} &= \int_{\mathbb{R}} \tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right)\left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d}\right)}{2}\right)\mathrm{d}\tilde{d} \\
\tilde{A}^{\pm} &= \sum_{j=A,B} \tilde{O^a}(\tilde{\Delta x}_z)\left(\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\right) - \tilde{V}\left(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x}_z\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\right)
\end{aligned}
$$

4 After Symmetrization and Perturbation

$$0 = \xi(D^+ + D^-) + \eta(C^+ + C^-) + (M^+ + M^-) + \zeta(A^+ + A^-)$$

$$0 = \xi(D^+ - D^-) + \eta(C^+ - C^-) + (M^+ - M^-) + \zeta(A^+ - A^-)$$

Where:

$$
\begin{aligned}
D^+ + D^- &= \frac{1}{l}(v_0\rho_0 - \bar{v}_0\bar{\rho}_0) + O(l^0) \\
D^+ - D^- &= \frac{1}{l}(v_0\bar{\rho}_0 - \bar{v}_0\rho_0) + O(l^0)
\end{aligned}
$$

$$
\begin{aligned}
C^+ + C^- &= -\frac{l^2}{12}\partial_x\left(\rho_0^2\partial_x\left(v_0 + \frac{\bar{\rho}_0\bar{\bar{v}}_0}{\rho_0}\right)\right) + O(l^3) \\
C^+ - C^- &= \bar{v}_0(\rho_0^2 - \bar{\rho}_0^2) + O(l^1)
\end{aligned}
$$

$$
\begin{aligned}
M^+ + M^- &= \frac{-2l}{3}(2\delta + l)\partial_x[\bar{\mu}_0(1 - \bar{v}_0)] + O(l^3) \\
M^+ - M^- &= -2\bar{\mu}_0(1 - \bar{v}_0) + O(l^1)
\end{aligned}
$$

$$
\begin{aligned}
A^+ + A^- &= \sum_{j=A,B}\rho_0\left(v_0\rho_0 - \bar{v}_0\bar{\rho}_0 - V_0^j\rho_0\right) + O(l^1) \\
A^+ - A^- &= \sum_{j=A,B}\rho_0\left(v_0\bar{\rho}_0 - \bar{v}_0\rho_0 - V_0^j\bar{\rho}_0\right) + O(l^1)
\end{aligned}
$$