Developing a Continuum model

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1 Agent Based Model

- let ρ in original model equal ζ in new model
- let $\frac{\mathrm{d}x_i}{\mathrm{d}t} = \dot{x}_i$, $\frac{\mathrm{d}y_k}{\mathrm{d}t} = \dot{y}_k$, $\frac{\mathrm{d}z_j}{\mathrm{d}t} = \dot{z}_j$

$$\begin{aligned}
\xi v_{xi} + \eta \sum_{j=1}^{N} O_{ij} \left(\dot{x}_i - \dot{x}_j\right) + \sum_{k=1}^{M} \Theta_{ik} \left(-F_s P_i + \frac{F_s}{V_m} \left(\dot{x}_i - \dot{y}_k\right)\right)& \\
+\zeta \sum_{j=A,B} O^a_{ij} \left(\dot{x}_i - \dot{z}_j\right) = 0 &\quad \text{for } i = 1, \dots, N \\
\sum_{i=1}^{N} \Theta_{ik} \left(F_s P_i - \frac{F_s}{V_m} \left(\dot{x}_i - \dot{y}_k\right)\right) = 0 &\quad \text{for } k = 1, \dots, M \\
k(z_B - z_A - L) - \zeta \sum_{i=1}^{N} O^a_{iB} \left(\dot{x}_i - \dot{z}_B\right) = 0& \\
-k(z_B - z_A - L) - \zeta \sum_{i=1}^{N} O^a_{iA} \left(\dot{x}_i - \dot{z}_A\right) = 0&
\end{aligned}$$

2 Continuum Model for Filaments and Adhesions

We initially formulate a model for an stress fiber of infinite length. After deriving the asymptotic limit model for short filaments we will restrict the fiber top a finite length and we assume that:

- All filaments are of equal length l
- When motors attach the initial distance between the motor attachment and the barbed end of the filament must be the same for both filaments

2.1 Book keeping

$$\partial_t \rho^{\pm}(t,x) + \partial_x (v^{\pm}(t,x) \rho^{\pm}(t,x)) = 0 \tag{1}$$

i.e. change in density at x over time is equal to decrease in flux as we move right along fiber

2.2 Forces acting on Actin Filaments

$$
\begin{array}{lcl}
\text{Agent Based Model} & & \text{Continuum Model} \\
\xi \dot{x}_i & \Longleftrightarrow & \xi \rho^{\pm}(t,x) v^{\pm}(t,x) \\
+\, \eta \displaystyle\sum_{j=1}^{N} O_{ij} \left(\dot{x}_i - \dot{x}_j \right) & \Longleftrightarrow & +\, \eta \displaystyle\sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y)(v^{\pm}(t,x) - v^{n}(t,y)) \rho^{\pm}(t,x) \rho^{n}(t,y) dy \\
-\displaystyle\sum_{k=1}^{M} \Theta_{ik} F_s \left(P_i - \frac{\dot{x}_i - \dot{y}_k}{V_m} \right) & \Longleftrightarrow & -\displaystyle\int_{\mathbb{R}} \Theta^{\pm}(t,x,y) F_s \left(\pm 1 - \frac{v^{\pm}(t,x) - v^{\mp}(t,y)}{2V_m} \right) dy \\
+\, \zeta \displaystyle\sum_{j=A,B} O^{a}_{ij} \left(\dot{x}_i - \dot{z}_j \right) & \Longleftrightarrow & +\, \zeta \displaystyle\sum_{j=A,B} O^{a}(x - z_j)(v^{\pm}(t,x) - V(t,z_j)) \rho^{\pm}(t,x) \\
= 0 \quad \text{for } i = 1, \ldots, N & \Longleftrightarrow & = 0 \quad \forall x \in \mathbb{R}
\end{array}
\tag{2}
$$

We construct several quantities to define the above equations:
$\rho^{+}(t,x)$ represents density of positive pointing (barbed end to the left) Actin filaments at point x and time t. And by the same definition $\rho^{-}(t,x)$ represents density of the negative pointing filaments.

$v^{+}(t,x)$ represents material velocity of positive pointing (barbed end to the left) Actin filaments at point x and time t. And by the same definition $v^{-}(t,x)$ represents velocity of the negative pointing filaments.

2.2.1 Cytoplasm viscosity

The first term is due to the fictional force acting on the filaments due to the viscosity of the cell's cytoplasm.

$$\xi \dot{x}_i \quad \Longleftrightarrow \quad \xi \rho^{\pm}(t,x) v^{\pm}(t,x)$$

In the first (positive) spacial argument:
The frictional force is equal to the drag coefficient ξ multiplied by the density of the positive filaments at x and the velocity of the positive filaments at x The same logic follows for the second spacial argument.

2.2.2 Crosslinker drag forces

The next term accounts for the effective drag force due to the protein friction that stems from the cross-linker proteins that connect each pair of filaments over the length of their overlap.

$$\eta \sum_{j=1}^{N} O_{ij} \left(\dot{x}_i - \dot{x}_j \right) \quad \Longleftrightarrow \quad \eta \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y)(v^{\pm}(t,x) - v^{n}(t,y)) \rho^{\pm}(t,x) \rho^{n}(t,y) dy$$

In the above equation:
$O(x-y)$ denotes the length of the region where two Actin filaments overlap given by $O(x-y) = \max(l - |x-y|, 0)$

In the first (positive) spacial argument:
The drag force is equal to the viscous drag coefficient η multiplied by the sum over n = positive, negative of: The relative velocity between the positive filaments at point x and n-direction filaments at point y multiplied by the density of positive filaments at point x and density of n-direction filaments at point y multiplied by the length of overlap between filaments at x and y integrated over all $y \in \mathbb{R}$. The same logic follows for the second spacial argument.

2.2.3 Myosin motors

This term takes into account active forces generated by the myosin motors acting on the Actin filaments.

$$-\sum_{k=1}^{M}\Theta_{ik}F_s\left(P_i-\frac{\dot{x}_i-\dot{y}_k}{V_m}\right)\quad\Longleftrightarrow\quad-\int_{\mathbb{R}}\Theta^{\pm}(t,x,y)F_s\left(\pm1-\frac{v^{\pm}(t,x)-v^{\mp}(t,y)}{2V_m}\right)dy$$

We introduce several quantities:
$\Theta^{+}(t,x,y)$ represents the density of the motors that are acting on positive filaments at position x and also connected to negative filaments in position y we define: $\Theta^{-}(t,y,x)=\Theta^{+}(t,x,y)=\Theta(t,x,y)$

In the first (positive) spacial argument:
The generated Myosin motor force is equal to the stall force of the motor F_s multiplied by: the density of the motors that are acting on positive filaments at position x and also connected to negative filaments in position y $\Theta^{+}(t,x,y)$ and the factor in brackets, integrated over all $y\in\mathbb{R}$. The factor in brackets describes the relative velocity between the motors acting on the filaments at x and the filaments at x. This factor is equal to 1 if the overlapping anti parallel Actin filaments are not moving relative to each other which decreases linearly as the relative velocity increases (i.e. the faster the filaments are trying to move away from each other the smaller the drag force). The same logic follows for the second spacial argument.

2.2.4 Focal Adhesions

The final term corresponds to the frictional force that prevents filaments overlapping the focal adhesion's from disconnecting easily.

$$\zeta\sum_{j=A,B}O^{a}_{ij}\left(\dot{x}_i-\dot{z}_j\right)\quad\Longleftrightarrow\quad\zeta\sum_{j=A,B}O^{a}(x-z_j)(v^{\pm}(t,x)-V(t,z_j))\rho^{\pm}(t,x)$$

In the above equation:
$O^{a}(x-z_j)$ denotes the length of the region where Actin filaments at x overlap with focal adhesion j given by $O(x-z_j)=\max(l-|x-z_j|,0)$ where z_j denotes the position of focal adhesion j.

$V(t,z_j)$ represents material velocity of focal adhesion j at time t

In the first (positive) spacial argument:
The frictional force is equal to the drag coefficient ζ multiplied by the sum over j = A, B of: the relative velocity between the positive filaments at point x and focal adhesion j multiplied by the density of the positive filaments at point x and the overlap between the positive filaments at point x and focal adhesion j. The same logic follows for the second spacial argument.

3 Continuum Model for Myosin Motors

We assume that:

- All filaments are of equal length l
- When motors attach the initial distance between the motor attachment and the barbed end of the filament must be the same for both filaments
- When the motor is distance δ away from barbed end of either filament the motor will slide off both filaments. Where we define δ as our 'motor drop off distance'
- When motors attach the initial distance between the motor attachment and the barbed end of the filament must be the same for both filaments. This leads to motors reaching the the point δ of both attached filaments at the same time.
- After sliding off motors will reattach to other actin filaments immediately at a distance $d\in[0,\delta)$ away from the barbed end of each filament
- As a result of this every motor will always be attached to exactly two filaments

3.1 Structured distributions of Myosin motors

$$\bar{\chi} = \bar{\chi}(t, x, d)$$

$\bar{\chi}(t, x, d)$ represents the density of motors at position x that are connected to anti parallel Actin filaments where both of filament's barbed ends are distance d away from x. $d \in [0, l]$

$$\chi^+ = \chi^+(t, x, d)$$
$$\chi^- = \chi^-(t, x, d)$$

$\chi^+(t, x, d)$ represents the density of motors at position x that are connected to parallel Actin filaments that are both pointing in the positive direction (barbed end to the left) where both of filament's barbed ends are distance d away from x. The same applies for χ^-.

3.2 Detachment of myosin motors

By our definitions we know that myosin motors will slide off Actin filaments when they reach a distance δ away for the barbed ends of the filaments. We let the pool of detached filaments be denoted by:

$$M_{\text{off}} = \frac{v^-(t, x - \delta') - v^+(t, x + \delta')}{2} \bar{\chi}(t, x, \delta) + V_m(\chi^+(t, x, \delta) + \chi^-(t, x, \delta))$$

Where $\delta' = \frac{l}{2} - \delta$ where l is the fixed length of each filament

This pool of detached myosin is immediately redistributed among pairs of actin filaments in the following distribution:

Number of motors reattached to:
Anti-parallel filaments $= \bar{R}M_{\text{off}}$
Positive parallel filaments $= R^+M_{\text{off}}$
Negative parallel filaments $= R^-M_{\text{off}}$

Where:

$$R^+ = \frac{(\rho^+)^2}{(\rho^+ + \rho^-)^2} \qquad R^- = \frac{(\rho^-)^2}{(\rho^+ + \rho^-)^2} \qquad \bar{R} = \frac{2\rho^+\rho^-}{(\rho^+ + \rho^-)^2}$$

3.3 Transport equations for anti parallel myosin motors

$$\begin{aligned} \partial_t \bar{\chi} + \partial_x \left(\frac{v^-(t, x - d') + v^+(t, x + d')}{2} \bar{\chi} \right) + \partial_d \left(\frac{v^-(t, x - d') - v^+(t, x + d')}{2} \bar{\chi} \right) \\ = \bar{R}M_{\text{off}} \frac{\rho^+(t, x + d')\rho^-(t, x - d')}{\int_\delta^l \rho^+(t, x + d')\rho^-(t, x - d') \mathrm{d}d} \end{aligned} \tag{3}$$

$$\bar{\chi}(t, x, l) = 0 \tag{4}$$

Where $d' = \frac{l}{2} - d$ where l is the fixed length of each filament

3.3.1 Myosin motor drift

The LHS of equation (3) describes the myosin motor drift in physical space. We know that change in density at x over time is equal to decrease in flux as we move right along fiber.

$$\partial_t \bar{\chi}$$

The first term represents the change in density over time.

$$\partial_x\left(\frac{v^-(t,x-d')+v^+(t,x+d')}{2}\bar{\chi}\right)$$

The second term represents the change in flux as the point x moves right along the fibre. Our velocity in this case is equal to the average of the velocity of the positive and negative filament that the motor at x is attached to, since this is the velocity that will move the motors at x over time.

$$\partial_d\left(\frac{v^-(t,x-d')-v^+(t,x+d')}{2}\bar{\chi}\right)$$

The third term represents the change in flux as the distance between the motor and its attached filaments increases. Our velocity in this case is equal to the relative velocity between the positive and negative filament that the motor at x is attached to, since this is the velocity that will move the attachment points i.e. distance d over time.

3.3.2 Myosin motor reattachment

The RHS of equation (3) describes the reattachment myosin motors to anti parallel filaments. The reattachment of motors at position x to two filaments that have their barbed ends a distance d away for x is proportional to:

$$\frac{\rho^+(t,x+d')\rho^-(t,x-d')}{\int_\delta^l \rho^+(t,x+d')\rho^-(t,x-d')\mathrm{d}d}$$

The density of positive filaments whose barbed end is a distance d to the left x, multiplied by the density of negative filaments whose barbed end is a distance d to the right of x. Divided by the total density of filaments that x could possibly reattach to, i.e. filaments at distances ranging from δ to l.

$$\bar{R}M_{\text{off}}$$

The proportionality constant is equal to the anti parallel filament reattachment constant multiplied by the number of detached myosin motors.

3.4 Transport equations for parallel myosin motors

$$\begin{aligned}\partial_t\chi^\pm+\partial_x((\mp V_m+v^\pm(t,x\pm d'))\chi^\pm)+\partial_d(\mp V_m\chi^\pm)\\ =R^\pm M_{\text{off}}\frac{\rho^\pm(t,x\pm d')^2}{\int_\delta^l \rho^\pm(t,x\pm d')^2\mathrm{d}d}\end{aligned} \tag{5}$$

$$\chi^\pm(t,x,l)=0 \tag{6}$$

3.4.1 Myosin motor drift

The LHS of equation (5) describes the myosin motor drift in physical space. We know that change in density at x over time is equal to decrease in flux as we move right along fiber.

$$\partial_t\chi^\pm$$

The first term represents the change in density over time.

$$\partial_x((\mp V_m+v^\pm(t,x\pm d'))\chi^\pm)$$

The second term represents the change in flux as the point x moves right along the fibre. Velocity in this case is equal to sum of the free moving velocity V_m and the material velocity of the actin filaments. Since this is the velocity that will move the motors at x over time.

$$\partial_d(V_m \chi^{\pm})$$

The third term represents the change in flux as the distance between the motor and its attached filaments increases. Velocity in this case is equal the free moving velocity V_m, as this is the relative velocity between the motor the fibers. Since this is the velocity that will move the attachment points (i.e. distance d) over time.

3.4.2 Myosin motor reattachment

The RHS of equation (5) describes the reattachment myosin motors to parallel filaments. We will consider the positive argument but a similar definition can be made for the negative argument. The reattachment of motors at position x to two filaments that have their barbed ends a distance d away for x is proportional to:

$$\frac{\rho^{\pm}(t, x \pm d')^2}{\int_{\delta}^{l} \rho^{\pm}(t, x \pm d')^2 \mathrm{d}d}$$

The density of positive filaments whose barbed end is a distance d to the left x squared. Divided by the total density of positive filaments squared that x could possibly reattach to, i.e. filaments at distances ranging from δ to l.

$$R^{\pm} M_{\text{off}}$$

The proportionality constant is equal to the parallel filament reattachment constant multiplied by the number of detached myosin motors.

4 Non-dimensionalization of model

4.1 Scaling of system

We scale our system of equations (1), (2), (3), (4), (5), (6) as follows:

$\tilde{x} = x/L$ where L is the equilibrium length of the stress fibre (A)

$\tilde{d} = d/l, \quad \tilde{\delta} = \delta/l$ where l is the actin filament length (B)

The approximations we derive below work in the limit $l \ll L$:

$$\tilde{t} = t/t_0 \qquad \text{where } t_0 = L/V_m \qquad (C)$$

$$\tilde{l} = l/L \qquad (D)$$

$$\tilde{v}^{\pm}(t/t_0, x/L) = v^{\pm}(t,x)/V_m \qquad \text{where } V_m \text{ is the myosin free velocity} \qquad (E)$$

$$\tilde{\rho}^{\pm}(t/t_0, x/L) = \rho^{\pm}(t,x)l \qquad (F)$$

$$\tilde{\bar{\chi}}\left(\frac{t}{t_0}, \frac{x}{L}, \frac{d}{l}\right) = \bar{\chi}(t,x,d)Ll \qquad \tilde{\chi^{\pm}}\left(\frac{t}{t_0}, \frac{x}{L}, \frac{d}{l}\right) = \chi^{\pm}(t,x,d)Ll \qquad (G)$$

$$\tilde{O}(\Delta x_y/L) = O(\Delta x_y)/l \qquad \tilde{O^a}(\Delta x_z/L) = O^a(\Delta x_z)/l \qquad (H)$$

$$\tilde{z_j} = z_j/L \qquad \text{For } j = A, B \qquad (I)$$

$$\tilde{V}(t/t_0, z_j/L) = V(t, z_j)/V_m \qquad \text{For } j = A, B \qquad (J)$$

We also apply the following transformations by defining:

$$(y - x) = l\Delta x_y, \qquad \frac{(x-y)}{2} = (l/2 - d) = d', \qquad (z_j - x) = l\Delta x_z$$

4.2 Continuum equation 1 after scaling

Equation (1) remains unchanged remained unchanged after scaling

4.3 Continuum equation 2

We observe that before scaling equation (2) has the form bellow where D, C, M, A are defined in the subsequent sections. In what follows we will scale and transform this equation.

$$D^{\pm} + C^{\pm} - M^{\pm} + A^{\pm} = 0$$

4.4 D - Cytoplasm viscous drag

Before scaling we originally started with:

$$D^{\pm} = \xi\rho^{\pm}(t,x)v^{\pm}(t,x)$$

We wish to convert to the scaled version of each parameter. From above we have: (A) - (H). By applying these to our above formula we have:

$$\begin{aligned}
&\xi\rho^{\pm}(t,x)v^{\pm}(t,x) && \text{Original Equation} \\
&\to \quad \xi\frac{\tilde{\rho}^{\pm}(t/t_0, x/L)}{l}\tilde{v}^{\pm}(t/t_0, x/L)V_m && \text{By (E) and (F)} \\
&= \xi\frac{L}{L}\frac{\tilde{\rho}^{\pm}(\tilde{t},\tilde{x})}{l}\tilde{v}^{\pm}(\tilde{t},\tilde{x})V_m && \text{By (A) and (C)} \\
&= \xi\frac{V_m}{L\tilde{l}}\tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{v}^{\pm}(\tilde{t},\tilde{x}) && \text{By (D)}
\end{aligned}$$

4.5 C - Cross linker friction

Before scaling we originally started with:

$$C^{\pm} = \eta \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y)(v^{\pm}(t,x) - v^{n}(t,y))\rho^{\pm}(t,x)\rho^{n}(t,y)\mathrm{d}y$$

We wish to convert to the scaled version of each parameter. From above we have: A - H. We can also apply the transformation, noting that:

- From first transformation: $y = x + l\Delta x_y$ 0
- 0 $\rightarrow \frac{y}{L} = \frac{x+l\Delta x_y}{L} \overset{A}{\rightarrow} \tilde{y} = \tilde{x} + \tilde{l}\tilde{\Delta x}_y$ 1
- And after scaling we have $\tilde{O}(\tilde{\Delta x}_y) = 1 - |\Delta x_y|$ Since: 2

$$\begin{aligned}
O(x-y) &= l - |x-y| \\
O(x - x + l\Delta x_y) &= l - |x - x + l\Delta x_y| && \text{By } 0 \\
O(\Delta x_y) &= l - |l\Delta x_y| \\
\tilde{O}(\Delta x_y/L)l &= l - |l\Delta x_y| && \text{By } H \\
\tilde{O}(\tilde{\Delta x}_y) &= 1 - |\Delta x_y| && \text{By } A
\end{aligned}$$

We also wish to apply the following change of variables to transform the integral from $y \rightarrow \tilde{\Delta x}_y$:

$$\text{Note that: by } A: \qquad \tilde{y} = \frac{y}{L} \quad \rightarrow \quad \frac{\mathrm{d}\tilde{y}}{\mathrm{d}y} = \frac{1}{L} \qquad 3$$

$$\begin{aligned}
(\tilde{y} - \tilde{x}) &= \tilde{l}\tilde{\Delta x}_y && \text{By } 1 \\
\frac{\tilde{y} - \tilde{x}}{\tilde{l}} &= \tilde{\Delta x}_y \\
\rightarrow \quad \frac{\mathrm{d}\tilde{\Delta x}_y}{\mathrm{d}y} &= \frac{\mathrm{d}\tilde{\Delta x}_y}{\mathrm{d}\tilde{y}} \frac{\mathrm{d}\tilde{y}}{\mathrm{d}y} && \text{By chain rule} \\
\frac{\mathrm{d}\tilde{\Delta x}_y}{\mathrm{d}y} &= \frac{1}{\tilde{l}} \frac{1}{L} && \text{By } 3 \\
\frac{\mathrm{d}\tilde{\Delta x}_y}{\mathrm{d}y} &= \frac{L}{l} \frac{1}{L} && \text{By } D \\
\frac{\mathrm{d}\tilde{\Delta x}_y}{\mathrm{d}y} &= \frac{1}{l} && \text{By } D
\end{aligned}$$

This gives us the transformation:

$$\mathrm{d}y = l\mathrm{d}\tilde{\Delta x}_y \qquad 4$$

By applying the scaling and change of variables to our above formula we have:

$$\eta \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y)(v^{\pm}(t,x) - v^{n}(t,y))\rho^{\pm}(t,x)\rho^{n}(t,y)\mathrm{d}y \qquad \text{Original Equation}$$

By equations E and F:

$$\to \eta \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y) \left(\tilde{v}^{\pm} \left(\frac{t}{t_0}, \frac{x}{L} \right) V_m - \tilde{v}^{n} \left(\frac{t}{t_0}, \frac{y}{L} \right) V_m \right) \frac{\tilde{\rho}^{\pm} \left(\frac{t}{t_0}, \frac{x}{L} \right)}{l} \frac{\tilde{\rho}^{n} \left(\frac{t}{t_0}, \frac{y}{L} \right)}{l} \mathrm{d}y$$

$$= \eta \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y) \left(\tilde{v}^{\pm} \left(\tilde{t}, \tilde{x} \right) V_m - \tilde{v}^{n} \left(\tilde{t}, \tilde{y} \right) V_m \right) \frac{\tilde{\rho}^{\pm} \left(\tilde{t}, \tilde{x} \right)}{l} \frac{\tilde{\rho}^{n} \left(\tilde{t}, \tilde{y} \right)}{l} \mathrm{d}y \qquad \text{By Ⓐ and Ⓒ}$$

$$= \frac{\eta V_m}{l^2} \sum_{n=-1,+1} \int_{\mathbb{R}} O(x-y)(\tilde{v}^{\pm}(\tilde{t}, \tilde{x}) - \tilde{v}^{n}(\tilde{t}, \tilde{y}))\tilde{\rho}^{\pm}(\tilde{t}, \tilde{x})\tilde{\rho}^{n}(\tilde{t}, \tilde{y})\mathrm{d}y \qquad \text{Rearranging}$$

By equations ①, ② and Ⓗ

$$= \frac{\eta V_m}{l^2} \sum_{n=-1,+1} \int_{\mathbb{R}} \tilde{O}(\tilde{\Delta x_y}) l (\tilde{v}^{\pm}(\tilde{t}, \tilde{x}) - \tilde{v}^{n}(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_y}))\tilde{\rho}^{\pm}(\tilde{t}, \tilde{x})\tilde{\rho}^{n}(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_y})\mathrm{d}y$$

$$= \frac{\eta V_m}{l} \sum_{n=-1,+1} \int_{\mathbb{R}} \tilde{O}(\tilde{\Delta x_y})(\tilde{v}^{\pm}(\tilde{t}, \tilde{x}) - \tilde{v}^{n}(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_y}))\tilde{\rho}^{\pm}(\tilde{t}, \tilde{x})\tilde{\rho}^{n}(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_y}) l \mathrm{d}\tilde{\Delta x_y} \qquad \text{By ④}$$

$$= \eta V_m \sum_{n=-1,+1} \int_{\mathbb{R}} \tilde{O}(\tilde{\Delta x_y})(\tilde{v}^{\pm}(\tilde{t}, \tilde{x}) - \tilde{v}^{n}(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_y}))\tilde{\rho}^{\pm}(\tilde{t}, \tilde{x})\tilde{\rho}^{n}(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_y})\mathrm{d}\tilde{\Delta x_y} \qquad \text{Rearranging}$$

4.6 M - Myosin Motor force

We before scaling we originally started with:

$$M^{\pm} = -\int_{\mathbb{R}} \Theta^{\pm}(t,x,y) F_s \left(\pm 1 - \frac{v^{\pm}(t,x) - v^{\mp}(t,y)}{2V_m} \right) dy$$

By noting that:

- From second transformation: $y = x - 2\left(\frac{l}{2} - d\right)$ ⓪
- $\overset{⓪}{\to} \frac{y}{L} = \frac{x - 2(\frac{l}{2} - d)}{L} \overset{Ⓐ}{\to} \tilde{y} = \tilde{x} - \tilde{l} + 2\frac{d}{L} \overset{Ⓑ}{\to} \tilde{y} = \tilde{x} - \tilde{l} + 2\frac{\tilde{d}l}{L} \overset{Ⓐ}{\to} \tilde{y} = \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d})$ ①
- By the definition of Θ: $\Theta(t,x,y) = \frac{1}{2}\bar{\chi}\left(t, \frac{x+y}{2}, \frac{l+(y-x)}{2}\right)$ ②

We first aim to apply ① and ② to Θ:

$$\begin{aligned}
\Theta^{+}(t,x,y) &= \frac{1}{2}\bar{\chi}\left(t, \frac{x+y}{2}, \frac{l+(y-x)}{2}\right) && \text{By ②} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\frac{t}{t_0}, \frac{x+y}{2L}, \frac{l+(y-x)}{2l}\right) && \text{By Ⓖ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{y}}{2}, \frac{1}{2} + \frac{(y-x)}{2l}\right) && \text{By Ⓐ and Ⓓ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{y}}{2}, \frac{1}{2} + \frac{L(\tilde{y}-\tilde{x})}{2l}\right) && \text{By Ⓐ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{y}}{2}, \frac{1}{2} + \frac{1}{2\tilde{l}}(\tilde{y}-\tilde{x})\right) && \text{By Ⓓ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{x}-2\tilde{l}(\frac{1}{2}-\tilde{d})}{2}, \frac{1}{2} + \frac{1}{2\tilde{l}}(\tilde{x}-2\tilde{l}(\frac{1}{2}-\tilde{d})-\tilde{x})\right) && \text{By ①} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \tilde{x}-\tilde{l}\left(\frac{1}{2}-\tilde{d}\right), \frac{1}{2} + \frac{1}{2\tilde{l}}\left(-2\tilde{l}\left(\frac{1}{2}-\tilde{d}\right)\right)\right) && \text{Rearranging} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \tilde{x}-\tilde{l}\left(\frac{1}{2}-\tilde{d}\right), \frac{1}{2} - \left(\frac{1}{2}-\tilde{d}\right)\right) && \text{Rearranging} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \tilde{x}-\tilde{l}\left(\frac{1}{2}-\tilde{d}\right), \tilde{d}\right) && \text{Rearranging}
\end{aligned}$$

$$\begin{aligned}
\Theta^{-}(t,x,y) &= \Theta^{-}(t,y,x) && \text{By definition of } \Theta \\
&= \frac{1}{2}\bar{\chi}\left(t, \frac{y+x}{2}, \frac{l+(x-y)}{2}\right) && \text{By ②} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\frac{t}{t_0}, \frac{x+y}{2L}, \frac{l+(x-y)}{2l}\right) && \text{By Ⓖ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{y}}{2}, \frac{1}{2} + \frac{(x-y)}{2l}\right) && \text{By Ⓐ and Ⓓ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{y}}{2}, \frac{1}{2} + \frac{L(\tilde{x}-\tilde{y})}{2l}\right) && \text{By Ⓐ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{y}}{2}, \frac{1}{2} + \frac{1}{2\tilde{l}}(\tilde{x}-\tilde{y})\right) && \text{By Ⓓ} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \frac{\tilde{x}+\tilde{x}-2\tilde{l}(\frac{1}{2}-\tilde{d})}{2}, \frac{1}{2} + \frac{1}{2\tilde{l}}\left(\tilde{x}-\left(\tilde{x}-2\tilde{l}\left(\frac{1}{2}-\tilde{d}\right)\right)\right)\right) && \text{By ①} \\
&= \frac{1}{2Ll}\tilde{\bar{\chi}}\left(\tilde{t}, \tilde{x}-\tilde{l}\left(\frac{1}{2}-\tilde{d}\right), \frac{1}{2} + \frac{1}{2\tilde{l}}\left(2\tilde{l}\left(\frac{1}{2}-\tilde{d}\right)\right)\right) && \text{Rearranging}
\end{aligned}$$

$$= \frac{1}{2Ll}\tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} + \left(\frac{1}{2} - \tilde{d}\right)\right) \qquad \text{Rearranging}$$

$$= \frac{1}{2Ll}\tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), 1 - \tilde{d}\right) \qquad \text{Rearranging}$$

Therefore we have:

$$\Theta^{\pm}(t,x,y) = \frac{1}{2Ll}\tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right) \qquad ③$$

We also wish to apply the following change of variables to transform the integral from $y \to \tilde{d}$. To do this we first express $\tilde{d}$ as a function of $\tilde{y}$ and then apply chain rule.

$$① \quad \to \quad \tilde{y} = \tilde{x} - 2\tilde{l}\left(\frac{1}{2} - \tilde{d}\right) \quad \to \quad \frac{\tilde{y} - \tilde{x}}{-2\tilde{l}} = \left(\frac{1}{2} - \tilde{d}\right) \quad \to \quad \frac{\tilde{y} - \tilde{x}}{2\tilde{l}} + \frac{1}{2} = \tilde{d}$$

$$\frac{\mathrm{d}\tilde{d}}{\mathrm{d}y} = \frac{\mathrm{d}\tilde{y}}{\mathrm{d}y}\frac{\mathrm{d}\tilde{d}}{\mathrm{d}\tilde{y}} \qquad \text{By Chain rule}$$

$$\frac{\mathrm{d}\tilde{d}}{\mathrm{d}y} = \frac{\mathrm{d}\tilde{y}}{\mathrm{d}y}\left(\frac{1}{2\tilde{l}}\right) \qquad \text{From above}$$

$$\frac{\mathrm{d}\tilde{d}}{\mathrm{d}y} = \left(\frac{1}{L}\right)\left(\frac{1}{2\tilde{l}}\right) \qquad \text{By Ⓐ}$$

$$\frac{\mathrm{d}\tilde{d}}{\mathrm{d}y} = \frac{1}{2L(l/L)} \qquad \text{By Ⓓ}$$

$$\frac{\mathrm{d}\tilde{d}}{\mathrm{d}y} = \frac{1}{2l} \qquad \text{By Ⓓ}$$

This gives us the transformation:

$$\mathrm{d}y = 2l\mathrm{d}\tilde{d} \qquad ④$$

By applying the scaling and change of variables to our above formula we have:

$$-\int_{\mathbb{R}} \Theta^{\pm}(t,x,y)F_s\left(\pm 1 - \frac{v^{\pm}(t,x) - v^{\mp}(t,y)}{2V_m}\right)dy \qquad \text{Original Equation}$$

By equation ③:

$$\to \quad -\int_{\mathbb{R}} \frac{1}{2Ll}\tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right) F_s\left(\pm 1 - \frac{v^{\pm}(t,x) - v^{\mp}(t,y)}{2V_m}\right)\mathrm{d}y$$

By equation Ⓔ and rearranging:

$$= -\frac{F_s}{2Ll}\int_{\mathbb{R}} \tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right)\left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\frac{t}{t_0}, \frac{x}{L}\right)V_m - \tilde{v}^{\mp}\left(\frac{t}{t_0}, \frac{y}{L}\right)V_m}{2V_m}\right)\mathrm{d}y$$

By equation Ⓐ and Ⓒ:

$$= -\frac{F_s}{2Ll}\int_{\mathbb{R}} \tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right)\left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{y}\right)}{2}\right)\mathrm{d}y$$

By equation ①:

$$= -\frac{F_s}{2Ll}\int_{\mathbb{R}} \tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right)\left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d})\right)}{2}\right) \mathrm{d}y$$

By equation ④:

$$= -\frac{F_s}{2Ll}\int_{\mathbb{R}} \tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right)\left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d})\right)}{2}\right) 2l\mathrm{d}\tilde{d}$$

$$= \frac{-F_s}{L}\int_{\mathbb{R}} \tilde{\tilde{\chi}}\left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right)\left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d})\right)}{2}\right) \mathrm{d}\tilde{d}$$

4.7 A - Focal Adhesion drag

Before scaling we originally started with:

$$A^{\pm} = \zeta \sum_{j=A,B} O^a(x - z_j)(v^{\pm}(t,x) - V(t,z_j))\rho^{\pm}(t,x)$$

We wish to convert to the scaled version of each parameter. From above we have: Ⓐ - Ⓗ. We can also apply the transformation, noting that:

- From third transformation: $z_j = x + l\Delta x_z$ ⓪
- ⓪ $\rightarrow \frac{z_j}{L} = \frac{x + l\Delta x_z}{L} \overset{Ⓐ,Ⓘ}{\rightarrow} \tilde{z_j} = \tilde{x} + \tilde{l}\tilde{\Delta x_z}$ ①
- And after scaling we have $\tilde{O^a}(\tilde{\Delta x_y}) = 1 - |\Delta x_y|$ Since: ②

$$\begin{aligned} O^a(x - z_j) &= l - |x - z_j| \\ O^a(x - x + l\Delta x_z) &= l - |x - x + l\Delta x_z| && \text{By ⓪} \\ O^a(\Delta x_z) &= l - |l\Delta x_z| \\ \tilde{O^a}(\Delta x_z/L)l &= l - |l\Delta x_z| && \text{By Ⓗ} \\ \tilde{O^a}(\tilde{\Delta x_z}) &= 1 - |\Delta x_z| && \text{By Ⓐ} \end{aligned}$$

By applying the scaling and transformation to our above formula we have:

$$\begin{aligned} &\zeta \sum_{j=A,B} O^a(x - z_j)(v^{\pm}(t,x) - V(t,z_j))\rho^{\pm}(t,x) && \text{Original Equation} \\ &\rightarrow \quad \zeta \sum_{j=A,B} O^a(x - z_j)\left(\tilde{v}^{\pm}\left(\frac{t}{t_0}, \frac{x}{L}\right)V_m - \tilde{V}\left(\frac{t}{t_0}, \frac{z_j}{L}\right)V_m\right)\tilde{\rho}^{\pm}\left(\frac{t}{t_0}, \frac{x}{L}\right)\frac{1}{l} && \text{By Ⓔ, Ⓕ and Ⓙ} \\ &= \zeta \sum_{j=A,B} O^a(x - z_j)\left(\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right)V_m - \tilde{V}\left(\tilde{t}, \tilde{z_j}\right)V_m\right)\tilde{\rho}^{\pm}\left(\tilde{t}, \tilde{x}\right)\frac{1}{l} && \text{By Ⓐ, Ⓒ and Ⓘ} \\ &= \frac{\zeta V_m}{l} \sum_{j=A,B} O^a(x - z_j)\left(\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{V}\left(\tilde{t}, \tilde{z_j}\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t}, \tilde{x}\right) && \text{Rearranging} \\ &= \frac{\zeta V_m}{l} \sum_{j=A,B} \tilde{O^a}(\tilde{\Delta x_z})l\left(\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{V}\left(\tilde{t}, \tilde{z_j}\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t}, \tilde{x}\right) && \text{By ② and Ⓗ} \\ &= \zeta V_m \sum_{j=A,B} \tilde{O^a}(\tilde{\Delta x_z})\left(\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{V}\left(\tilde{t}, \tilde{z_j}\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t}, \tilde{x}\right) && \text{Rearranging} \\ &= \zeta V_m \sum_{j=A,B} \tilde{O^a}(\tilde{\Delta x_z})\left(\tilde{v}^{\pm}\left(\tilde{t}, \tilde{x}\right) - \tilde{V}\left(\tilde{t}, \tilde{x} + \tilde{l}\tilde{\Delta x_z}\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t}, \tilde{x}\right) && \text{By ①} \end{aligned}$$

4.8 Continuum equation 2 after scaling

Before scaling we originally started with:

$$D^{\pm} + C^{\pm} - M^{\pm} + A^{\pm} = 0$$

And therefore as seen above after scaling we have:

$$\begin{aligned}
&\xi \frac{V_m}{L\tilde{l}} \tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{v}^{\pm}(\tilde{t},\tilde{x}) \\
&+ \eta V_m \sum_{n=-1,+1} \int_{\mathbb{R}} \tilde{O}(\Delta \tilde{x}_y)(\tilde{v}^{\pm}(\tilde{t},\tilde{x}) - \tilde{v}^{n}(\tilde{t},\tilde{x}+\tilde{l}\Delta\tilde{x}_y))\tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{\rho}^{n}(\tilde{t},\tilde{x}+\tilde{l}\Delta\tilde{x}_y)\mathrm{d}\Delta\tilde{x}_y \\
&- \frac{-F_s}{L} \int_{\mathbb{R}} \tilde{\tilde{\chi}} \left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right) \left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d}\right)}{2}\right) \mathrm{d}\tilde{d} \\
&+ \zeta V_m \sum_{j=A,B} \tilde{O^a}(\Delta \tilde{x}_z) \left(\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\right) - \tilde{V}\left(\tilde{t},\tilde{x}+\tilde{l}\Delta\tilde{x}_z\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\right) \\
&= 0
\end{aligned}$$

By setting the following definitions:

$$\begin{aligned}
\tilde{D}^{\pm} &= \frac{1}{\tilde{l}} \tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{v}^{\pm}(\tilde{t},\tilde{x}) \\
\tilde{C}^{\pm} &= \sum_{n=-1,+1} \int_{\mathbb{R}} \tilde{O}(\Delta \tilde{x}_y)(\tilde{v}^{\pm}(\tilde{t},\tilde{x}) - \tilde{v}^{n}(\tilde{t},\tilde{x}+\tilde{l}\Delta\tilde{x}_y))\tilde{\rho}^{\pm}(\tilde{t},\tilde{x})\tilde{\rho}^{n}(\tilde{t},\tilde{x}+\tilde{l}\Delta\tilde{x}_y)\mathrm{d}\Delta\tilde{x}_y \\
\tilde{M}^{\pm} &= \int_{\mathbb{R}} \tilde{\tilde{\chi}} \left(\tilde{t}, \tilde{x} - \tilde{l}\left(\frac{1}{2} - \tilde{d}\right), \frac{1}{2} \mp \left(\frac{1}{2} - \tilde{d}\right)\right) \left(\pm 1 - \frac{\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\right) - \tilde{v}^{\mp}\left(\tilde{t}, \tilde{x} - 2\tilde{l}(\frac{1}{2} - \tilde{d}\right)}{2}\right) \mathrm{d}\tilde{d} \\
\tilde{A}^{\pm} &= \sum_{j=A,B} \tilde{O^a}(\Delta \tilde{x}_z) \left(\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\right) - \tilde{V}\left(\tilde{t},\tilde{x}+\tilde{l}\Delta\tilde{x}_z\right)\right)\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\right)
\end{aligned}$$

Therefore our new equation is written as:

$$\begin{aligned}
\xi \frac{V_m}{L} \tilde{D}^{\pm} + \eta V_m \tilde{C}^{\pm} - \frac{F_s}{L} \tilde{M}^{\pm} + \zeta V_m \tilde{A}^{\pm} = 0 \\
\xi \frac{V_m}{F_s} \tilde{D}^{\pm} + \eta \frac{V_m L}{F_s} \tilde{C}^{\pm} - \tilde{M}^{\pm} + \zeta \frac{V_m L}{F_s} \tilde{A}^{\pm} = 0
\end{aligned}$$

By defining our scaled constants as follows:

$$\tilde{\xi} = \xi \frac{V_m}{F_s}, \qquad \tilde{\eta} = \eta \frac{V_m L}{F_s}, \qquad \tilde{\zeta} = \eta \frac{V_m L}{F_s}$$

We have our final scaled version of equation 2 defined as:

$$\tilde{\xi}\tilde{D}^{\pm} + \tilde{\eta}\tilde{C}^{\pm} - \tilde{M}^{\pm} + \tilde{\zeta}\tilde{A}^{\pm} = 0 \tag{7}$$

4.9 Myosin motor drop off after scaling

Before scaling we originally started with:

$$M_{\text{off}} = \frac{v^-\left(t, x-(l/2-\delta)\right) - v^+\left(t, x+(l/2-\delta)\right)}{2}\bar{\chi}(t,x,\delta) + V_m(\chi^+(t,x,\delta)+\chi^-(t,x,\delta))$$

We then wish to apply the scaling above as follows:

$$\frac{v^-\left(t, x-(l/2-\delta)\right) - v^+\left(t, x+(l/2-\delta)\right)}{2}\bar{\chi}(t,x,\delta) + V_m(\chi^+(t,x,\delta)+\chi^-(t,x,\delta)) \qquad \text{Original}$$

By equations Ⓔ and Ⓖ

$$\begin{aligned}\rightarrow \quad & \frac{1}{2}\left(V_m\tilde{v}^-\left(\frac{t}{t_0}, \frac{x}{L}-\left(\frac{l}{2L}-\frac{\delta}{L}\right)\right) - V_m\tilde{v}^+\left(\frac{t}{t_0}, \frac{x}{L}+\left(\frac{l}{2L}-\frac{\delta}{L}\right)\right)\right)\frac{1}{Ll}\tilde{\bar{\chi}}\left(\frac{t}{t_0},\frac{x}{L},\frac{\delta}{l}\right) \\ & + V_m\left(\frac{1}{Ll}\tilde{\chi}^+\left(\frac{t}{t_0},\frac{x}{L},\frac{\delta}{l}\right) + \frac{1}{Ll}\tilde{\chi}^-\left(\frac{t}{t_0},\frac{x}{L},\frac{\delta}{l}\right)\right)\end{aligned}$$

By equations Ⓐ, Ⓑ, Ⓒ and Ⓓ

$$\begin{aligned}= & \frac{V_m}{2Ll}\left(\tilde{v}^-\left(\tilde{t}, \tilde{x}-\left(\frac{\tilde{l}}{2}-\frac{\delta}{L}\right)\right) - \tilde{v}^+\left(\tilde{t}, \tilde{x}+\left(\frac{\tilde{l}}{2}-\frac{\delta}{L}\right)\right)\right)\tilde{\bar{\chi}}\left(\tilde{t},\tilde{x},\frac{\delta}{l}\right) \\ & + \frac{V_m}{Ll}\left(\tilde{\chi}^+\left(\tilde{t},\tilde{x},\frac{\delta}{l}\right) + \tilde{\chi}^-\left(\tilde{t},\tilde{x},\frac{\delta}{l}\right)\right)\end{aligned}$$

By rearranging

$$\begin{aligned}= & \frac{V_m}{2Ll}\left(\tilde{v}^-\left(\tilde{t}, \tilde{x}-\left(\frac{\tilde{l}}{2}-\frac{\delta l}{Ll}\right)\right) - \tilde{v}^+\left(\tilde{t}, \tilde{x}+\left(\frac{\tilde{l}}{2}-\frac{\delta l}{Ll}\right)\right)\right)\tilde{\bar{\chi}}\left(\tilde{t},\tilde{x},\frac{\delta}{l}\right) \\ & + \frac{V_m}{Ll}\left(\tilde{\chi}^+\left(\tilde{t},\tilde{x},\frac{\delta}{l}\right) + \tilde{\chi}^-\left(\tilde{t},\tilde{x},\frac{\delta}{l}\right)\right)\end{aligned}$$

By equations Ⓑ and Ⓓ

$$\begin{aligned}= & \frac{V_m}{2Ll}\left(\tilde{v}^-\left(\tilde{t}, \tilde{x}-\left(\frac{\tilde{l}}{2}-\tilde{l}\tilde{\delta}\right)\right) - \tilde{v}^+\left(\tilde{t}, \tilde{x}+\left(\frac{\tilde{l}}{2}-\tilde{l}\tilde{\delta}\right)\right)\right)\tilde{\bar{\chi}}\left(\tilde{t},\tilde{x},\tilde{\delta}\right) \\ & + \frac{V_m}{Ll}\left(\tilde{\chi}^+\left(\tilde{t},\tilde{x},\tilde{\delta}\right) + \tilde{\chi}^-\left(\tilde{t},\tilde{x},\tilde{\delta}\right)\right)\end{aligned}$$

By rearranging

$$\begin{aligned}= & \frac{V_m}{2Ll}\left(\tilde{v}^-\left(\tilde{t}, \tilde{x}-\tilde{l}\left(1/2-\tilde{\delta}\right)\right) - \tilde{v}^+\left(\tilde{t}, \tilde{x}+\tilde{l}\left(1/2-\tilde{\delta}\right)\right)\right)\tilde{\bar{\chi}}\left(\tilde{t},\tilde{x},\tilde{\delta}\right) \\ & + \frac{V_m}{Ll}\left(\tilde{\chi}^+\left(\tilde{t},\tilde{x},\tilde{\delta}\right) + \tilde{\chi}^-\left(\tilde{t},\tilde{x},\tilde{\delta}\right)\right)\end{aligned}$$

Therefore M_{off} after scaling now reads:

$$\begin{aligned}\tilde{M}_{\text{off}} = & \frac{V_m}{2Ll}\left(\tilde{v}^-\left(\tilde{t}, \tilde{x}-\tilde{l}\left(1/2-\tilde{\delta}\right)\right) - \tilde{v}^+\left(\tilde{t}, \tilde{x}+\tilde{l}\left(1/2-\tilde{\delta}\right)\right)\right)\tilde{\bar{\chi}}\left(\tilde{t},\tilde{x},\tilde{\delta}\right) \\ & + \frac{V_m}{Ll}\left(\tilde{\chi}^+\left(\tilde{t},\tilde{x},\tilde{\delta}\right) + \tilde{\chi}^-\left(\tilde{t},\tilde{x},\tilde{\delta}\right)\right) \qquad ①\end{aligned}$$

However for simplicity in future definitions we define:

$$\begin{aligned}
\tilde{M}^{\text{off}} &= \frac{L^2 l}{V_m}\tilde{M}_{\text{off}} \\
\tilde{M}^{\text{off}} &= \frac{L}{2}\left(\tilde{v}^-\left(\tilde{t},\tilde{x}-\tilde{l}\left(1/2-\tilde{\delta}\right)\right)-\tilde{v}^+\left(\tilde{t},\tilde{x}+\tilde{l}\left(1/2-\tilde{\delta}\right)\right)\right)\tilde{\bar{\chi}}\left(\tilde{t},\tilde{x},\tilde{\delta}\right) \\
&\quad + L\left(\tilde{\chi}^+\left(\tilde{t},\tilde{x},\tilde{\delta}\right)+\tilde{\chi}^-\left(\tilde{t},\tilde{x},\tilde{\delta}\right)\right)
\end{aligned} \qquad ②$$

Additionally our motor reattachment coefficients were originally defined as:

$$R^+ = \frac{(\rho^+)^2}{(\rho^+ + \rho^-)^2} \qquad R^- = \frac{(\rho^-)^2}{(\rho^+ + \rho^-)^2} \qquad \bar{R} = \frac{2\rho^+\rho^-}{(\rho^+ + \rho^-)^2}$$

However after scaling, by equation Ⓕ, they are redefined as:

$$\tilde{R}^+ = \frac{(\tilde{\rho}^+)^2}{(\tilde{\rho}^+ + \tilde{\rho}^-)^2} \qquad \tilde{R}^- = \frac{(\tilde{\rho}^-)^2}{(\tilde{\rho}^+ + \tilde{\rho}^-)^2} \qquad \tilde{\bar{R}} = \frac{2\tilde{\rho}^+\tilde{\rho}^-}{(\tilde{\rho}^+ + \tilde{\rho}^-)^2} \qquad ③$$

4.10 Continuum equation 3 and 4 after scaling

4.10.1 Continuum equation 3 after scaling

Before scaling we originally started with:

$$\begin{aligned}
&\partial_t \bar{\chi}(t,x,d) \\
&+ \partial_x \left(\frac{v^-(t, x-(l/2-d)) + v^+(t, x+(l/2-d))}{2} \bar{\chi}(t,x,d) \right) \\
&+ \partial_d \left(\frac{v^-(t, x-(l/2-d)) - v^+(t, x+(l/2-d))}{2} \bar{\chi}(t,x,d) \right) \\
&= \bar{R} M_{\text{off}} \frac{\rho^+(t, x+(l/2-d))\rho^-(t, x-(l/2-d))}{\int_\delta^l \rho^+(t, x+(l/2-d))\rho^-(t, x-(l/2-d)) \mathrm{d}d}
\end{aligned}$$

We then wish to apply the scaling above as follows:

By equations (E), (F) and (G):

$$\begin{aligned}
\rightarrow \quad &\partial_t \frac{1}{Ll} \tilde{\bar{\chi}}(\frac{t}{t_0}, \frac{x}{L}, \frac{d}{l}) \\
&+ \partial_x \left(\frac{V_m \tilde{v}^-(\frac{t}{t_0}, \frac{x}{L} - (l/2L - d/L)) + V_m \tilde{v}^+(\frac{t}{t_0}, \frac{x}{L} + (l/2L - d/L))}{2} \frac{1}{Ll} \tilde{\bar{\chi}}(\frac{t}{t_0}, \frac{x}{L}, d/l) \right) \\
&+ \partial_d \left(\frac{V_m \tilde{v}^-(\frac{t}{t_0}, \frac{x}{L} - (l/2L - d/L)) - V_m \tilde{v}^+(\frac{t}{t_0}, \frac{x}{L} + (l/2L - d/L))}{2} \frac{1}{Ll} \tilde{\bar{\chi}}(\frac{t}{t_0}, \frac{x}{L}, d/l) \right) \\
&= \bar{R} M_{\text{off}} \frac{\frac{1}{l}\tilde{\rho}^+(\frac{t}{t_0}, \frac{x}{L} + (l/2L - d/L))\frac{1}{l}\tilde{\rho}^-(\frac{t}{t_0}, \frac{x}{L} - (l/2L - d/L))}{\int_\delta^l \frac{1}{l}\tilde{\rho}^+(\frac{t}{t_0}, \frac{x}{L} + (l/2L - d/L))\frac{1}{l}\tilde{\rho}^-(\frac{t}{t_0}, \frac{x}{L} - (l/2L - d/L)) \mathrm{d}d}
\end{aligned}$$

By equations (A), (B), (C) and (D):

$$\begin{aligned}
&\partial_t \frac{1}{Ll} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \\
&+ \partial_x \frac{V_m}{Ll} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - (l/2L - ld/lL)) + \tilde{v}^+(\tilde{t}, \tilde{x} + (l/2L - ld/lL))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&+ \partial_d \frac{V_m}{Ll} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - (l/2L - ld/lL)) - \tilde{v}^+(\tilde{t}, \tilde{x} + (l/2L - ld/lL))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&= \bar{R} M_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + (l/2L - ld/lL))\tilde{\rho}^-(\tilde{t}, \tilde{x} - (l/2L - ld/lL))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + (l/2L - ld/lL))\tilde{\rho}^-(\tilde{t}, \tilde{x} - (l/2L - ld/lL)) \mathrm{d}d}
\end{aligned}$$

By equations (B) and (D):

$$\begin{aligned}
&\partial_t \frac{1}{Ll} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \\
&+ \partial_x \frac{V_m}{Ll} \left(\frac{v^-(\tilde{t}, \tilde{x} - (\tilde{l}/2 - \tilde{l}\tilde{d})) + v^+(\tilde{t}, \tilde{x} + (\tilde{l}/2 - \tilde{l}\tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&+ \partial_d \frac{V_m}{Ll} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - (\tilde{l}/2 - \tilde{l}\tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + (\tilde{l}/2 - \tilde{l}\tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&= \bar{R} M_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + (\tilde{l}/2 - \tilde{l}\tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - (\tilde{l}/2 - \tilde{l}\tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + (\tilde{l}/2 - \tilde{l}\tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - (\tilde{l}/2 - \tilde{l}\tilde{d})) \mathrm{d}d}
\end{aligned}$$

By rearranging:

$$\partial_t \frac{1}{Ll} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d})$$

$$+ \partial_x \frac{V_m}{Ll} \left(\frac{v^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) + v^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$+ \partial_d \frac{V_m}{Ll} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$= \bar{R} M_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d})))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))\mathrm{d}d}$$

By chain rule:

$$\frac{\mathrm{d}\tilde{t}}{\mathrm{d}t} \partial_{\tilde{t}} \frac{1}{Ll} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d})$$
$$+ \frac{\mathrm{d}\tilde{x}}{\mathrm{d}x} \partial_{\tilde{x}} \frac{V_m}{Ll} \left(\frac{v^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) + v^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$+ \frac{\mathrm{d}\tilde{d}}{\mathrm{d}d} \partial_{\tilde{d}} \frac{V_m}{Ll} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$= \bar{R} M_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d})))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))\mathrm{d}d}$$

By equations Ⓐ, Ⓑ and Ⓒ:

$$\frac{1}{t_0} \partial_{\tilde{t}} \frac{1}{Ll} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d})$$
$$+ \frac{1}{L} \partial_{\tilde{x}} \frac{V_m}{Ll} \left(\frac{v^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) + v^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$+ \frac{1}{l} \partial_{\tilde{d}} \frac{V_m}{Ll} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$= \bar{R} M_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d})))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))\mathrm{d}d}$$

By definition of t_0:

$$\partial_{\tilde{t}} \frac{V_m}{L^2 l} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d})$$
$$+ \partial_{\tilde{x}} \frac{V_m}{L^2 l} \left(\frac{v^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) + v^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$+ \partial_{\tilde{d}} \frac{V_m}{L l^2} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$
$$= \bar{R} M_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d})))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))\mathrm{d}d}$$

By applying motor drop off definitions ① and ③ above and rearranging:

$$\partial_{\tilde{t}} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d})$$
$$+ \partial_{\tilde{x}} \left(\frac{v^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) + v^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right)$$

$$
\begin{aligned}
&+ \partial_{\tilde{d}} \frac{L}{l} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&= \frac{L^2 l}{V_m} \tilde{\bar{R}} \tilde{M}_{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d})))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))\mathrm{d}d}
\end{aligned}
$$

By definition ② and equation Ⓓ:

$$
\begin{aligned}
&\partial_{\tilde{t}} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \\
&+ \partial_{\tilde{x}} \left(\frac{v^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) + v^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&+ \frac{1}{\tilde{l}} \partial_{\tilde{d}} \left(\frac{\tilde{v}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d})) - \tilde{v}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))}{2} \tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, \tilde{d}) \right) \\
&= \tilde{\bar{R}} \tilde{M}^{\text{off}} \frac{\tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d})))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))}{\int_\delta^l \tilde{\rho}^+(\tilde{t}, \tilde{x} + \tilde{l}(1/2 - \tilde{d}))\tilde{\rho}^-(\tilde{t}, \tilde{x} - \tilde{l}(1/2 - \tilde{d}))\mathrm{d}d} \qquad (8)
\end{aligned}
$$

4.10.2 Continuum equation 4 after scaling

Before scaling we originally started with:

$$\bar{\chi}(t, x, l) = 0$$

After scaling, by equations Ⓐ, Ⓒ, Ⓓ and Ⓖ we have:

$$\tilde{\bar{\chi}}(\tilde{t}, \tilde{x}, 0) = 0 \qquad (9)$$

4.11 Continuum equation 5 and 6 after scaling

4.11.1 Continuum equation 5 after scaling

Before scaling we originally started with:

$$\begin{aligned}&\partial_t \chi^{\pm}(t,x,d)\\&+\partial_x\left(\left(\mp V_m+v^{\pm}(t,x\pm(l/2-d))\right)\chi^{\pm}(t,x,d)\right)\\&+\partial_d\left(\mp V_m\chi^{\pm}(t,x,d)\right)\\&=R^{\pm}M_{\text{off}}\frac{\rho^{\pm}(t,x\pm(l/2-d))^2}{\int_\delta^l \rho^{\pm}(t,x\pm(l/2-d))^2 \mathrm{d}d}\end{aligned}$$

We then wish to apply the scaling above as follows:

By equations Ⓔ, Ⓕ and Ⓖ:

$$\begin{aligned}&\partial_t \frac{1}{Ll}\tilde{\chi}^{\pm}\left(\frac{t}{t_0},\frac{x}{L},\frac{d}{l}\right)\\&+\partial_x\left(\left(\mp V_m+V_m\tilde{v}^{\pm}\left(\frac{t}{t_0},\frac{x}{L}\pm\left(\frac{l}{2L}-\frac{d}{L}\right)\right)\right)\frac{1}{Ll}\tilde{\chi}^{\pm}\left(\frac{t}{t_0},\frac{x}{L},\frac{d}{l}\right)\right)\\&+\partial_d\left(\mp V_m\frac{1}{Ll}\tilde{\chi}^{\pm}\left(\frac{t}{t_0},\frac{x}{L},\frac{d}{l}\right)\right)\\&=R^{\pm}M_{\text{off}}\frac{\frac{1}{l^2}\tilde{\rho}^{\pm}\left(\frac{t}{t_0},\frac{x}{L}\pm\left(\frac{l}{2L}-\frac{d}{L}\right)\right)^2}{\int_\delta^l \frac{1}{l^2}\tilde{\rho}^{\pm}\left(\frac{t}{t_0},\frac{x}{L}\pm\left(\frac{l}{2L}-\frac{d}{L}\right)\right)^2 \mathrm{d}d}\end{aligned}$$

By equations Ⓐ, Ⓑ, Ⓒ and Ⓓ:

$$\begin{aligned}&\partial_t \frac{1}{Ll}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\\&+\partial_x\left(\left(\mp V_m+V_m\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\left(\frac{\tilde{l}}{2}-\frac{ld}{lL}\right)\right)\right)\frac{1}{Ll}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\&+\partial_d\left(\mp V_m\frac{1}{Ll}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\&=R^{\pm}M_{\text{off}}\frac{\frac{1}{l^2}\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\left(\frac{\tilde{l}}{2}-\frac{ld}{lL}\right)\right)^2}{\int_\delta^l \frac{1}{l^2}\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\left(\frac{\tilde{l}}{2}-\frac{ld}{lL}\right)\right)^2 \mathrm{d}d}\end{aligned}$$

By equation Ⓓ and rearranging:

$$\begin{aligned}&\partial_t \frac{1}{Ll}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\\&+\partial_x\frac{V_m}{Ll}\left(\left(\mp 1+\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\left(\frac{\tilde{l}}{2}-\tilde{l}\tilde{d}\right)\right)\right)\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\&+\partial_d\frac{V_m}{Ll}\left(\mp\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\&=R^{\pm}M_{\text{off}}\frac{\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\left(\frac{\tilde{l}}{2}-\tilde{l}\tilde{d}\right)\right)^2}{\int_\delta^l \tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\left(\frac{\tilde{l}}{2}-\tilde{l}\tilde{d}\right)\right)^2 \mathrm{d}d}\end{aligned}$$

By chain rule and rearranging:

$$\frac{\mathrm{d}\tilde{t}}{\mathrm{d}t}\partial_{\tilde{t}}\frac{1}{Ll}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)$$

$$
\begin{aligned}
&+\frac{\mathrm{d}\tilde{x}}{\mathrm{d}x}\partial_{\tilde{x}}\frac{V_m}{Ll}\left(\left(\mp 1+\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)\right)\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&+\frac{\mathrm{d}\tilde{d}}{\mathrm{d}d}\partial_{\tilde{d}}\frac{V_m}{Ll}\left(\mp\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&=R^{\pm}M_{\mathrm{off}}\frac{\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2}{\int_\delta^l\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2\mathrm{d}d}
\end{aligned}
$$

By equations Ⓐ, Ⓑ and Ⓒ:

$$
\begin{aligned}
&\frac{1}{t_0}\partial_{\tilde{t}}\frac{1}{Ll}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\\
&+\frac{1}{L}\partial_{\tilde{x}}\frac{V_m}{Ll}\left(\left(\mp 1+\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)\right)\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&+\frac{1}{l}\partial_{\tilde{d}}\frac{V_m}{Ll}\left(\mp\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&=R^{\pm}M_{\mathrm{off}}\frac{\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2}{\int_\delta^l\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2\mathrm{d}d}
\end{aligned}
$$

By definition of t_0:

$$
\begin{aligned}
&\partial_{\tilde{t}}\frac{V_m}{L^2l}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\\
&+\partial_{\tilde{x}}\frac{V_m}{L^2l}\left(\left(\mp 1+\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)\right)\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&+\partial_{\tilde{d}}\frac{V_m}{Ll^2}\left(\mp\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&=R^{\pm}M_{\mathrm{off}}\frac{\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2}{\int_\delta^l\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2\mathrm{d}d}
\end{aligned}
$$

By applying motor drop off definitions ① and ③ above and rearranging:

$$
\begin{aligned}
&\partial_{\tilde{t}}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\\
&+\partial_{\tilde{x}}\left(\left(\mp 1+\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)\right)\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&+\partial_{\tilde{d}}\frac{L}{l}\left(\mp\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&=\frac{L^2l}{V_m}\tilde{R}^{\pm}\tilde{M}_{\mathrm{off}}\frac{\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2}{\int_\delta^l\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^2\mathrm{d}d}
\end{aligned}
$$

By definition ② and equation Ⓓ:

$$
\begin{aligned}
&\partial_{\tilde{t}}\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\\
&+\partial_{\tilde{x}}\left(\left(\mp 1+\tilde{v}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)\right)\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)\\
&+\frac{1}{\tilde{l}}\partial_{\tilde{d}}\left(\mp\tilde{\chi}^{\pm}\left(\tilde{t},\tilde{x},\tilde{d}\right)\right)
\end{aligned}
$$

$$= \tilde{R}^{\pm}\tilde{M}^{\mathrm{off}}\frac{\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^{2}}{\int_{\delta}^{l}\tilde{\rho}^{\pm}\left(\tilde{t},\tilde{x}\pm\tilde{l}\left(1/2-\tilde{d}\right)\right)^{2}\mathrm{d}d} \tag{10}$$

4.11.2 Continuum equation 6 after scaling

Before scaling we originally started with:

$$\chi^{\pm}(t,x,l)=0$$

After scaling, by equations A, C, D and G we have:

$$\tilde{\chi}^{\pm}(\tilde{t},\tilde{x},0)=0 \tag{11}$$

5 Perturbation approximation

Note that in all the following sections tildes will be omitted

5.1 Symmetrization

5.1.1 Symmetrization Definitions

We write the equations for the total and differential densities and velocities of actin filaments as:

$$v = \frac{v^+ + v^-}{2}, \qquad \bar{v} = \frac{v^+ - v^-}{2}$$
$$\rho = \rho^+ + \rho^-, \qquad \bar{\rho} = \rho^+ - \rho^-$$

5.2 Sum and difference equations of (7)

We can derive a sum and difference balance equation from the 2 arguments (positive and negative) of equation (8):

$$\xi(D^+ + D^-) + \eta(C^+ + C^-) + (M^+ + M^-) + \zeta(A^+ + A^-) = 0$$
$$\xi(D^+ - D^-) + \eta(C^+ - C^-) + (M^+ - M^-) + \zeta(A^+ - A^-) = 0$$

5.3 Macroscopic myosin densities

We first define the total density of myosin motors as:

$$\chi = \bar{\chi} + \chi^+ + \chi^-$$

And then define the macroscopic myosin densities as:

$$\mu = \int_\delta^1 \chi(t, x, d)\mathrm{d}d$$

$$\bar{\mu} = \int_\delta^1 \bar{\chi}(t, x, d)\mathrm{d}d$$

$$\mu^\pm = \int_\delta^1 \chi^\pm(t, x, d)\mathrm{d}d$$

5.4 Expansions of dependent quantities

We now introduce expansions of all the dependent quantities with respect to the small parameter l as $l \to 0$ (actin filaments are much shorter than the length of the whole bundle). We use subscript 0 to denote the zeroth order approximations with respect to this small parameter. For example:

$$\rho = \rho_0 + O(l), \quad \bar{\rho} = \bar{\rho}_0 + O(l),$$

$$v = v_0 + O(l), \quad \bar{v} = \bar{v}_0 + O(l),$$

$$\bar{\chi} = \bar{\chi}_0 + O(l), \quad \bar{\mu} = \bar{\mu}_0 + O(l),$$

ect.

Our goal is to use perturbation theory in the limit $l \to 0$ and to obtain a system of equations for $\rho_0, \bar{\rho}_0, v_0, \bar{v}_0$ and $\bar{\mu}_0$.

5.5 Filament Density transport equations

From equation (1) we have:

$$\partial_t \rho^{\pm}(t,x) + \partial_x(v^{\pm}(t,x)\rho^{\pm}(t,x)) = 0$$

Therefore the transport equations for ρ and $\bar{\rho}$ are:

$$\partial_t(\rho^+ + \rho^-) + \partial_x(v^+\rho^+ + v^-\rho^-) = 0 \quad \rightarrow \quad \partial_t\rho + \partial_x(v\rho - \bar{v}\bar{\rho}) = 0$$
$$\partial_t(\rho^+ - \rho^-) + \partial_x(v^+\rho^+ - v^-\rho^-) = 0 \quad \rightarrow \quad \partial_t\bar{\rho} + \partial_x(v\bar{\rho} - \bar{v}\rho) = 0$$

Applying Taylor expansion with respect to l to equations above we have:

$$\begin{aligned}
0 &= \partial_t\rho + \partial_x(v\rho - \bar{v}\bar{\rho}) \\
0 &= \partial_t(\rho_0 + O(l)) + \partial_x((v_0 + O(l))(\rho_0 + O(l)) - (\bar{v}_0 + O(l))(\bar{\rho}_0 + O(l))) \\
0 &= \partial_t\rho_0 + \partial_x(v_0\rho_0 - \bar{v}_0\bar{\rho}_0) + O(l)
\end{aligned} \tag{12}$$

$$\begin{aligned}
0 &= \partial_t\bar{\rho} + \partial_x(v\bar{\rho} - \bar{v}\rho) \\
0 &= \partial_t(\bar{\rho}_0 + O(l)) + \partial_x((v_0 + O(l))(\bar{\rho}_0 + O(l)) - (\bar{v}_0 + O(l))(\rho_0 + O(l))) \\
0 &= \partial_t\bar{\rho}_0 + \partial_x(v_0\bar{\rho}_0 - \bar{v}_0\rho_0) + O(l)
\end{aligned} \tag{13}$$

5.6 Myosin Density transport equations

By omitting tildes from equation (8) we have:

$$\begin{aligned}
&\partial_t\bar{\chi}(t,x,d) \\
&+ \partial_x\left(\frac{v^-(t,x-l(1/2-d)) + v^+(t,x+l(1/2-d))}{2}\bar{\chi}(t,x,d)\right) \\
&+ \frac{1}{l}\partial_d\left(\frac{v^-(t,x-l(1/2-d)) - v^+(t,x+l(1/2-d))}{2}\bar{\chi}(t,x,d)\right) \\
&= \bar{R}M^{\mathrm{off}}\frac{\rho^+(t,x+l(1/2-d))\rho^-(t,x-l(1/2-d))}{\int_\delta^l \rho^+(t,x+l(1/2-d))\rho^-(t,x-l(1/2-d))\mathrm{d}d}
\end{aligned}$$

Second Term of equation
By applying Taylor expansion with respect to $x - l(1/2-d)$ and $x + l(1/2-d)$

$$\begin{aligned}
&\partial_x\left(\frac{v^-(t,x-l(1/2-d)) + v^+(t,x+l(1/2-d))}{2}\bar{\chi}(t,x,d)\right) \\
=&\partial_x\left(\frac{v^-(t,x) - l(1/2-d)\partial_x v^-(t,x) + v^+(t,x) + l(1/2-d)\partial_x v^+(t,x)}{2}\bar{\chi}(t,x,d)\right) + O(l^2) \\
=&\partial_x\left(\frac{v^-(t,x) + v^+(t,x) + l(1/2-d)\partial_x(v^+(t,x) - v^-(t,x))}{2}\bar{\chi}(t,x,d)\right) + O(l^2)
\end{aligned}$$

By applying symmetrization listed above we have:

$$=\partial_x\left([v + l(1/2-d)\partial_x\bar{v}]\,\bar{\chi}(t,x,d)\right) + O(l^2)$$

Third Term of equation

By applying Taylor expansion with respect to $x - l(1/2-d)$ and $x + l(1/2-d)$

$$
\begin{aligned}
&\frac{1}{l}\partial_d\left(\frac{v^-(t,x-l(1/2-d)) - v^+(t,x+l(1/2-d))}{2}\bar{\chi}(t,x,d)\right)\\
=&\frac{1}{l}\partial_d\left(\frac{[v^-(t,x) - l(1/2-d)\partial_x v^-(t,x)] - [v^+(t,x) + l(1/2-d)\partial_x v^+(t,x)]}{2}\bar{\chi}(t,x,d)\right) + O(l^2)\\
=&\frac{1}{l}\partial_d\left(\frac{-[v^+(t,x) - v^-(t,x)] - l(1/2-d)\partial_x[v^+(t,x) + v^-(t,x)]}{2}\bar{\chi}(t,x,d)\right) + O(l^2)
\end{aligned}
$$

By applying symmetrization listed above we have:

$$
\begin{aligned}
=&\frac{1}{l}\partial_d\left([-\bar{v} - l(1/2-d)\partial_x v]\bar{\chi}(t,x,d)\right) + O(l^2)\\
=&-\frac{1}{l}\partial_d\left([\bar{v} + l(1/2-d)\partial_x v]\bar{\chi}(t,x,d)\right) + O(l^2)
\end{aligned}
$$

RHS of equation
By applying Taylor expansion with respect to $x - l(1/2-d)$ and $x + l(1/2-d)$

$$
\begin{aligned}
&\bar{R}M^{\text{off}}\frac{\rho^+(t,x+l(1/2-d)))\rho^-(t,x-l(1/2-d))}{\int_\delta^l \rho^+(t,x+l(1/2-d))\rho^-(t,x-l(1/2-d))\mathrm{d}d}\\
=&\bar{R}M^{\text{off}}\frac{[\rho^+(t,x) + l(1/2-d)\partial_x\rho^+(t,x)][\rho^-(t,x) - l(1/2-d)\partial_x\rho^-(t,x)]}{\int_\delta^l[\rho^+(t,x) + l(1/2-d)\partial_x\rho^+(t,x)][\rho^-(t,x) - l(1/2-d)\partial_x\rho^-(t,x)]\mathrm{d}d} + O(l^3)\\
=&\bar{R}M^{\text{off}}\frac{[(\rho+\bar{\rho})/2 + l(1/2-d)\partial_x(\rho+\bar{\rho})/2][(\rho-\bar{\rho})/2 - l(1/2-d)\partial_x(\rho-\bar{\rho})/2]}{\int_\delta^l[(\rho+\bar{\rho})/2 + l(1/2-d)\partial_x(\rho+\bar{\rho})/2][(\rho-\bar{\rho})/2 - l(1/2-d)\partial_x(\rho-\bar{\rho})/2]\mathrm{d}d} + O(l^3)\\
=&\bar{R}M^{\text{off}}\frac{[(\rho+\bar{\rho}) + l(1/2-d)\partial_x(\rho+\bar{\rho})][(\rho-\bar{\rho}) - l(1/2-d)\partial_x(\rho-\bar{\rho})]}{\int_\delta^l[(\rho+\bar{\rho}) + l(1/2-d)\partial_x(\rho+\bar{\rho})][(\rho-\bar{\rho}) - l(1/2-d)\partial_x(\rho-\bar{\rho})]\mathrm{d}d} + O(l^3)
\end{aligned}
$$

5.7 Filament Forces

5.7.1 D - Cytoplasm viscous drag

By omitting tildes from Equation (7) we have

$$D^\pm = \frac{1}{l}\rho^\pm(t,x)v^\pm(t,x)$$

We can now calculate the following quantities:

$$
\begin{aligned}
D^+ + D^- =&\frac{1}{l}\rho^+(t,x)v^+(t,x) + \frac{1}{l}\rho^-(t,x)v^-(t,x)\\
=&\frac{1}{l}(\rho^+(t,x)v^+(t,x) + \rho^-(t,x)v^-(t,x))\\
=&\frac{1}{l}(v\rho - \bar{v}\bar{\rho})\\
=&\frac{1}{l}((v_0 + O(l))(\rho_0 + O(l)) - (\bar{v}_0 + O(l))(\bar{\rho}_0 + O(l)))\\
=&\frac{1}{l}(v_0\rho_0 - \bar{v}_0\bar{\rho}_0) + O(l^0)
\end{aligned}
$$

$$
\begin{aligned}
D^+ - D^- =& \frac{1}{l}\rho^+(t,x)v^+(t,x) - \frac{1}{l}\rho^-(t,x)v^-(t,x) \\
=& \frac{1}{l}(\rho^+(t,x)v^+(t,x) - \rho^-(t,x)v^-(t,x)) \\
=& \frac{1}{l}(v\bar{\rho} - \bar{v}\rho) \\
=& \frac{1}{l}(v_0\bar{\rho}_0 - \bar{v}_0\rho_0) + O(l^0)
\end{aligned}
$$

5.7.2 C - Cross linker friction

By omitting tildes from Equation (7) we have

$$C^\pm = \sum_{n=-1,+1} \int_{\mathbb{R}} O(\Delta x_y)(v^\pm(t,x) - v^n(t,x+l\Delta x_y))\rho^\pm(t,x)\rho^n(t,x+l\Delta x_y)\mathrm{d}\Delta x_y$$

We first define I^m as follows:

$$I^m = \sum_{n=-1,+1} (v^m(t,x) - v^n(t,y))\rho^m(t,x)\rho^n(t,y)$$

Where we have $v^m = v + m\bar{v}$ and $\rho^m = (\rho + m\bar{\rho})/2$ for $m = -1, +1$. This gives us the result that:

$$I^+ + I^- = (v(t,x) - v(t,y))\rho(t,y)\rho(t,x) - \bar{v}(t,y)\bar{\rho}(t,y)\rho(t,x) + \bar{v}(t,x)\rho(t,y)\bar{\rho}(t,x) \qquad (1)$$

$$I^+ - I^- = 4(v(t,x) - v(t,y))\rho(t,y)\bar{\rho}(t,x) - \bar{v}(t,y)\bar{\rho}(t,y)\bar{\rho}(t,x) + \bar{v}(t,x)\rho(t,y)\rho(t,x) \qquad (2)$$

Now observe that moments of the length of the overlapping region $O(\Delta x_y)$, where Δx is the distance between the two center points of the overlapping actin filaments are given by:

$$
\begin{aligned}
&\int_{-\infty}^{\infty} O(\Delta x_y)\mathrm{d}\Delta x_y = \int_{-1}^{1}(1 - |\Delta x_y|)\mathrm{d}\Delta x_y = 1^2 \\
&\int_{-\infty}^{\infty} O(\Delta x_y)\Delta x_y\mathrm{d}\Delta x_y = \int_{-1}^{1}(1 - |\Delta x_y|)\Delta x_y\mathrm{d}\Delta x_y = 0 \\
&\int_{-\infty}^{\infty} O(\Delta x_y){\Delta x_y}^2\mathrm{d}\Delta x_y = \int_{-1}^{1}(1 - |\Delta x_y|){\Delta x_y}^2\mathrm{d}\Delta x_y = \frac{1^4}{6} \\
&\int_{-\infty}^{\infty} O(\Delta x_y){\Delta x_y}^3\mathrm{d}\Delta x_y = \int_{-1}^{1}(1 - |\Delta x_y|){\Delta x_y}^3\mathrm{d}\Delta x_y = 0 \qquad \text{ect.}
\end{aligned}
\qquad (3)
$$

This is done by applying the definition of $O(\Delta x_y)$ and noting that $O(\Delta x_y)$ is only non-zero between -1 and 1.

Given all these definitions we can now find that by (1):

$$
\begin{aligned}
C^+ + C^- &= \int_{\mathbb{R}} I^+|_{y=x+l\Delta x_y} O(\Delta x_y)\mathrm{d}\Delta x_y + \int_{\mathbb{R}} I^-|_{y=x+l\Delta x_y} O(\Delta x_y)\mathrm{d}\Delta x_y \\
C^+ + C^- &= \int_{\mathbb{R}} (I^+|_{y=x+l\Delta x_y} + I^-|_{y=x+l\Delta x_y}) O(\Delta x_y)\mathrm{d}\Delta x_y
\end{aligned}
$$

By equation (1) we have:

$$
\begin{aligned}
C^+ + C^- = &\int_{\mathbb{R}} O(\Delta x_y)(v(t,x) - v(t,x+l\Delta x_y))\rho(t,x+l\Delta x_y)\rho(t,x)\mathrm{d}\Delta x_y \\
&- \int_{\mathbb{R}} O(\Delta x_y)\bar{v}(t,x+l\Delta x_y)\bar{\rho}(t,x+l\Delta x_y)\rho(t,x)\mathrm{d}\Delta x_y \\
&+ \int_{\mathbb{R}} O(\Delta x_y)\bar{v}(t,x)\rho(t,x+l\Delta x_y)\bar{\rho}(t,x)\mathrm{d}\Delta x_y
\end{aligned}
$$

First Term of equation:
By Taylor expansion with respect to $x + l\Delta x_y$:

$$\int_{\mathbb{R}} O(\Delta x_y)(v(t,x) - v(t, x + l\Delta x_y))\rho(t, x + l\Delta x_y)\rho(t,x)\mathrm{d}\Delta x_y$$

$$= \int_{\mathbb{R}} O(\Delta x_y) \left(v(t,x) - \left[v(t,x) + l\Delta x_y \partial_x v(t,x) + \frac{(l\Delta x_y)^2}{2}\partial_{xx} v(t,x) \right] \right)$$
$$\times [\rho(t,x) + l\Delta x_y \partial_x \rho(t,x)]\rho(t,x)\mathrm{d}\Delta x_y + O(l^4)$$

$$= \int_{\mathbb{R}} O(\Delta x_y) \left[-l\Delta x_y \partial_x v(t,x) - \frac{(l\Delta x_y)^2}{2}\partial_{xx} v(t,x) \right]$$
$$\times [\rho(t,x) + l\Delta x_y \partial_x \rho(t,x)]\rho(t,x)\mathrm{d}\Delta x_y + O(l^4)$$

$$= \int_{\mathbb{R}} O(\Delta x_y) [-l\Delta x_y \partial_x v(t,x)\rho(t,x) - l\Delta x_y \partial_x v(t,x) l\Delta x_y \partial_x \rho(t,x)$$
$$\left. - \frac{(l\Delta x_y)^2}{2}\partial_{xx} v(t,x)\rho(t,x) - \frac{(l\Delta x_y)^2}{2}\partial_{xx} v(t,x) l\Delta x_y \partial_x \rho(t,x) \right] \rho(t,x)\mathrm{d}\Delta x_y + O(l^4)$$

Since all odd moments are equal to 0 by 3, and by the 2nd moment definition in 3:

$$= \int_{\mathbb{R}} O(\Delta x_y) \left[-(l\Delta x_y)^2 \partial_x v(t,x)\partial_x \rho(t,x) - \frac{(l\Delta x_y)^2}{2}\rho(t,x)\partial_{xx} v(t,x) \right] \rho(t,x)\mathrm{d}\Delta x_y + O(l^4)$$
$$= -l^2 \int_{\mathbb{R}} O(\Delta x_y){\Delta x_y}^2 \left[\partial_x v(t,x)\partial_x \rho(t,x) + \frac{1}{2}\rho(t,x)\partial_{xx} v(t,x) \right] \rho(t,x)\mathrm{d}\Delta x_y + O(l^4)$$
$$= -l^2 \frac{1}{6} \left[\partial_x v(t,x)\partial_x \rho(t,x) + \frac{1}{2}\rho(t,x)\partial_{xx} v(t,x) \right] \rho(t,x) + O(l^4)$$
$$= -\frac{l^2}{6} \left[\partial_x v \partial_x \rho + \frac{1}{2}\rho \partial_{xx} v \right] \rho + O(l^4)$$
$$= -\frac{l^2}{12}\partial_x(\rho^2 \partial_x v) + O(l^4)$$

Second Term of equation:
By Taylor expansion with respect to $x + l\Delta x_y$:

$$-\int_{\mathbb{R}} O(\Delta x_y)\bar{v}(t, x + l\Delta x_y)\bar{\rho}(t, x + l\Delta x_y)\rho(t,x)\mathrm{d}\Delta x_y$$
$$= -\int_{\mathbb{R}} O(\Delta x_y) \left[\bar{v}(t,x) + l\Delta x_y \partial_x \bar{v}(t,x) + \frac{(l\Delta x_y)^2}{2}\partial_{xx}\bar{v}(t,x) \right]$$
$$\times \left[\bar{\rho}(t,x) + l\Delta x_y \partial_x \bar{\rho}(t,x) + \frac{(l\Delta x_y)^2}{2}\partial_{xx}\bar{\rho}(t,x) \right] \rho(t,x)\mathrm{d}\Delta x_y + O(l^5)$$

Since all odd moments are equal to 0 by 3 and by the definition of the 0th and 2nd moment:

$$= -\int_{\mathbb{R}} O(\Delta x_y) \left[\bar{v}\bar{\rho} + \bar{v}\frac{(l\Delta x_y)^2}{2}\partial_{xx}\bar{\rho} + (l\Delta x_y)^2 \partial_x \bar{v}\bar{\rho} + \bar{\rho}\frac{(l\Delta x_y)^2}{2}\partial_{xx}\bar{v} \right] \rho \mathrm{d}\Delta x_y + O(l^4)$$
$$= -\bar{v}\bar{\rho}\rho - \frac{l^2}{6}\left[\rho\bar{v}\frac{1}{2}\partial_{xx}\bar{\rho} + \rho\partial_x \bar{v}\bar{\rho} + \rho\bar{\rho}\frac{1}{2}\partial_{xx}\bar{v} \right] + O(l^4)$$
$$= -\bar{v}\bar{\rho}\rho - \frac{l^2}{12}[\rho\bar{v}\partial_{xx}\bar{\rho} + 2\rho\partial_x \bar{v}\bar{\rho} + \rho\bar{\rho}\partial_{xx}\bar{v}] + O(l^4)$$

$$=-\bar{v}\bar{\rho}\rho-\frac{l^2}{12}(\rho\partial_{xx}(\bar{\rho}\bar{v}))+O(l^3)$$

Third Term of equation:
By Taylor expansion with respect to $x+l\Delta x_y$:

$$\int_{\mathbb{R}} O(\Delta x_y)\bar{v}(t,x)\rho(t,x+l\Delta x_y)\bar{\rho}(t,x)\mathrm{d}\Delta x_y$$
$$=\int_{\mathbb{R}} O(\Delta x_y)\bar{v}(t,x)\left[\rho(t,x)+l\Delta x_y\partial_x\rho(t,x)+\frac{(l\Delta x_y)^2}{2}\partial_{xx}\rho(t,x)\right]\bar{\rho}(t,x)\mathrm{d}\Delta x_y+O(l^3)$$

Since all odd moments are equal to 0 by ③ and by the definition of the 0th and 2nd moment:

$$=\int_{\mathbb{R}} O(\Delta x_y)\left[\bar{v}(t,x)\bar{\rho}(t,x)\rho(t,x)+\bar{v}(t,x)\bar{\rho}(t,x)\frac{(l\Delta x_y)^2}{2}\partial_{xx}\rho(t,x)\right]\mathrm{d}\Delta x_y+O(l^3)$$
$$=(1)\bar{v}(t,x)\bar{\rho}(t,x)\rho(t,x)+\left(\frac{1}{6}\right)l^2\left(\bar{v}(t,x)\bar{\rho}(t,x)\frac{1}{2}\partial_{xx}\rho(t,x)\right)+O(l^3)$$
$$=\bar{v}(t,x)\bar{\rho}(t,x)\rho(t,x)+\frac{l^2}{12}\left(\bar{v}(t,x)\bar{\rho}(t,x)\partial_{xx}\rho(t,x)\right)+O(l^3)$$
$$=\bar{v}\bar{\rho}\rho+\frac{l^2}{12}\left(\bar{v}\bar{\rho}\partial_{xx}\rho\right)+O(l^3)$$

Combining the above terms:
We now have:

$$\begin{aligned}
C^++C^- &=-\frac{l^2}{12}\partial_x(\rho^2\partial_x v)-\bar{v}\bar{\rho}\rho-\frac{l^2}{12}(\rho\partial_{xx}(\bar{\rho}\bar{v}))+\bar{v}\bar{\rho}\rho+\frac{l^2}{12}\left(\bar{v}\bar{\rho}\partial_{xx}\rho\right)+O(l^3)\\
&=-\frac{l^2}{12}\partial_x(\rho^2\partial_x v)-\frac{l^2}{12}(\rho\partial_{xx}(\bar{\rho}\bar{v}))+\frac{l^2}{12}\left(\bar{v}\bar{\rho}\partial_{xx}\rho\right)+O(l^3)\\
&=-\frac{l^2}{12}\left(\partial_x(\rho^2\partial_x v)+\rho\partial_{xx}(\bar{\rho}\bar{v})-\bar{v}\bar{\rho}\partial_{xx}\rho\right)+O(l^3)\\
&=-\frac{l^2}{12}\partial_x\left(\rho^2\partial_x\left(v+\frac{\bar{\rho}\bar{\bar{v}}}{\rho}\right)\right)+O(l^3)\\
&=-\frac{l^2}{12}\partial_x\left(\rho_0^2\partial_x\left(v_0+\frac{\bar{\rho}_0\bar{\bar{v}}_0}{\rho_0}\right)\right)+O(l^3)
\end{aligned}$$

similarly we also have:

$$C^+-C^-=\bar{v}_0(\rho_0^2-\bar{\rho}_0^2)+O(l^1)$$

5.7.3 M - Myosin Motor force

By omitting tildes from Equation (7) we have

$$M^\pm=\int_{\mathbb{R}}\bar{\chi}\left(t,x-l\left(\frac{1}{2}-d\right),\frac{1}{2}\mp\left(\frac{1}{2}-d\right)\right)\left(\pm 1-\frac{v^\pm(t,x)-v^\mp\left(t,x-2l(\frac{1}{2}-d)\right)}{2}\right)\mathrm{d}d$$

First we will consider M^+

$$M^+=\int_{\mathbb{R}}\bar{\chi}\left(t,x-l\left(\frac{1}{2}-d\right),d\right)\left(1-\frac{v^+(t,x)-v^-\left(t,x-2l(\frac{1}{2}-d)\right)}{2}\right)\mathrm{d}d$$

Applying Taylor expansion with respect to $x - l(1/2 - d)$ and $x - 2l(1/2 - d)$ and using chain rule

$$M^{+} = \int_{\mathbb{R}} \left(\bar{\chi}\,(t,x,d) - l\left(\frac{1}{2} - d\right) \partial_x \bar{\chi}\,(t,x,d) \right) \left(1 - \frac{v^{+}\,(t,x) - v^{-}\,(t,x) + 2l(\frac{1}{2} - d)\partial_x v^{-}\,(t,x)}{2} \right) \mathrm{d}d + O(l^2)$$

Applying Symmetrization definitions stated above

$$M^{+} = \int_{\mathbb{R}} \left(\bar{\chi}\,(t,x,d) - l\left(\frac{1}{2} - d\right) \partial_x \bar{\chi}\,(t,x,d) \right) \left(1 - \bar{v} - l\left(\frac{1}{2} - d\right) \partial_x v^{-}\,(t,x) \right) \mathrm{d}d + O(l^2)$$

We will now consider M^{-}

$$M^{-} = \int_{\mathbb{R}} \bar{\chi}\left(t, x - l\left(\frac{1}{2} - d\right), 1 - d\right) \left(-1 - \frac{v^{-}\,(t,x) - v^{+}\left(t, x - 2l(\frac{1}{2} - d)\right)}{2} \right) \mathrm{d}d$$

Applying Taylor expansion with respect to $x - l(1/2 - d)$, $1 - d$ and $x - 2l(1/2 - d)$ and using chain rule

$$\begin{aligned} M^{-} = \int_{\mathbb{R}} & \left(\bar{\chi}\,(t,x,d) - l\left(\frac{1}{2} - d\right) \partial_x \bar{\chi}\,(t,x,d) - (1 - 2d)\partial_d \bar{\chi}\,(t,x,d) \right) \\ & \times \left(-1 - \frac{v^{-}\,(t,x) - v^{+}\,(t,x) + 2l(\frac{1}{2} - d)\partial_x v^{+}\,(t,x)}{2} \right) \mathrm{d}d + O(l^2) \end{aligned}$$

Applying Symmetrization definitions stated above

$$\begin{aligned} M^{-} = \int_{\mathbb{R}} & \left(\bar{\chi}\,(t,x,d) - l\left(\frac{1}{2} - d\right) \partial_x \bar{\chi}\,(t,x,d) - (1 - 2d)\partial_d \bar{\chi}\,(t,x,d) \right) \\ & \times \left(-1 + \bar{v} + l\left(\frac{1}{2} - d\right) \partial_x v^{+}\,(t,x) \right) \mathrm{d}d + O(l^2) \end{aligned}$$

Using the definitions above and noting that $\bar{\chi}$ is constant with respect to d we now have:

$$
\begin{aligned}
M^{+}+M^{-}=&\int_{\mathbb{R}}\left(\bar{\chi}\left(t,x,d\right)+l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\left(t,x,d\right)\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{-}\left(t,x\right)\right)\\
&+\left(\bar{\chi}\left(t,x,d\right)-l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\left(t,x,d\right)\right)\left(-1+\bar{v}+l\left(\frac{1}{2}-d\right)\partial_x v^{+}\left(t,x\right)\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{-}\right)\\
&-\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(-1+\bar{v}+l\left(\frac{1}{2}-d\right)\partial_x v^{+}\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{-}\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{+}\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(2-2\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x(v^{-}+v^{+})\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x 2\bar{v}\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(2-2\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x 2v\right)\mathrm{d}d+O(l^2)\\
=&\,2l\int_{\mathbb{R}}\left(\frac{1}{2}-d\right)\left[\bar{\chi}\left(\partial_x\bar{v}\right)+\left(\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v\right)\right]\mathrm{d}d+O(l^2)\\
=&\,2l\int_{\mathbb{R}}\left(\frac{1}{2}-d\right)\left[\partial_x(\bar{\chi}(1-\bar{v}))\right]\mathrm{d}d+O(l^2)\\
=&\,2l\partial_x\int_{\mathbb{R}}\left(\frac{1}{2}-d\right)\left[(\bar{\chi}(1-\bar{v}))\right]\mathrm{d}d+O(l^2)\\
=&\,2l\partial_x\left(\frac{1}{3}(2\delta+l)\right)\bar{\mu}(1-\bar{v})+O(l^3)\\
=&\,\frac{2l}{3}(2\delta+l)\bar{\mu}\partial_x(1-\bar{v})+O(l^3)\\
=&-2l\left(\frac{1}{3}(2\delta+l)\right)\partial_x(\bar{\mu}(1-\bar{v}))+O(l^3)\\
=&\,\frac{-2l}{3}(2\delta+l)\partial_x[\bar{\mu}_0(1-\bar{v}_0)]+O(l^3)
\end{aligned}
$$

Similarly we can find that:

$$
\begin{aligned}
M^{+}-M^{-}=&\int_{\mathbb{R}}\left(\bar{\chi}\left(t,x,d\right)+l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\left(t,x,d\right)\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{-}\left(t,x\right)\right)\\
&\qquad-\left(\bar{\chi}\left(t,x,d\right)-l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\left(t,x,d\right)\right)\left(-1+\bar{v}+l\left(\frac{1}{2}-d\right)\partial_x v^{+}\left(t,x\right)\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{-}\right)\\
&-\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{+}\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(1-\bar{v}-l\left(\frac{1}{2}-d\right)\partial_x v^{-}\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(-1+\bar{v}+l\left(\frac{1}{2}-d\right)\partial_x v^{+}\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(l\left(\frac{1}{2}-d\right)\partial_x(v^{+}-v^{-})\right)\mathrm{d}d+O(l^2)\\
=&\int_{\mathbb{R}}\bar{\chi}\left(l\left(\frac{1}{2}-d\right)\partial_x 2\bar{v}\right)\\
&+\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{\chi}\right)\left(l\left(\frac{1}{2}-d\right)\partial_x 2\bar{v}\right)\mathrm{d}d+O(l^2)\\
=&\,2l\int_{\mathbb{R}}\left(\frac{1}{2}-d\right)\left[\bar{\chi}\left(\partial_x\bar{v}\right)+\left(\partial_x\bar{\chi}\right)\left(l\left(\frac{1}{2}-d\right)\partial_x\bar{v}\right)\right]\mathrm{d}d+O(l^2)\\
=&\,2l\partial_x\bar{v}\int_{\mathbb{R}}\left(\frac{1}{2}-d\right)\left[\bar{\chi}+\left(\partial_x\bar{\chi}\right)\left(l\left(\frac{1}{2}-d\right)\right)\right]\mathrm{d}d+O(l^2)\\
=&-2\bar{\mu}_0(1-\bar{v}_0)+O(l^1)
\end{aligned}
$$

5.7.4 A - Focal Adhesion drag

By omitting tildes from Equation (7) we have

$$
A^{\pm}=\sum_{j=A,B}O^{a}(\Delta x_z)\left(v^{\pm}\left(t,x\right)-V\left(t,x+l\Delta x_z\right)\right)\rho^{\pm}\left(t,x\right)
$$

We can now calculate the following quantities:

$$
\begin{aligned}
A^{+}+A^{-} = & \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{+}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{+}(t,x)\\
& + \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{-}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{-}(t,x)\\
= & \sum_{j=A,B} O^{a}(\Delta x_z)\left(\left(v^{+}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{+}(t,x)+\left(v^{-}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{-}(t,x)\right)\\
= & \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{+}(t,x)\rho^{+}(t,x)-V(t,x+l\Delta x_z)\rho^{+}(t,x)+v^{-}(t,x)\rho^{-}(t,x)-V(t,x+l\Delta x_z)\rho^{-}(t,x)\right)\\
= & \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{+}(t,x)\rho^{+}(t,x)+v^{-}(t,x)\rho^{-}(t,x)-V(t,x+l\Delta x_z)(\rho^{+}(t,x)+\rho^{-}(t,x))\right)
\end{aligned}
$$

By applying symmetrization definitions, letting $V(t,x)=V_j$ and considering that O^a will be equal to the density of filaments: ρ at that point x

$$
\begin{aligned}
& = \sum_{j=A,B} O^{a}(\Delta x_z)\left(v\rho-\bar{v}\bar{\rho}-V(t,x+l\Delta x_z)\rho\right)\\
& = \sum_{j=A,B} \rho\left(v\rho-\bar{v}\bar{\rho}-V_j\rho\right)
\end{aligned}
$$

$$
\begin{aligned}
A^{+}-A^{-} = & \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{+}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{+}(t,x)\\
& - \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{-}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{-}(t,x)\\
= & \sum_{j=A,B} O^{a}(\Delta x_z)\left(\left(v^{+}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{+}(t,x)-\left(v^{-}(t,x)-V(t,x+l\Delta x_z)\right)\rho^{-}(t,x)\right)\\
= & \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{+}(t,x)\rho^{+}(t,x)-V(t,x+l\Delta x_z)\rho^{+}(t,x)-v^{-}(t,x)\rho^{-}(t,x)+V(t,x+l\Delta x_z)\rho^{-}(t,x)\right)\\
= & \sum_{j=A,B} O^{a}(\Delta x_z)\left(v^{+}(t,x)\rho^{+}(t,x)-v^{-}(t,x)\rho^{-}(t,x)-V(t,x+l\Delta x_z)(\rho^{+}(t,x)-\rho^{-}(t,x))\right)
\end{aligned}
$$

By applying symmetrization definitions, letting $V(t,x)=V_j$ and considering that O^a will be equal to the density of filaments: ρ at that point x

$$
\begin{aligned}
& = \sum_{j=A,B} O^{a}(\Delta x_z)\left(v\bar{\rho}-\bar{v}\rho-V(t,x+l\Delta x_z)\bar{\rho}\right)\\
& = \sum_{j=A,B} \rho\left(v\bar{\rho}-\bar{v}\rho-V_j\bar{\rho}\right)
\end{aligned}
$$

5.7.5 Final equations

This leaves us with the following equations:

$$0=\xi(D^{+}+D^{-})+\eta(C^{+}+C^{-})+(M^{+}+M^{-})+\zeta(A^{+}+A^{-})$$

$$0=\xi(D^{+}-D^{-})+\eta(C^{+}-C^{-})+(M^{+}-M^{-})+\zeta(A^{+}-A^{-})$$