Q1 (1).

Let M be a ~~w~~ doxastic model, and w be a world in M s.t.

$$w \models B_a B_a p.$$

We need to show (?) $w \models B_a p$ (?)

For this, let s be any world s.t.

$$w \xrightarrow{a} s.$$

It's enough to show (??) $s \models p$ (??)

(since s arbitrary with $w \xrightarrow{a} s$, so conclusion follows by semantics of B_a)

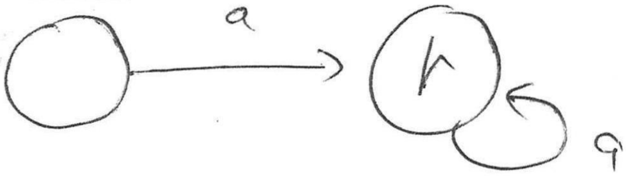
By Euclideanness, $w \xrightarrow{a} s$ and $w \xrightarrow{a} s$ implies $s \xrightarrow{a} s$.

$\boxed{\text{Then: } (w \xrightarrow{a} s \text{ and } w \models B_a B_a p) \text{ give us } s \models B_a p} \Longrightarrow$

$\Rightarrow s \models p.$

DONE.

Q1 (a) (2). Counterexample



Q2. 4 (1). 10 worlds

(2). I skip loops and arrows' directions, or arrows obtain by transitivity:

$$\begin{tikzcd}
(2,3) & (2,4) & & \\
(2,5) \arrow[r, dash, "S"] & (3,4) \arrow[r, dash, "P"] & (2,6) \arrow[r, dash, "S"] & (3,5) \\
(3,6) \arrow[r, dash, "S"] & (4,5) & (4,6) & (5,6)
\end{tikzcd}$$

$\|Odd(x)\| = \{(3,4), (3,5), (3,6), (5,6)\}$

$\|Odd(y)\| = \{(2,3), (2,5), (3,5), (4,5)\}$

(3). $\neg(K_S x \wedge K_S y)$. Also: $\neg K_S x \wedge \neg K_S y$ (different solution!)

(4).

$$\begin{tikzcd}
(2,5) \arrow[r, dash, "S"] & (3,4) \arrow[r, dash, "P"] & (2,6) \arrow[r, dash, "S"] & (3,5) \\
(3,6) \arrow[r, dash, "S"] & (4,5) & &
\end{tikzcd}$$

(5). $\neg(K_p x \wedge K_p y)$, Similar issue

$(3,4) \stackrel{p}{\text{———}} (2,6)$

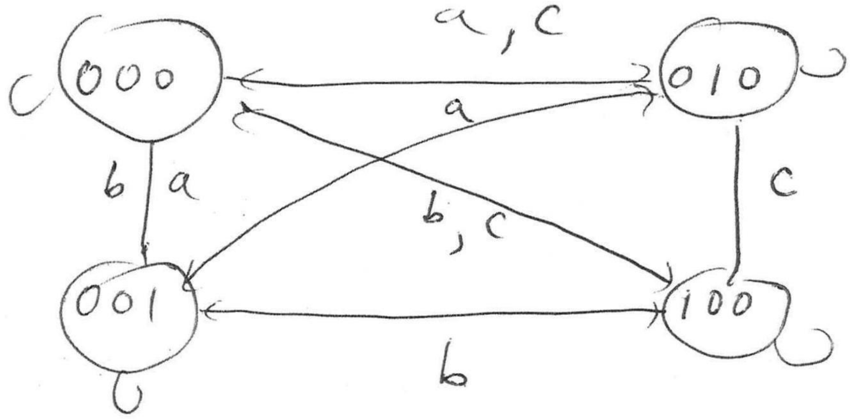
(6). $K_S x \wedge K_S y \wedge \underbrace{(K_S)}_{\text{optional}} \, Odd(x)$

$(3,4)$

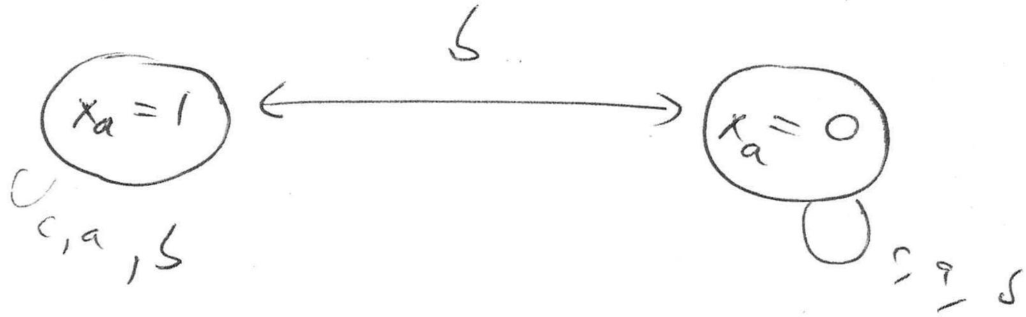
Answer: $x = 3, \; y = 4$

Q 3.

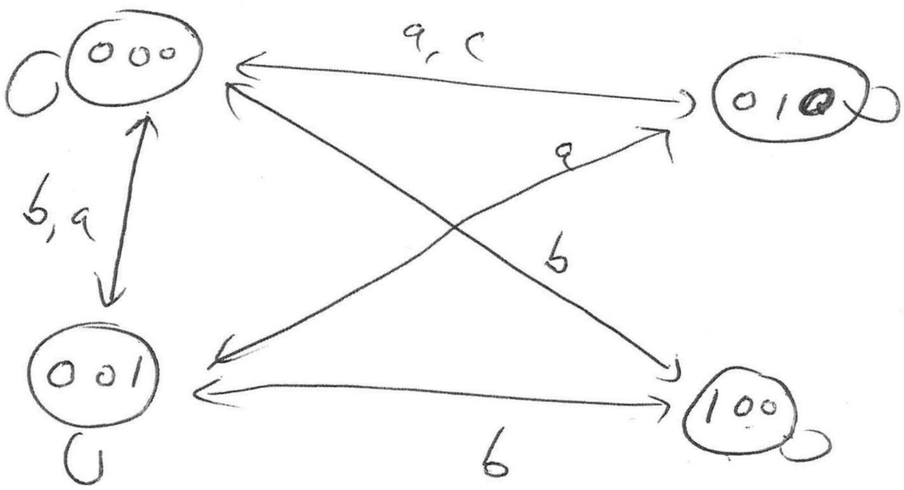
(1)


000
010
001
100
a, c
a
b
a
b, c
c
b

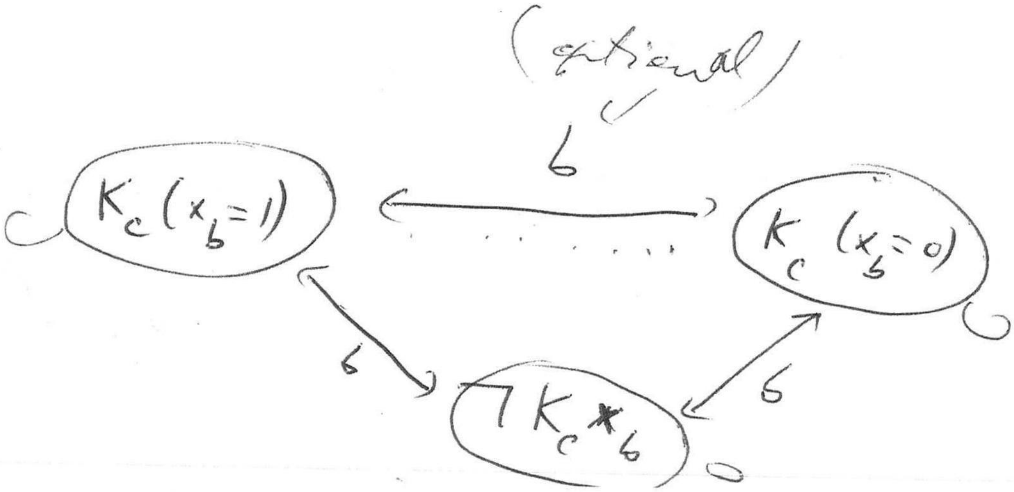
(2)


b
$x_a = 1$
$x_a = 0$
c, a, b
c, a, b

(3)

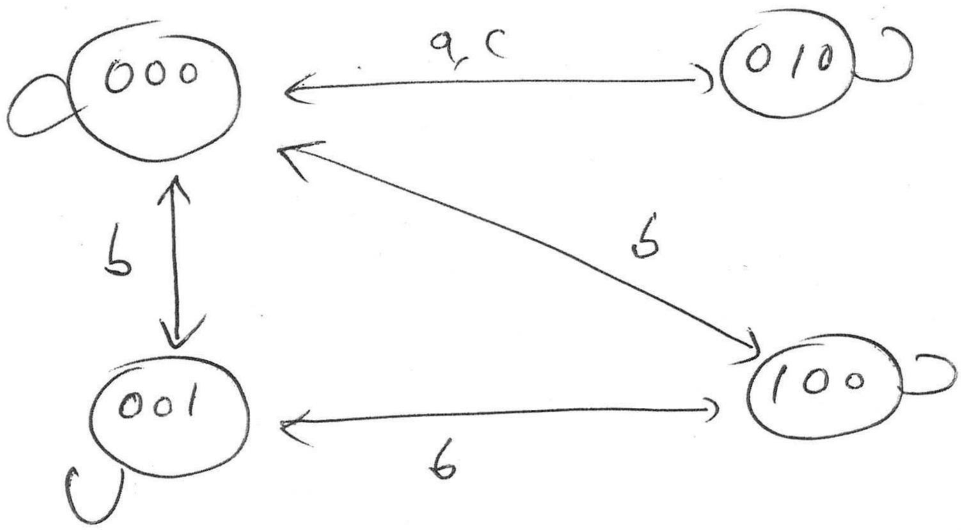

000
010
001
100
a, c
a
b, a
b
b

(4).

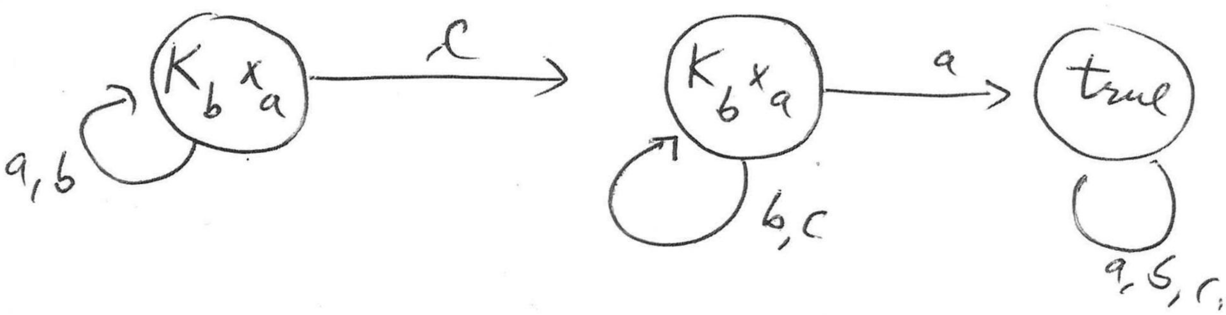

(optional)
b
$K_c(x_b=1)$
$K_c(x_b=0)$
$\neg K_c x_b$
b
b

Q3.

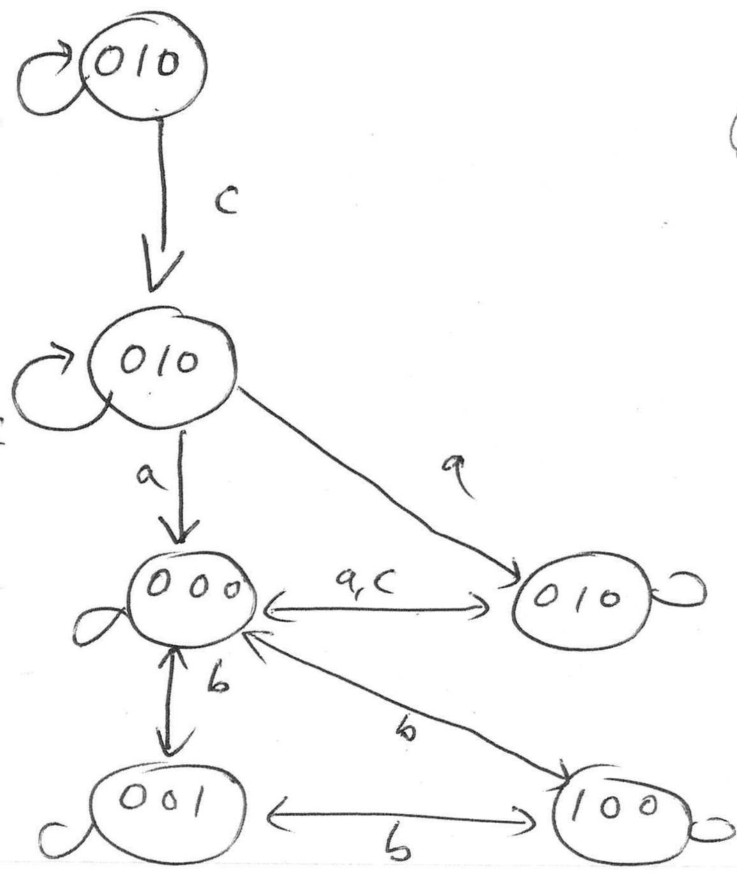
(5).



(6).



(7).



6 (8) $\boxed{(0,1,0)}$