Momentum Distribution Properties of Ultracold Hard-Core Anyons in One Dimension

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Motivation

Anyons: mostly theoretical but can model real systems

Anyons:

- Are theoretical particles
- Used in 2D systems to realize topological quantum computers
- Help us understand real 1D systems of spinful particles

Background

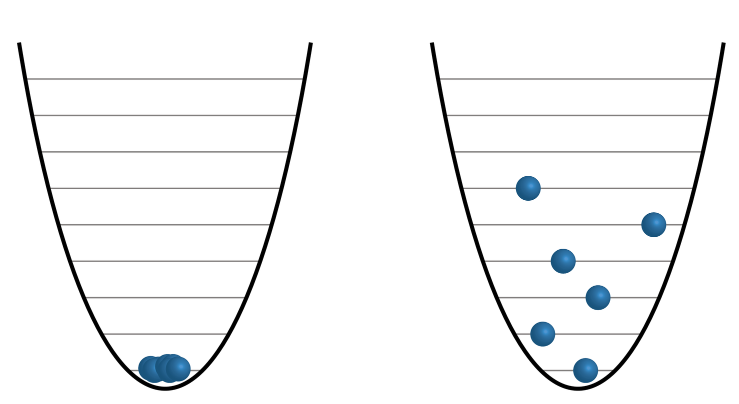
Anyon is "mixture" between boson and fermion

Bosons vs. Fermions:

- Two types of particles in nature: bosons and fermions
- Different because they have different spin
- Bosons: spin quantum number is an integer (0, 1, 2, …)
- Fermions: spin quantum number is odd multiple of 1/2 (1/2, 3/2, …)

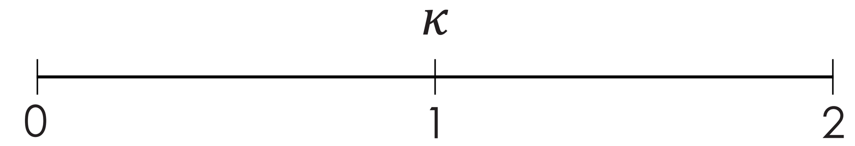
Bosons & Fermions behave differently:

- Fermions follow Pauli Exclusion Principle (PEP), bosons do not
- PEP: two fermions cannot have same quantum state or position



Anyons:

- Not experimentally observed like bosons & fermions
- Described by anyon parameter κ between 0 and 2
- Can be bosons, fermions, or something in between as κ varies
- Need not follow PEP, unless κ = 1



Methods

Hard core gives particle fermi-like properties

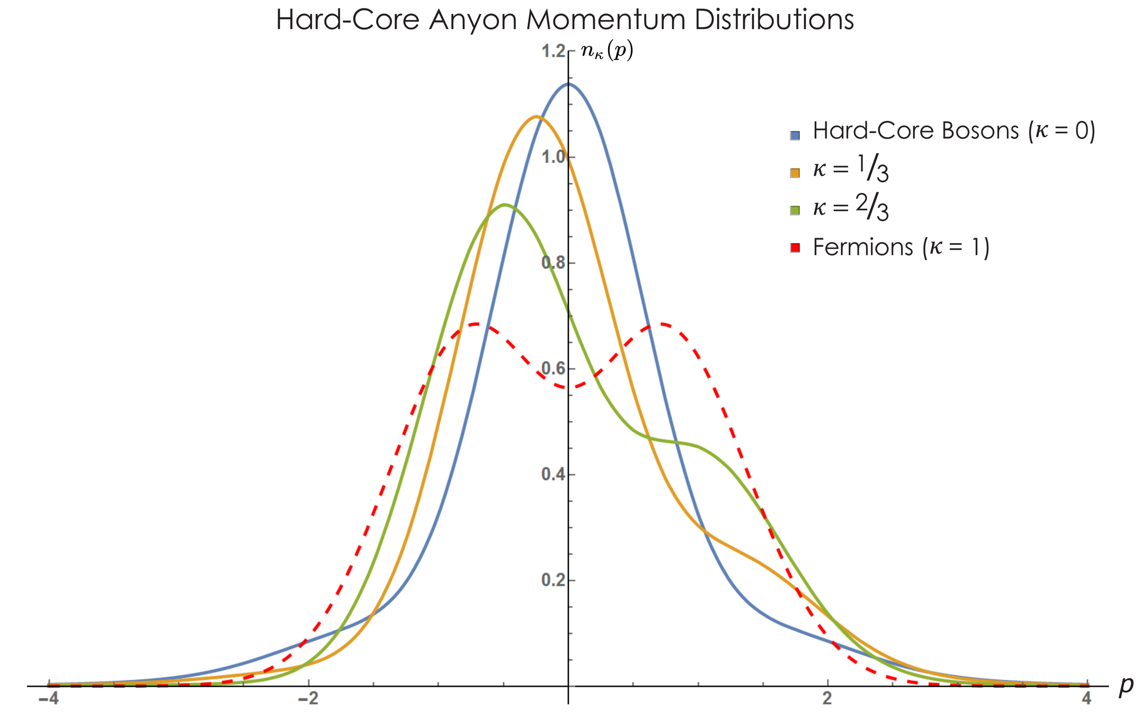
What is a hard core?

- Anyons differ from fermions — e.g., only the latter obeys PEP
- Particles with hard cores cannot occupy same position
- So, hard-core anyons (**HCA**) are similar to fermions
  → known as "Fermionization"

The system we study comprises:

- Two spinless, hard-core anyons in harmonic potential: $V(x) = \frac{1}{2}m\omega^2 x^2$
- Ultracold HCA: this means HCA are in lowest energy configuration

Anyon-fermion mapping lets us study HCA



Each $n_\kappa(p)$ was plotted using numerical integration

Note: all the following equations use natural units — i.e., $\hbar = m = \omega = 1$

Step 1: Wave function for two fermions → Anyon-Fermion Mapping → Step 2: Wave function for two hard-core anyons → Fourier Transform → Step 3: Momentum Distribution

$$\psi_\kappa(x_1, x_2) = e^{i\pi(1-\kappa)\theta(x_2 - x_1)}\psi_F(x_1, x_2)$$

w.f. for two HCA with parameter κ; w.f. for two fermions

where $\theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

$$n_\kappa(p) = \frac{N}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' e^{ip(x-x')}\rho(x, x')$$

$$\rho(x, x') = \int_{-\infty}^{\infty} dx_2 \psi_\kappa^*(x, x_2)\psi_\kappa(x', x_2)$$

Anyon-fermion mapping:

- Transforms fermi wave function into HCA wave function
- Multiplies fermi w.f. by exponential function

Momentum distribution $n_\kappa(p)$:

- Gives number density of particles in momentum space
- Calculated using the one-body density matrix $\rho(x, x')$

Results

HCA momentum distribution decays $\mathcal{O}(p^{-4})$

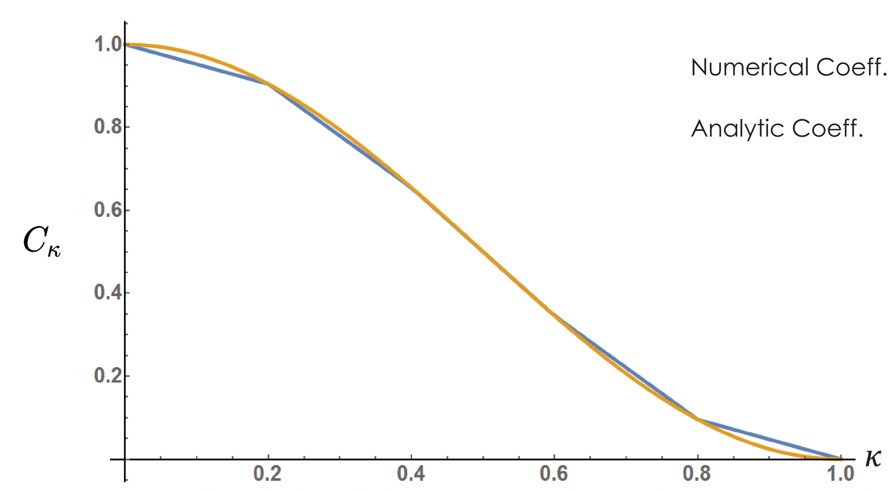
For large momentum p:

- Fermi momentum distribution $n_F(p)$ decays to zero exponentially
- HCA momentum distribution has power law decay

We show using asymptotic analysis:

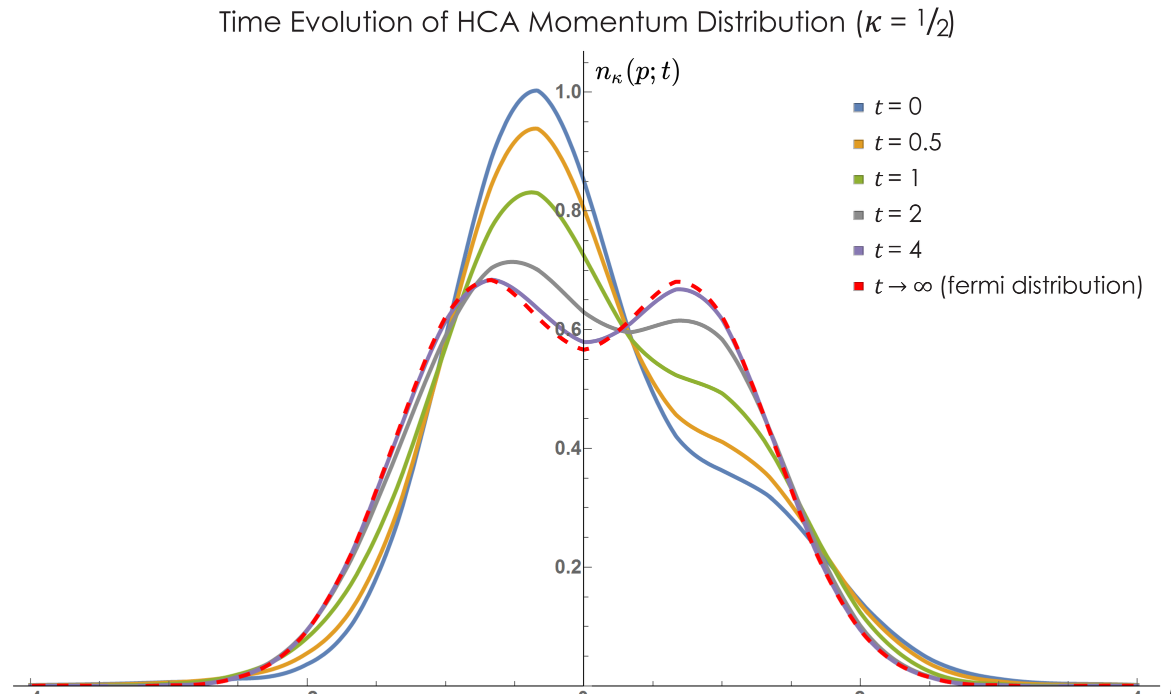
$$\lim_{p\to\infty} = \left(\frac{2}{\pi}\right)^{3/2}\frac{\cos^2\left(\frac{\pi\kappa}{2}\right)}{p^4} = \frac{C_\kappa}{p^4}$$

Numerically calculate C_κ to confirm our analytic result:



Numerically calculate C_κ for various κ and normalize to unity at $\kappa = 0$

Expanding HCA mimic fermi momentum distribution



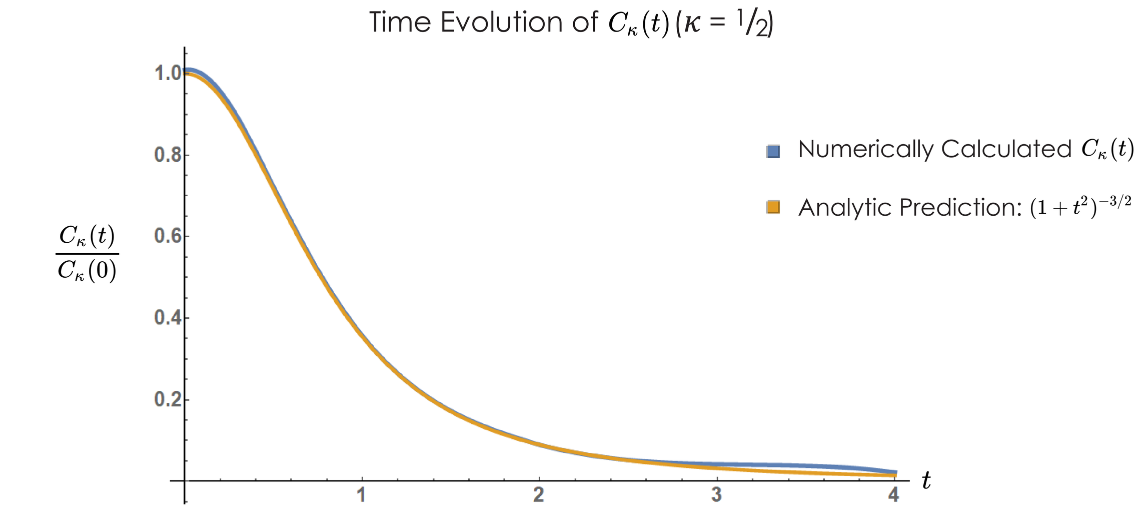
What happens if the potential is turned off? (i.e., $\omega = 1 \to \omega = 0$)

- We show numerically that $\lim_{t\to\infty} n_\kappa(p; t) = n_F(p)$
- As HCA gas expands, its momentum dist. becomes the fermi dist.
- This phenomenon is known as "dynamical fermionization"
- We show analytically: $C_\kappa(t) = \left(\frac{2}{\pi}\right)^{3/2}\cos^2\left(\frac{\pi\kappa}{2}\right)\frac{1}{(1+t^2)^{3/2}}$

Conclusions

HCA undergo dynamical fermionization

- Other papers have shown that dynamical fermionization occurs in systems of hard-core bosons
- We show that it also occurs in systems of hard-core anyons
- The momentum tail coefficient for hard-core anyons shares same time dependence of hard-core bosons



Numerically calculated time dependence of momentum tail coefficient matches analytic prediction

Future Work

Still need to study case where particles have spin

- We have studied the two hard-core anyon case
- We are working on asking some of the same questions for the case where N hard-core anyons are confined in a harmonic trap
  - Less explored area because expressions for the momentum distribution grow more complex with more particles
- So far we have only studied spinless hard-core anyons, which means the spin of each particle is constrained and cannot change
  - When the spin of each particle is allowed to change, the dynamics are more sophisticated

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