Momentum Distribution Properties of Ultracold Hard-Core Anyons in One Dimension

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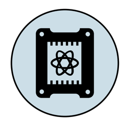
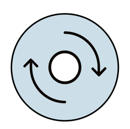
Motivation

Anyons: mostly theoretical but can model real systems

Anyons:


Are theoretical particles


Used in 2D systems to realize topological quantum computers


Help us understand real 1D systems of spinful particles

Background

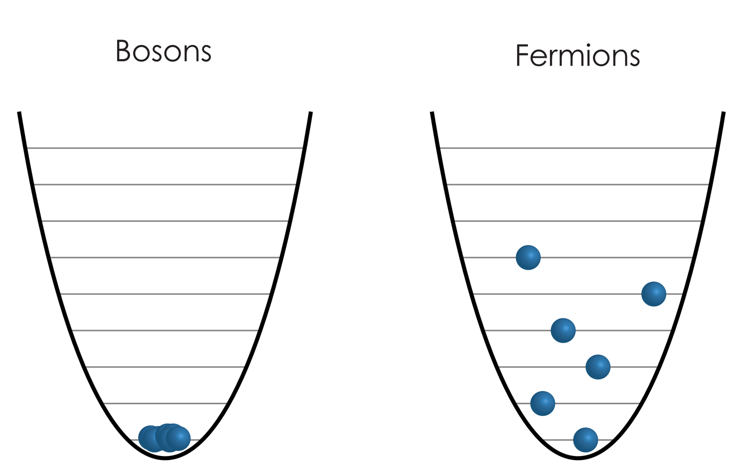
Anyon is "mixture" between boson and fermion

Bosons vs. Fermions:

- Two types of particles in nature: bosons and fermions
- Different because they have different spin
- Bosons: spin quantum number is an integer (0, 1, 2, …)
- Fermions: spin quantum number is odd multiple of $1/2$ ($1/2$, $3/2$, …)

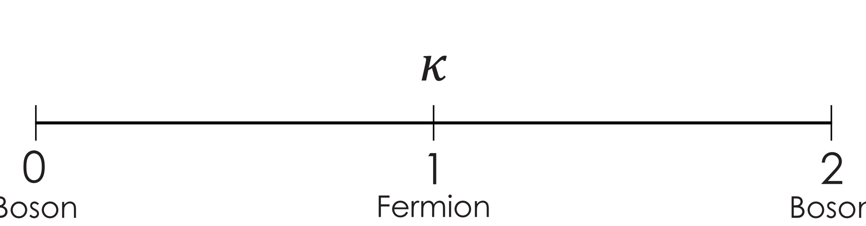
Bosons & Fermions behave differently:

- Fermions follow Pauli Exclusion Principle (PEP), bosons do not
- PEP: two fermions cannot have same quantum state or position



Anyons:

- Not experimentally observed like bosons & fermions
- Described by anyon parameter κ between 0 and 2
- Can be bosons, fermions, or something in between as κ varies
- Need not follow PEP, unless $\kappa = 1$



Methods

Hard core gives particle fermi-like properties

What is a hard core?

- Anyons differ from fermions — e.g., only the latter obeys PEP
- Particles with hard cores cannot occupy same position
- So, hard-core anyons (**HCA**) are similar to fermions

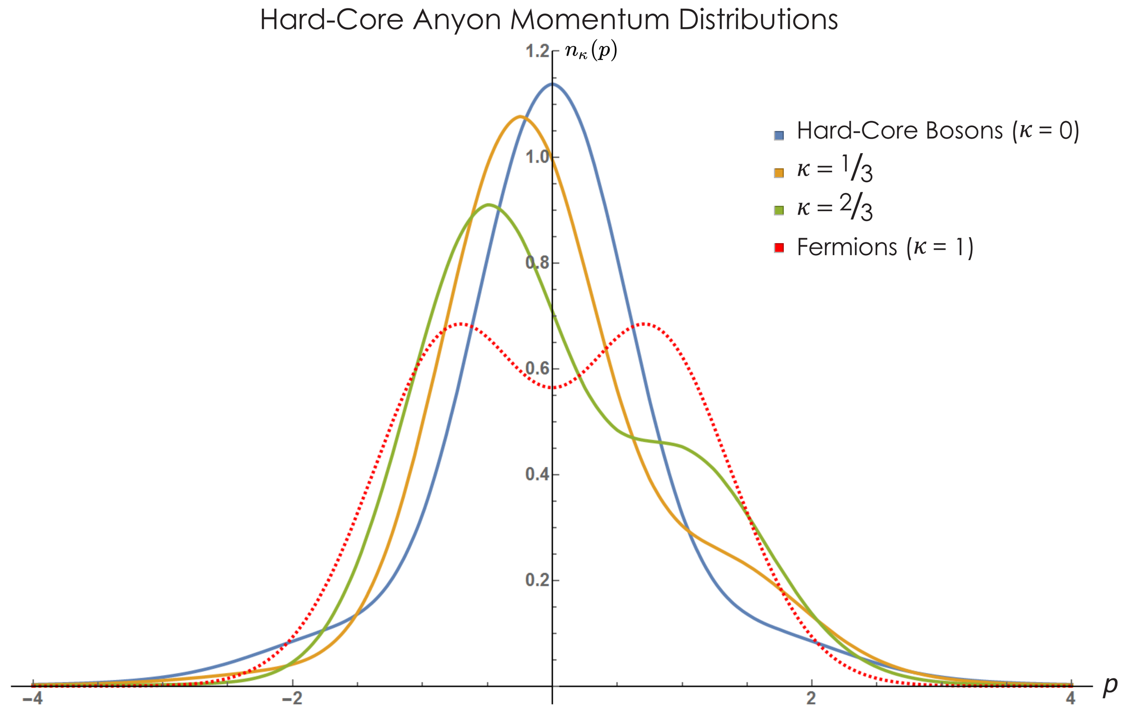
→ known as "Fermionization"

The system we study comprises:

- Two spinless, hard-core anyons in harmonic potential: $V(x) = \frac{1}{2}m\omega^2 x^2$
- Ultracold HCA: particles are in lowest energy configuration

Note: all the following equations use natural units — i.e., $\hbar = m = \omega = 1$

Anyon-fermion mapping lets us study HCA



Each $n_\kappa(p)$ was plotted using numerical integration

Step 1: Wave function for two fermions — Anyon-Fermion Mapping → Step 2: Wave function for two hard-core anyons — Fourier Transform → Step 3: Momentum Distribution

$$\psi_\kappa(x_1, x_2) = \overbrace{e^{i\pi(1-\kappa)\theta(x_2 - x_1)}}^{\text{Anyon-Fermion Mapping}} \psi_F(x_1, x_2)$$

w.f. for two HCA with parameter κ

w.f. for two fermions

where $\theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

$$n_\kappa(p) = \frac{N}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' e^{ip(x-x')}\rho(x, x')$$

$$\rho(x, x') = \int_{-\infty}^{\infty} dx_2 \psi_\kappa^*(x, x_2)\psi_\kappa(x', x_2)$$

Anyon-fermion mapping:

- Transforms fermi wave function into HCA wave function
- Multiplies fermi w.f. by exponential function

Momentum distribution $n_\kappa(p)$**:**

- Gives number density of particles in momentum space
- Calculated using the one-body density matrix $\rho(x, x')$

Results

HCA momentum distribution decays $\mathcal{O}(p^{-4})$

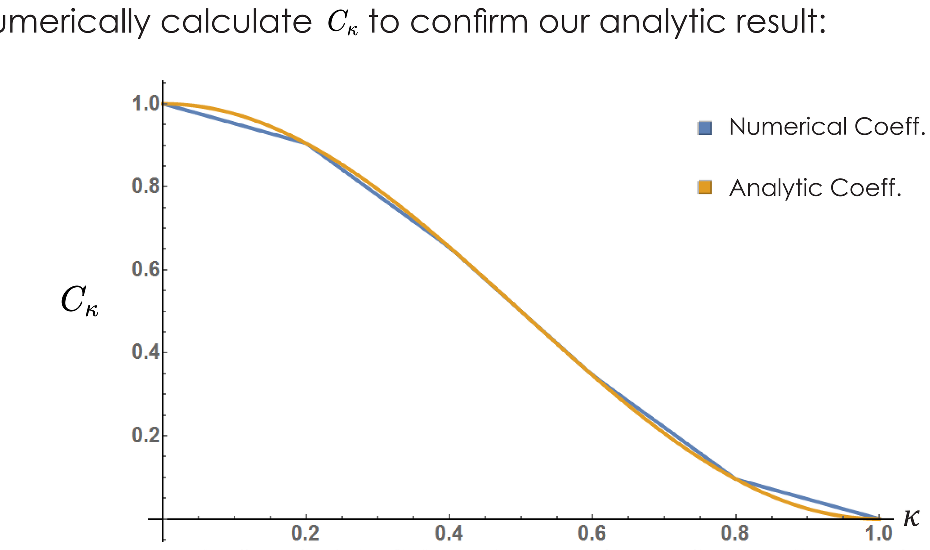
For large momentum *p*:

- Fermi momentum distribution $n_F(p)$ decays to zero exponentially
- HCA momentum distribution has power law decay

We show using asymptotic analysis:

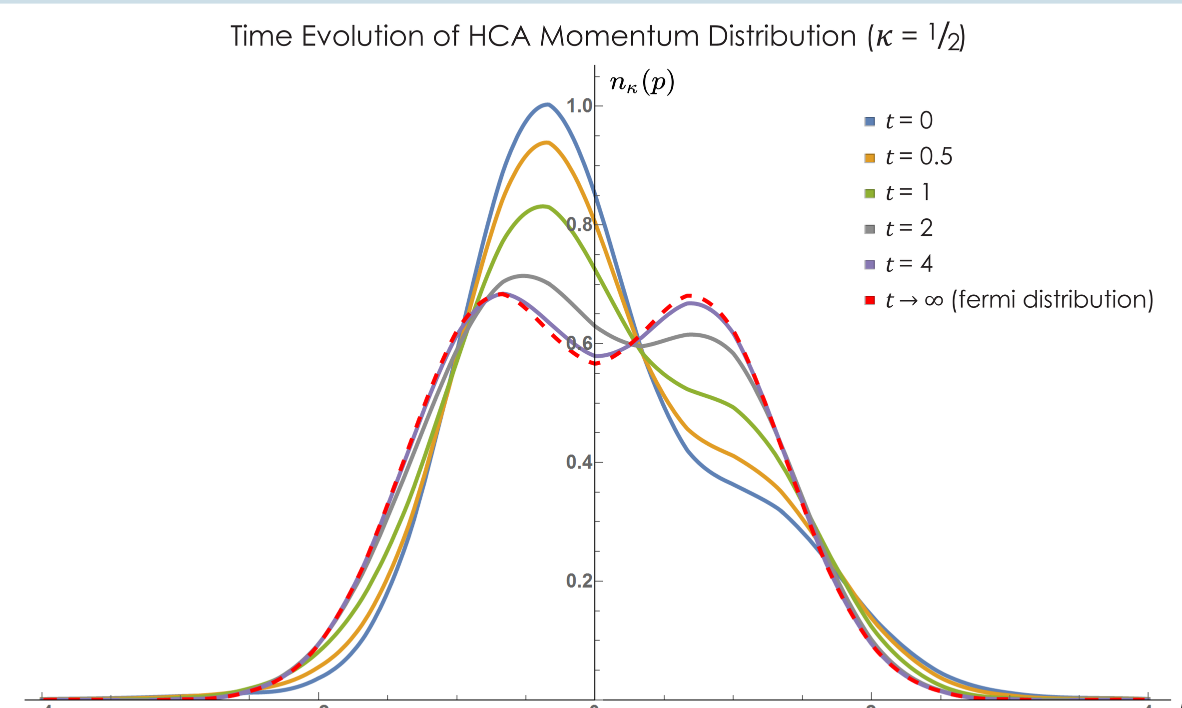
$$\lim_{p\to\infty} = \left(\frac{2}{\pi}\right)^{3/2} \frac{\cos^2\left(\frac{\pi\kappa}{2}\right)}{p^4} = \frac{C_\kappa}{p^4}$$

Numerically calculate C_κ to confirm our analytic result:



Numerically calculate C_κ for various κ and normalize to unity at $\kappa = 0$

Expanding HCA mimic fermi momentum distribution



What happens if the potential is turned off? (i.e., $\omega = 1 \to \omega = 0$)

- We show numerically that $n_\kappa(p)$

Conclusions

Future Work

$$\psi_F(x_1, x_2)$$

Explain momentum distribution, how it varies with kappa, the momentum tail result

Trap is turned off: show gif if possible to illustrate time evolution, and also show time evolution of momentum tail coefficient

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