HA 1

5a) (1) $f(x) = \sqrt{x} \quad \Rightarrow \quad a_0 = \frac{\sqrt{x_0}}{0!}(x-x_0)^0 = \sqrt{x_0}$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad \Rightarrow \quad a_1 = \frac{1}{2\sqrt{x_0}\cdot 1!}(x-x_0)^1 = \frac{1}{2\sqrt{x_0}}(x-x_0)$$

$$f''(x) = -\frac{1}{4x^{3/2}} \quad \Rightarrow \quad a_2 = -\frac{1}{4x_0^{3/2}\cdot 2!}(x-x_0)^2 = -\frac{1}{8x_0^{3/2}}(x-x_0)^2$$

$$\frac{a_1}{a_0} = \frac{1}{2x_0}(x-x_0) \qquad \frac{a_2}{a_1} = -\frac{1}{4x_0}(x-x_0)^2$$

rekursiv: $a_0 = y_0 = \sqrt{x_0}$

$$a_1 = \left(\frac{3}{2\cdot 1} - 1\right)\left(\frac{x}{x_0} - 1\right)\sqrt{x_0}$$

$$y_1 = y_0 + a_1$$

$$\Rightarrow \sqrt{x_0} + \left(\frac{3}{2} - 1\right)\left(\frac{x}{x_0} - 1\right)\sqrt{x_0}$$

$$a_2 = \left[\left(\frac{3}{2\cdot 2} - 1\right)\left(\frac{x}{x_0} - 1\right)\right]\cdot\left[\left(\frac{3}{2} - 1\right)\left(\frac{x}{x_0} - 1\right)\sqrt{x_0}\right]$$

$$y_2 = y_1 + a_2 = y_0 + a_1 + a_2$$

$$\Rightarrow \sqrt{x_0} + \left(\frac{3}{2} - 1\right)\left(\frac{x}{x_0} - 1\right)\sqrt{x_0} + \left[\left(\frac{3}{2\cdot 2} - 1\right)\left(\frac{x}{x_0} - 1\right)\right]\cdot\left[\left(\frac{3}{2} - 1\right)\left(\frac{x}{x_0} - 1\right)\sqrt{x_0}\right]$$

$$\stackrel{!}{=} a_0 + a_0 \cdot \frac{a_1}{a_0} + a_0 \cdot \frac{a_1}{a_0} \cdot \frac{a_2}{a_1}$$

... immer ein Schritt mehr

Taylor: $\sum_{i=1}^{k} a_i = a_0 + a_1 + a_2 + \ldots = a_0 + a_0 \cdot \frac{a_1}{a_0} + a_1 \cdot \frac{a_2}{a_1} \ldots$