Hausaufgabe 10

Tuesday, June 9, 2020 11:15 AM

2)

i	0	1	2
x_i	-1	0	2
y_i	16	8	16

a) Bedingungsgleichungen:

$$a_0 + b_0 x_0 + c_0 x_0^2 + d_0 x_0^3 = y_0 \quad \Rightarrow a_0 - b_0 + c_0 - d_0 = 16$$
$$a_0 + b_0 x_1 + c_0 x_1^2 + d_0 x_1^3 = y_1 \quad \Rightarrow a_0 = 8$$
$$a_1 + b_1 x_1 + c_1 x_1^2 + d_1 x_1^3 = y_1 \quad \Rightarrow a_1 = 8$$
$$a_1 + b_1 x_2 + c_1 x_2^2 + d_1 x_2^3 = y_2 \quad \Rightarrow a_1 + 2b_1 + 4c_1 + 8d_1 = 16$$
$$b_0 + 2c_0 x_1 + 3d_0 x_1^2 = b_1 + 2c_1 x_1 + 3d_1 x_1^2 \quad \Rightarrow b_0 = b_1$$
$$2c_0 + 6d_0 x_1 = 2c_1 + 6d_1 x_1^2 \quad \Rightarrow 2c_0 = 2c_1$$
$$2c_0 + 6d_0 x_0 = 0 \quad \Rightarrow 2c_0 - 6d_0 = 0$$
$$2c_1 + 6d_1 x_2 = 0 \quad \Rightarrow 2c_1 + 12d_1 = 0$$

$\Rightarrow$ LGS

...

$a_0 = 8,\ b_0 = -4,\ c_0 = 6,\ d_0 = 2$

$a_1 = 8,\ b_1 = -4,\ c_1 = 6,\ d_1 = -1$

$$\Rightarrow s_3(x) = \begin{cases} 2x^3 + 6x^2 - 4x + 8 & x < 0 \\ -x^3 + 6x^2 - 4x + 8 & x \geq 0 \end{cases}$$

b)

i	0	1	2
x_i	-1	0	2
y_i	16	8	16

y_i	16	8	16
h_i	1	2	-

Gamma bestimmen

$$\gamma_1 = 6\left(\frac{y_2 - y_1}{h_1} - \frac{y_1 - y_0}{h_0}\right) = 6\left(\frac{16-8}{2} - \frac{8-16}{1}\right) = 72$$

Matrix bestimmen

$$A = (2(h_0 + h_1)) = (2(2+1)) = (6)$$

Beta bestimmen

$$A \cdot \beta = \gamma \Rightarrow (6) \cdot \beta = (72) \Rightarrow \beta = (12)$$

$\Rightarrow$ mit $\beta_0 = \beta_n = 0$:

i	0	1	2
β_i	0	12	0

Alpha bestimmen

$$\alpha_0 = \frac{y_1 - y_0}{h_0} - \frac{1}{3}\beta_0 h_0 - \frac{1}{6}\beta_1 h_0 = \frac{8-16}{1} - \frac{1}{3}\cdot 0 \cdot 1 - \frac{1}{6}\cdot 12 \cdot 1 = -10$$

$$\alpha_1 = \frac{y_2 - y_1}{h_1} - \frac{1}{3}\beta_1 h_1 - \frac{1}{6}\beta_2 h_1 = \frac{16-8}{2} - \frac{1}{3}\cdot 12 \cdot 2 - \frac{1}{6}\cdot 0 \cdot 2 = -4$$

P bestimmen

$$p_0(x) = y_0 + \alpha_0(x - x_0) + \frac{\beta_0}{2}(x - x_0)^2 + \frac{\beta_1 - \beta_0}{6h_0}(x - x_0)^3$$
$$= 16 - 10(x+1) + 0(x+1)^2 + \frac{12}{6}(x+1)^3$$
$$= 8 - 4x + 6x^2 + 2x^3$$

$$p_1(x) = y_1 + a_1(x - x_1) + \frac{\beta_1}{2}(x - x_1)^2 + \frac{\beta_2 - \beta_1}{6h_1}(x - x_1)^3$$
$$= 8 - 4x + 6x^2 - x^3$$

$$\Rightarrow S_2(x) = \begin{cases} 2x^3 + 6x^2 - 4x + 8 & x < 0 \\ -x^3 + 6x^2 - 4x + 8 & x \geq 0 \end{cases}$$