1a)

$$A = D - E - F = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B_J = D^{-1}(E+F) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1/2 \\ 0 & 0 & 0 \\ -1/2 & 0 & 0 \end{pmatrix}$$

$$\|B_J\|_\infty = 1/2$$

$\rho(B_J)$: Eigenwerte $0 = \det(B_J - \lambda I) = \det \begin{pmatrix} -\lambda & 0 & 1/2 \\ 0 & -\lambda & 0 \\ -1/2 & 0 & -\lambda \end{pmatrix}$

$$= -\lambda^3 + \frac{1}{4}\lambda = \lambda\left(-\lambda^2 + \frac{1}{4}\right)$$

$$\Rightarrow \lambda_1 = 0 \quad \lambda_2 = \pm\frac{1}{2}$$

$$\Rightarrow |\lambda_2| = \frac{1}{2} \Rightarrow \rho(B_J) = \frac{1}{2} < 1$$

$$B_{GS} = (D-E)^{-1} \cdot F = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ -1/4 & 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$$

$$\|B_{GS}\|_\infty = \frac{1}{2}$$

$\rho(B_{GS})$: Eigenwerte $0 = \det \begin{pmatrix} -\lambda & 0 & -1/2 \\ 0 & -\lambda & 0 \\ 0 & 0 & 1/4 - \lambda \end{pmatrix}$

$$\Leftrightarrow 0 = \lambda^2\left(\frac{1}{4} - \lambda\right)$$

$$\Rightarrow \lambda_{1/2} = 0 \quad \lambda_2 = \frac{1}{4}$$

$$\rho(B_{GS}) = \frac{1}{4} < 1$$

NE:

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & -1 & 0 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & 0 & 1/2 \end{array}\right)$$

b) $x_1 = \frac{1}{2}(4 - x_3) = 2 - \frac{1}{2}x_3$

$x_2 = 0$

$x_3 = \frac{1}{2}(5 - x_1) = \frac{5}{2} - \frac{1}{2}x_1$

Jacobi:

$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $x_1 = \begin{pmatrix} 2 \\ 0 \\ 5/2 \end{pmatrix}$ $x_2 = \begin{pmatrix} 3/4 \\ 0 \\ 3/2 \end{pmatrix}$ $x_3 = \begin{pmatrix} 5/4 \\ 0 \\ 17/8 \end{pmatrix}$ $x_4 = \begin{pmatrix} 19/16 \\ 0 \\ 19/8 \end{pmatrix}$

GS:

$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $x_1 = \begin{pmatrix} 2 \\ 0 \\ 3/2 \end{pmatrix}$ $x_2 = \begin{pmatrix} 5/4 \\ 0 \\ 19/8 \end{pmatrix}$ $x_3 = \begin{pmatrix} 17/16 \\ 0 \\ 63/32 \end{pmatrix}$ $x_4 = \begin{pmatrix} 65/64 \\ 0 \\ 253/128 \end{pmatrix}$