1)

$$A = \begin{pmatrix} 10 & 15 & 20 \\ 15 & -50 & 25 \\ 20 & 25 & -75 \end{pmatrix} \quad \bar{S}_1 = \begin{pmatrix} 15 \\ 20 \end{pmatrix} \quad \rightsquigarrow \quad H = Q_1 A Q_1^T$$

$$c = \frac{15}{\sqrt{15^2+20^2}} = \frac{3}{5} \qquad s = \frac{20}{\sqrt{20^2+15^2}} = \frac{4}{5}$$

$$\tilde{Q}_1 = \begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix} \Rightarrow Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3/4 & 4/5 \\ 0 & -4/5 & 3/5 \end{pmatrix}$$

$$Q_1 \cdot A = \begin{pmatrix} 10 & 15 & 20 \\ 25 & -10 & -45 \\ 0 & 55 & -65 \end{pmatrix} \qquad A Q_1^T = \begin{pmatrix} 10 & 15 & 0 \\ 25 & 10 & -45 \\ 0 & 55 & -65 \end{pmatrix} = H$$

4 a) $f(x) = x + \ln(x) - 2 \qquad x \in [1,2]$

$f(1) = -1 \qquad f(2) = \ln(2) \approx 0{,}6931$

$f'(x) = 1 + \frac{1}{x} > 0 \quad \forall x \in X \Rightarrow$ streng monoton wachsend

$\Rightarrow$ genau eine Nst in X

$$\Phi(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x + \ln(x) - 2}{1 + \frac{1}{x}} = \frac{x + 1 - x - \ln(x) + 2}{1 + 1/x}$$

$$= \frac{3 - \ln(x)}{1 + \frac{1}{x}}$$

i) $\Phi(x) = x \quad \forall x \in [1,2] \Rightarrow \Phi(x) \in [1,2]$

$$\left.\begin{array}{l} \Phi(1) = 3/2 \\ \Phi(2) = \frac{3 - \ln(2)}{3/2} \approx 1{,}5379 \end{array}\right\} \in [1,2] \checkmark$$

$$\Phi'(x) = \frac{f(x) \cdot f''(x)}{(f'(x))^2} \Leftrightarrow \Phi'(x) = 0 \Leftrightarrow f(x) = 0 \wedge f''(x) = 0$$

$f(x) = 0 \Rightarrow x = \Phi(x) \in [1,2]$

$f''(x) = -\frac{1}{x^2} \neq 0 \quad \forall x \in [1,2]$

$$\alpha \leq \max_{\xi \in [1,2]} |\Phi'(\xi)|$$

$$\Phi'(x) = \frac{-\frac{1}{x}\left(1+\frac{1}{x}\right) - (3-\ln(x))\cdot\frac{1}{x^2}}{\left(1+\frac{1}{x}\right)^2} = \frac{-x-1+3-\ln(x)}{\left(1+\frac{1}{x}\right)^2 x^2}$$

$$= \frac{2-x-\ln(x)}{(x+1)^2} =$$

$$\max_{\xi\in[1,2]} |\Phi'(\xi)| \leq \frac{\max_{x\in[1,2]} |2-x-\ln(x)|}{\min_{x\in[1,2]} (x+1)^2} = \frac{1}{4}$$

$z(x) = 2-x-\ln(x) \qquad z(1) = 1 \qquad z(2) = -\ln(2)$

$z'(x) = -1 - \frac{1}{x}$ $\Rightarrow$ streng monoton fallend

$\Rightarrow \max |z(x)| = z(1) = 1$

b) a-priori: $|x - x_k| \leq \frac{a^k}{1-a}|x_1 - x_0| \leq \frac{1/4^k}{1-\frac{1}{4}} \left|\frac{3}{2} - 1\right| \leq 10^{-6}$

$k = 9 \Rightarrow 2{,}543 \cdot 10^{-6} > 10^{-6}$

$k = 10 \Rightarrow 6{,}3578 \cdot 10^{-7} < 10^{-6}$

$\Rightarrow$ $k = 10$ Schranke unter 10^{-6}