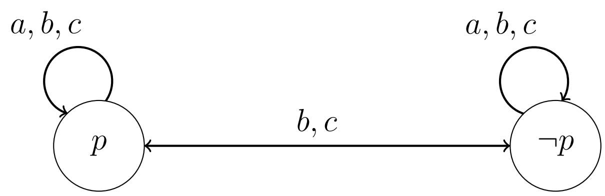
Warming up Week 3

November 14, 2018

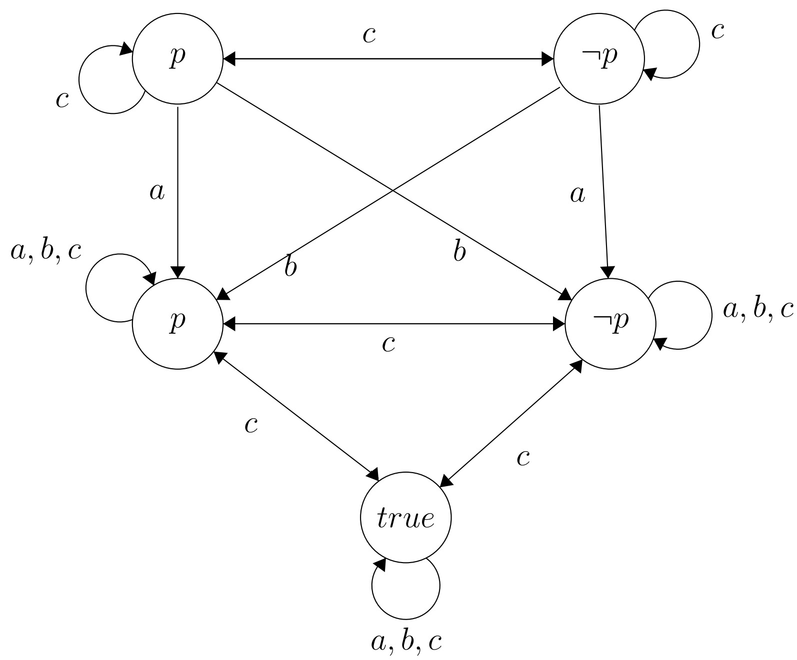
Recall the story of slides 23-24 of Week 3, Hoorcollege 2:

Agent A sends a message to agent B. This message is either p or $\neg p$ and A knows that message. Agent C intercepts the message, but he can't read it (so it doesn't know whether the message is p or $\neg p$). What he does however is to modify the content of the message (so if the message is p, it becomes $\neg p$, and if the message was $\neg p$, it is now p). B receives the message and announces to A that he got the message. Neither A nor B suspects that C could have intercepted the message. They think that C thinks that either both A and B know the content of the message, or that they both don't know.

The initial model is the following model.



The event model is the following graph.



We call α the left upper world and we call β the right upper world. We also call α' the left world in the middle row and β' the right world in the middle row.

Exercise slide p.23-24.

(a) Show that $[\alpha]\Box_A\Box_B p$.

Hint. Use the Knowledge-Action axiom to push $[\alpha]$ inside the modalities. Use also that $(\phi \Rightarrow \phi) \iff \mathit{True}$ and $(\phi \Rightarrow \mathit{True}) \iff \mathit{True}$ and $\Box_B \mathit{True} \iff \mathit{True}$.

(b) Derive that $[\alpha](\Box_a\Box_B p \vee \Box_A\Box_B\neg p)$.

Hint. You can use the equivalence $[\alpha]\phi \Rightarrow [\alpha](\phi \vee \psi)$.

(c) Similarly to (a), show that $[\beta]\Box_A\Box_B\neg p$.

(d) Derive that $[\beta](\Box_a\Box_B p \vee \Box_A\Box_B\neg p)$.

(e) Using (b) and (d), show that $[\alpha]\Box_C(\Box_A\Box_B p \vee \Box_A\Box_B\neg p)$.

Hint. Use that $\mathit{True} \wedge \mathit{True} \iff \mathit{True}$.