Statistiek 2018

Huiswerk # 3
Student: Tim Stolp
Group: D

1. Voor $-1 \leq x \leq 1$ hebben we

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy = \int_0^1 \frac{3}{5}(x^2+y)dy = \frac{3}{5}[\frac{1}{2}y^2 + x^2y]_0^1 = \frac{3}{5}(\frac{1}{2}+x^2) - 0 = \frac{3}{10} + \frac{3}{5}x^2$$

   Voor $0 \leq y \leq 1$ hebben we

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx = \int_{-1}^1 \frac{3}{5}(x^2+y)dx = \frac{3}{5}[\frac{1}{3}x^3 + xy]_{-1}^1 = \frac{3}{5}(\frac{1}{3}+y) - (-\frac{1}{3}-y) = \frac{6}{10} + \frac{6}{5}y$$

2.

$$Var(Y) = \int_0^1 y^2(\frac{6}{10} + \frac{6}{5}y)dy - (\int_0^1 y(\frac{6}{10} + \frac{6}{5}y)dy)^2$$

$$= [\frac{6}{30}y^3 + \frac{6}{20}y^4]_0^1 - ([\frac{6}{20}y^2 + \frac{6}{15}y^3]_0^1])^2 = (\frac{6}{30} + \frac{6}{20}) - (\frac{6}{20} + \frac{6}{15})^2 = \frac{1}{2} - \frac{49}{100} = \frac{1}{100}$$

3. Als $f(x,y) = f_X(x)f_Y(y)$ dan zijn X en Y onafhankelijk

$$f(x,y) = \frac{3}{5}(x^2+y)$$

$$f_X(x)f_Y(y) = (\frac{3}{10} + \frac{3}{5}x^2)(\frac{6}{10} + \frac{6}{5}y) = \frac{18}{25}x^2y + \frac{9}{25}x^2 + \frac{9}{25}y + \frac{9}{50}$$

$$\frac{3}{5}(x^2+y) \neq \frac{18}{25}x^2y + \frac{9}{25}x^2 + \frac{9}{25}y + \frac{9}{50}$$

   Dus zijn X en Y niet onafhankelijk

4.

$$E(X+Y) = E(X) + E(Y)$$

$$E(X) = \int_{-1}^{1} x(\frac{3}{10} + \frac{3}{5}x^2)dx = [\frac{3}{20}x^2 + \frac{3}{20}x^4]_{-1}^{1} = (\frac{3}{20} + \frac{3}{20}) - (\frac{3}{20} + \frac{3}{20}) = 0$$

$$E(Y) = \int_{0}^{1} y(\frac{6}{10} + \frac{6}{5}y)dy = [\frac{6}{20}y^2 + \frac{6}{15}y^3]_{0}^{1} = (\frac{6}{20} + \frac{6}{15}) - (0+0) = \frac{7}{10}$$

$$E(X+Y) = 0 + \frac{7}{10} = \frac{7}{10}$$