Dynamical Fermionization in Ultracold Systems

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Research Summary

- We study strongly-interacting, ultracold particles confined in a one-dimensional harmonic trap
- Strongly-interacting 1D systems often exhibit peculiar quantum behavior

Central Question:

- What happens to momentum distribution when trapping potential is turned off?

Summary

Background
- Types of Particles
- Strong Interaction

Methods
- Bose-Fermi Mapping
- One Body Density Matrix
- Density Profiles
- Dynamical Fermionization

Results
- Density Profiles
- Dynamical Fermionization

Future Work

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Bosons and Fermions

- In nature, we only observe bosons and fermions
- Distinguishing characteristic is how their wavefunctions behave when two particles are exchanged

$$\psi_B(x_1, x_2) = \psi_B(x_2, x_1)$$
$$\psi_F(x_1, x_2) = -\psi_F(x_2, x_1)$$

- Exchange Statistics: how wave function behaves under particle exchange
- Identical particles $\rightarrow$ exchanging particles can only change ψ by complex phase

- We consider systems of strongly interacting particles: repulsive, contact interactions
- We can write the Hamiltonian

$$H = \sum_{j=1}^{N} \left[-\frac{1}{2} \frac{\partial^2}{\partial x_j^2} + \frac{1}{2} x_j^2 \right] + g \sum_{j<\ell} \delta(x_j - x_\ell)$$

  where $\hbar = m = \omega = 1$ and g is large but finite
- Our research will consider strongly-interacting bosons with a spin degree of freedom

- Previously we studied spinless, hard-core particles, which corresponds to limit $g \to \infty$
- When g is infinite for spinors, ground state is degenerate because energy is independent of spin configuration
- For finite but large g, g.s. is governed by an effective Hamiltonian

$$H_{\text{eff}} = -\frac{1}{g} \sum_{i=1}^{N-1} C_i (1 + \mathcal{E}_{i,i+1})$$

where each C_i is a positive constant and $\mathcal{E}_{i,i+1}$ is the exchange operator (Yang (2015))

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Bose-Fermi Mapping

- Bosons and fermions are very different, but hard-core bosons (HCB) are similar to fermions (e.g., no two can occupy same position)
- M. Girardeau (1960) showed there was a mapping between these two systems
- Procedure for turning N-particle fermion solution into boson solution

$$\psi_B(x_1, \ldots, x_N) = A(x_1, \ldots, x_n)\psi_F(x_1, \ldots, x_N)$$

$$A(x_1, , \ldots, x_N) = \prod_{1 \le \ell < j \le N} \epsilon(x_j - x_\ell)$$

Methods: One Body Density Matrix

- After obtaining the wave function for our system using the mappings, we study its properties using the One-Body Density Matrix (OBDM)
  Spinless:

$$\rho(x, x') = \int_{-\infty}^{\infty} dx_2 \cdots dx_N \psi^*(x, x_2, \ldots, x_N)\psi(x', x_2, \ldots, x_N)$$

  Spinful:

$$\begin{aligned} \rho_{\sigma,\sigma'}(x, x') = \sum_{\sigma_2,\ldots,\sigma_N} \int_{-\infty}^{\infty} & dx_2 \cdots dx_N \\ & \times \psi^*(x, x_2, \ldots, x_N; \sigma, \sigma_2, \ldots, \sigma_N) \\ & \times \psi(x', x_2, \ldots, x_N; \sigma', \sigma_2, \ldots, \sigma_N) \end{aligned}$$

Real Space Density Profile (RSDP)

- Number density of particles in real space

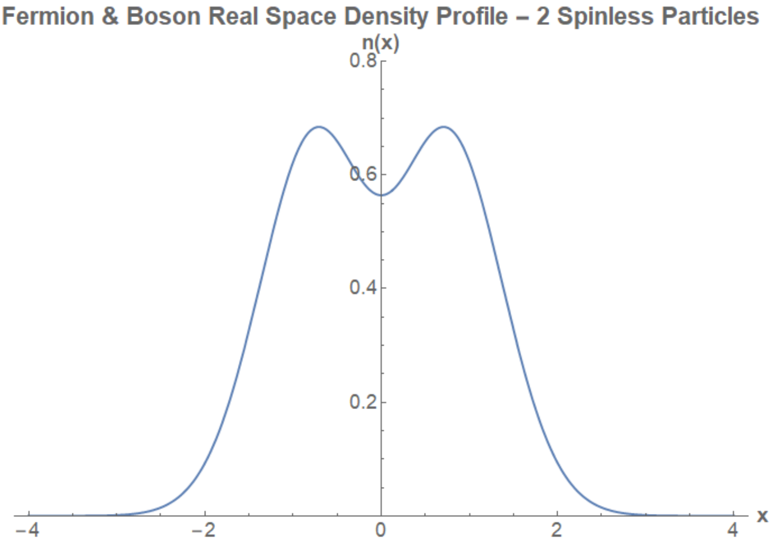
$$n_\sigma(x) = N\rho_{\sigma,\sigma}(x,x)$$

Momentum Space Density Profile (MSDP)

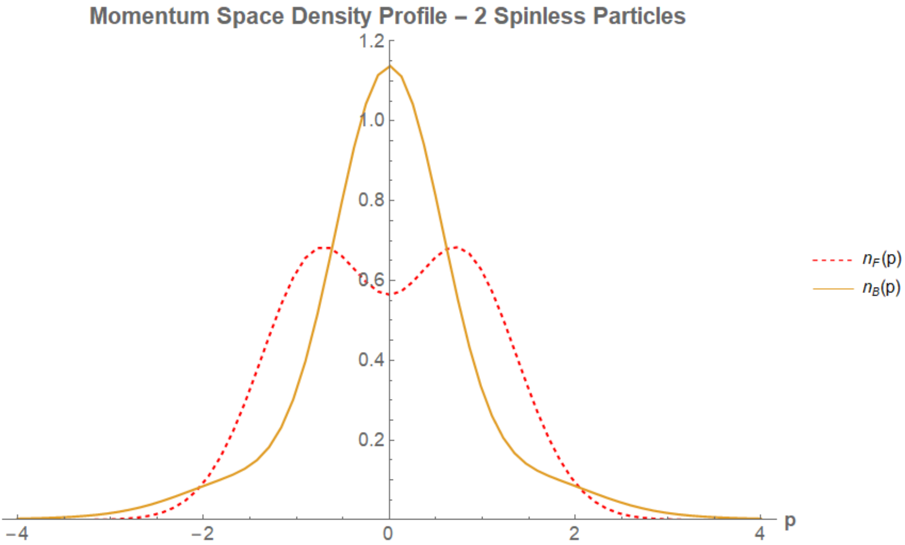
- Number density of particles in momentum space

$$n_\sigma(p) = \frac{N}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' e^{ip(x-x')}\rho_{\sigma,\sigma}(x,x')$$

Methods: Real Space Density Profile



Methods: Momentum Space Density Profile



Dynamical Fermionization

- Hard-core bosons and fermions have different momentum distributions
- When trapping potential is turned off for hard-core bosons, Minguzzi et al. (2005) showed that

$$\lim_{t\to\infty} n_B(p;t) = n_F(p;0)$$

using a scaling transformation

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- I have studied a non-separable wave function for two spin-$\frac{1}{2}$ bosons

$$\Psi(x_1, x_2; \sigma_1, \sigma_2) = \frac{1}{\sqrt{2}}[\varphi_F \chi_S(\sigma_1, \sigma_2) + \varphi_B \chi_T(\sigma_1, \sigma_2)]$$

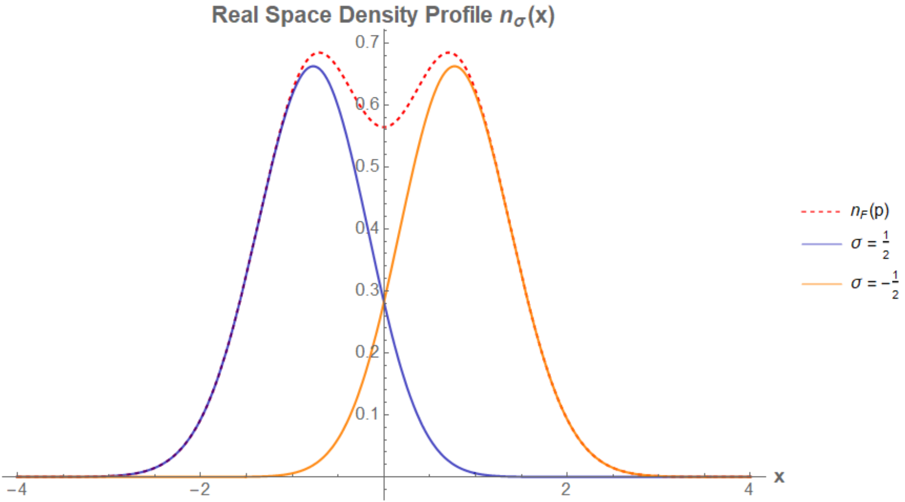
$$|\chi_S\rangle = \frac{1}{\sqrt{2}}(|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle)$$

$$|\chi_T\rangle = \frac{1}{\sqrt{2}}(|\uparrow, \downarrow\rangle + |\downarrow, \uparrow\rangle)$$

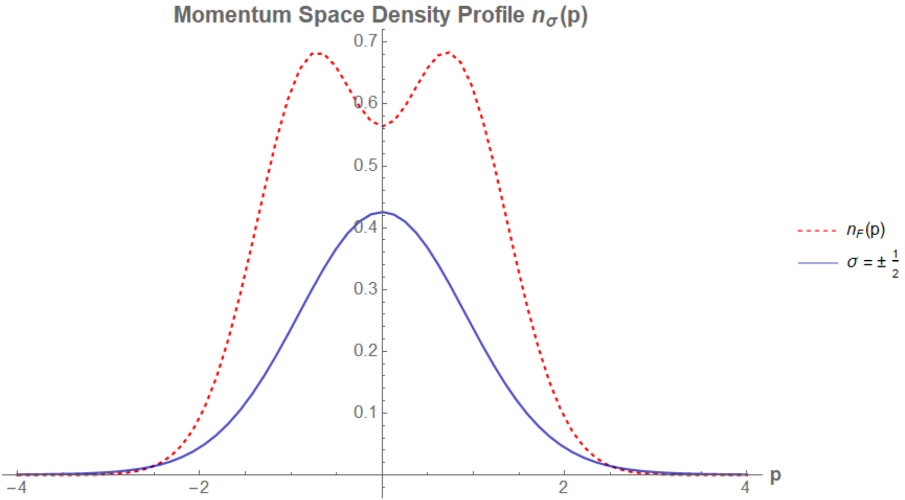
$$\varphi_F(x_1, x_2) : \text{Solution for Two Spinless Fermions}$$

$$\varphi_B(x_1, x_2) : \text{Solution for Two Spinless Bosons}$$

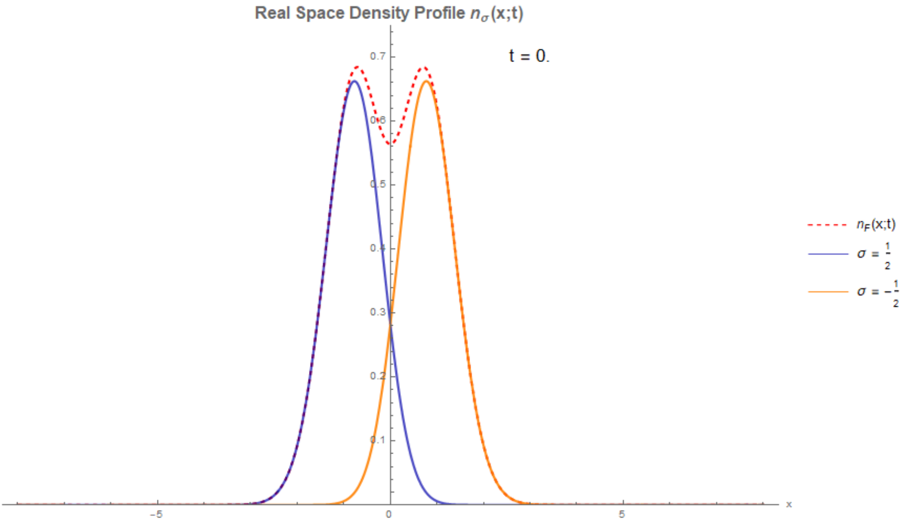
Results: Real Space Density Profile



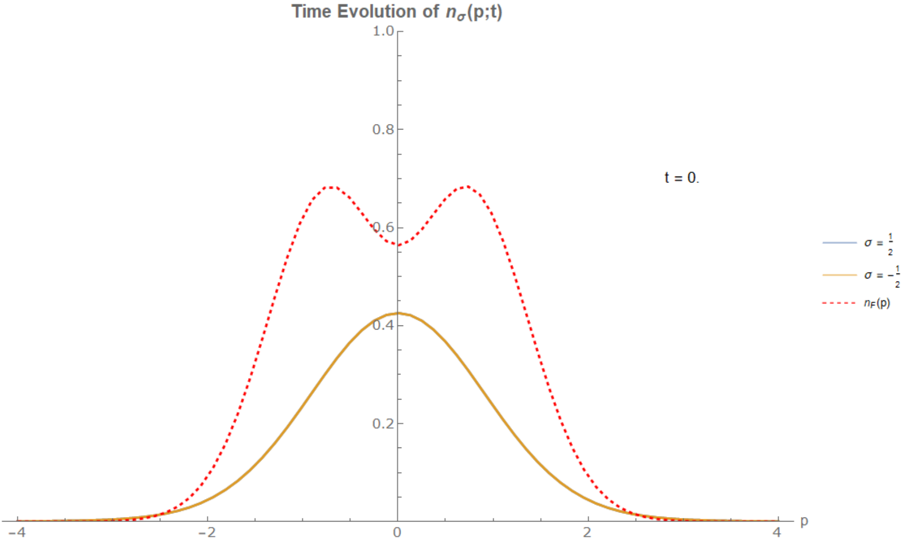
Results: Momentum Space Density Profile



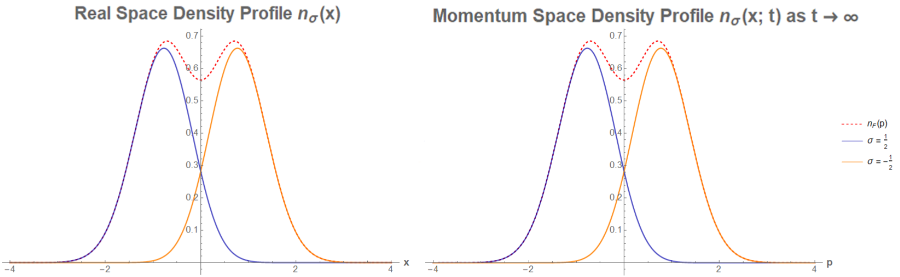
Results: Real Space Density Profile $t \to \infty$



Results: Momentum Space Density Profile $t \to \infty$



Results: Initial RSDP vs. Final MSDP



Dynamical Fermionization

- When trapping potential is turned off

$$\lim_{t\to\infty} \sum_{\sigma} n_{\sigma}(p;t) = n_F(p;0)$$

- Showed this theoretically using stationary-phase approximation
- Confirmed this result by numerically computing momentum distribution

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Past Research:

- Studied spinless systems of bosons, fermions, anyons
- Showed dynamical fermionization occurs in spinor systems

Future Work:

- Murmann et al. (2015) can experimentally create a three particle spinor state
- Calculate long term dynamics of MSDP for this state to make our work better connected with experiment

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References

[1] M. Girardeau.
Relationship between systems of impenetrable bosons and fermions in one dimension.
Journal of Mathematical Physics, 1(6):516–523, 1960.

[2] M. D. Girardeau.
Anyon-fermion mapping and applications to ultracold gases in tight waveguides.
Physical Review Letters, 97(10):1–4, 2006.

[3] Y. Hao and Y. Song.
One-dimensional hard-core anyon gas in a harmonic trap at finite temperature.
The European Physical Journal D, 71(6):135, 2017.

[4] A. Minguzzi and D. M. Gangardt.
Exact coherent states of a harmonically confined tonks-girardeau gas.
Physical Review Letters, 94(24):1–4, 2005.

[5] A. Minguzzi, P. Vignolo, and M. P. Tosi.
High-momentum tail in the Tonks gas under harmonic confinement.
Physics Letters A, 294(3-4):222–226, 2002.

[6] S. Murmann, F. Deuretzbacher, G. Zürn, J. Bjerlin, S. M. Reimann, L. Santos, T. Lompe, and S. Jochim.
Antiferromagnetic heisenberg spin chain of a few cold atoms in a one-dimensional trap.
Physical review letters, 115(21):215301, 2015.

[7] S. Tan.
Large momentum part of a strongly correlated fermi gas.
Annals of Physics, 323(12):2971 – 2986, 2008.

[8] P. Vignolo and A. Minguzzi.
Universal Contact for a Tonks-Girardeau Gas at Finite Temperature.
Physical Review Letters, 110(2):1–5, 2013.

[9] L. Yang, L. Guan, and H. Pu.
Strongly interacting quantum gases in one-dimensional traps.
Physical Review A, 91(4):043634, 2015.