Answers to Final Exam 2017/18

January 5, 2018

1 Question 1

Prove via a counterexample that:

$$\neg\Box_a p \not\Rightarrow B_a \neg\Box_a p$$

Counterexample:

$$\begin{tikzcd} \neg p \arrow[r, "a"] & p \\ w & s \end{tikzcd}$$

is a plausibility model such that:

$$w \models \neg\Box_a p$$

(since $w \leq_a w$, but $w \not\models p$). However, $w \not\models B_a\neg\Box_a p$, since $best_a w(a) = \{s\}$, and

$$s \not\models \neg\Box_a p$$

(because in fact $s \models \Box_a p$), hence $w \not\models B_a\neg\Box_a p$ and so

$$\neg\Box_a p \not\Rightarrow B_a \neg\Box_a p$$

2 Question 2

1. **M**: $$\begin{tikzcd} \neg m, \neg h, s \arrow[r] & m, h, \neg s \arrow[r] & m, \neg h, s \arrow[r] & \neg m, h, \neg s \end{tikzcd}$$

2. After $\Uparrow m$, we get:
$$\begin{tikzcd} \neg m, \neg h, s \arrow[r] & \neg m, h, \neg s \arrow[r] & m, h, \neg s \arrow[r] & m, \neg h, s \end{tikzcd}$$

3. After $\uparrow \neg s$, we get:
$$\begin{tikzcd} \neg m, \neg h, s \arrow[r] & \neg m, h, \neg s \arrow[r] & m, \neg h, s \arrow[r] & m, h, \neg s \end{tikzcd}$$

   So the car believes h, that there is a human ahead.

4. If we perform the upgrades in the reverse order, we get the following:

   First apply $\uparrow \neg s$ to $\mathbf{M}$ - the model remains unchanged.

$$\boxed{\neg m, \neg h, s} \longrightarrow \boxed{m, h, \neg s} \longrightarrow \boxed{m, \neg h, s} \longrightarrow \boxed{\neg m, h, \neg s}$$

   Secondly apply $\Uparrow m$ to obtain the same model as part (2):

$$\boxed{\neg m, \neg h, s} \longrightarrow \boxed{\neg m, h, \neg s} \longrightarrow \boxed{m, h, \neg s} \longrightarrow \boxed{m, \neg h, s}$$

   In this situation the car believes $\neg h$ and s, that there is no human and that it is safe to drive.

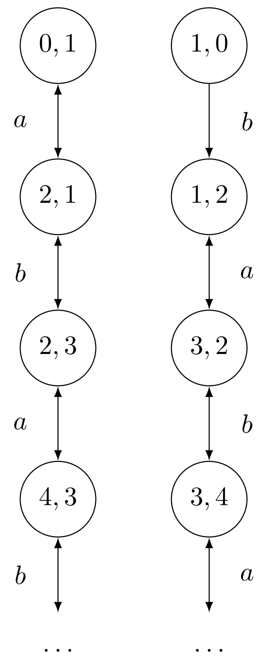
3 Question 3

1. (a) $\mathbf{M}$:

$$\begin{tikzcd}
0,1 \arrow[d, leftrightarrow, "a"'] & 1,0 \arrow[d, leftrightarrow, "b"] \\
2,1 \arrow[d, leftrightarrow, "b"'] & 1,2 \arrow[d, leftrightarrow, "a"] \\
2,3 \arrow[d, leftrightarrow, "a"'] & 3,2 \arrow[d, leftrightarrow, "b"] \\
4,3 \arrow[d, leftrightarrow, "b"'] & 3,4 \arrow[d, leftrightarrow, "a"] \\
\dots & \dots
\end{tikzcd}$$

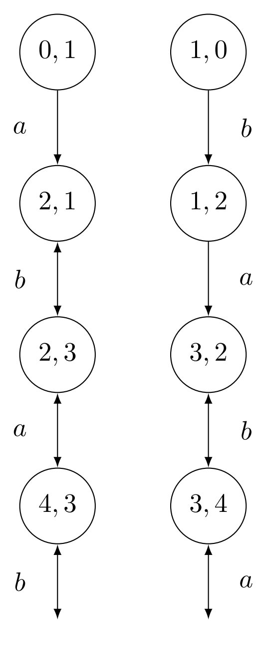
   (b) A answers $\neg B_a(n_a > n_b) \wedge \neg B_a(n_a < n_b)$. This is true in all worlds in the model $\mathbf{M}$ except in world $(1, 0)$. So after $\Uparrow (\neg B_a(n_a > n_b) \wedge \neg B_a(n_a < n_b))$, we obtain:

$\mathbf{M}'$:



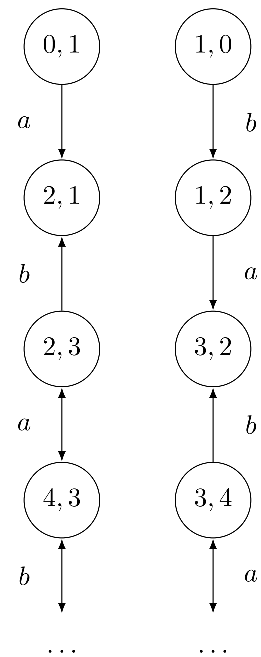
(c) B says $\neg B_a(n_a > n_b) \wedge \neg B_a(n_a < n_b)$. This is true in all worlds in the model $\mathbf{M}'$, EXCEPT for $(0,1), (1,0), (1,2)$. So after $\Uparrow (\neg B_a(n_a > n_b) \wedge \neg B_a(n_a < n_b))$, we obtain:

$\mathbf{M}''$:



(d) A says $B_a(n_a > n_b)$. This is true in worlds $(0,1), (2,1), (1,0), (1,2), (3,2)$, and false in all other worlds in $\mathbf{M}''$. Updating with $\Uparrow B_a(n_a > n_b)$, we get model $\mathbf{M}'''$:

$\mathbf{M}''$:

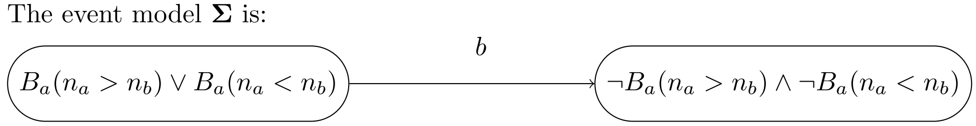


(e) The first (true) answer excludes $(1, 0)$, and the second (true) answer excludes $(0, 1), (1, 0), (1, 2)$. The third answer being true implies that theactual world is among: $(0, 1), (2, 1), (1, 0), (1, 2), (3, 2)$. So, the actual world belongs to:

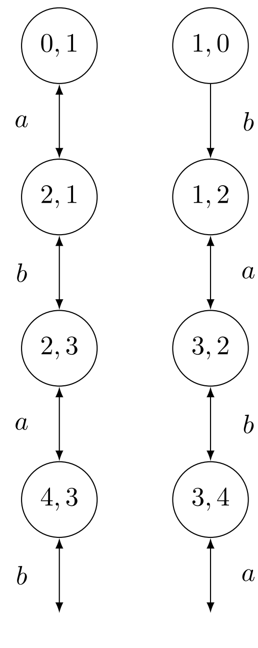
$$\{(0, 1), (2, 1), (1, 0), (1, 2), (3, 2)\} - \{(0, 1), (1, 0), (1, 2)\}$$

So the real world is either $(2, 1)$ or $(3, 2)$.

2. (a) The event model $\mathbf{\Sigma}$ is:

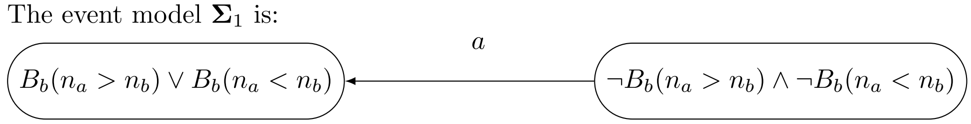


and the updated model $\mathbf{M}_1 = \mathbf{M} \otimes \mathbf{\Sigma}$ is:

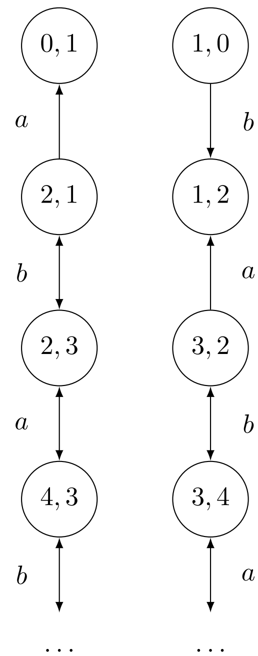


i.e. exactly the same as in part (2b).

(b) The event model $\mathbf{\Sigma}_1$ is:



and the updated model $\mathbf{M}_2 = \mathbf{M}_1 \otimes \mathbf{\Sigma}_1$ is:



(c) Assuming the real world is the same as in the previous scenario, i.e. either $(2,1)$ or $(3,2)$, Alexandru, being sincere, will now answer "I believe my number is smaller":

$$B_a(n_a < n_b)$$

since this is true in both $(2,1)$ or $(3,2)$. This belief is FALSE, since in both $(2,1)$ and $(3,2)$, Alexandru's number is in fact bigger than Bob's.