Statistiek 2018

Huiswerk # 4
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1. Verwachting:

$$E(X_1) = \int_1^{1+\theta} xf(x)dx = \int_1^{1+\theta} x(\frac{2}{\theta^2}(x-1))dx$$

$$= \frac{2}{\theta^2}[1/3x^3 - 1/2x^2]_1^{1+\theta} = \frac{2}{\theta^2}(1/3(\theta^3+3\theta^2+3\theta+1) - 1/2(\theta^2+2\theta+1) - (-1/6))$$

$$= \frac{2}{\theta^2}(1/3\theta^3 + 1/2\theta^2) = \frac{2}{3}\theta + 1$$

Verdelingsfunctie:

$$\int_1^x f(x)dx = \int_1^x \frac{2}{\theta^2}(x-1)dx = \frac{2}{\theta^2}[1/2x^2 - x]_1^x$$

$$= \frac{2}{\theta^2}(1/2x^2 - x + 1/2) = \frac{x^2}{\theta^2} - \frac{2x}{\theta^2} + \frac{1}{\theta^2}$$

2.

$$E(\bar{X_n}) = E(\frac{1}{n}(X_1 + ... + X_n)) = \frac{1}{n}(E(X_1) + ... + E(X_n))$$

$$= \frac{1}{n}((\frac{2}{3}\theta + 1) + ... + (\frac{2}{3}\theta + 1)) = \frac{1}{n}n(\frac{2}{3}\theta + 1) = \frac{2}{3}\theta + 1$$

$$\frac{2}{3}\theta + 1 \neq \theta$$

Dus geen zuivere schatter van θ

3.

$$T = a(X_1 + ...X_n) + b = \theta$$

$$T = a(n(\frac{2}{3}\theta + 1)) + b$$

Neem a = $\frac{3}{2n}$

$$T = \frac{3}{2n}(n(\frac{2}{3}\theta + 1)) + b$$

$$T = \theta + \frac{3}{2} + b$$

Neem b = $-\frac{3}{2}$

$$T = \theta + \frac{3}{2} - \frac{3}{2}$$

$$T = \theta$$