Monday, June 15, 2020 4:27 PM

1)

i	0	1	2	3
x_i	-2	0	1	2
y_i	4	0	1	-4

mit $p(x) = ax^2 + b$

$\Rightarrow p(-2) = 4 \Rightarrow 4a + b = 4$

$p(0) = 0 \Rightarrow b = 0$

$p(1) = 1 \Rightarrow a + b = 1$

$p(2) = -4 \Rightarrow 4a + b = -4$

$\Rightarrow A = \begin{pmatrix} 4 & 1 \\ 0 & 1 \\ 1 & 1 \\ 4 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ 0 \\ 1 \\ -4 \end{pmatrix}$, $A^T = \begin{pmatrix} 4 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$A^T A = \begin{pmatrix} 4 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 0 & 1 \\ 1 & 1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 33 & 9 \\ 9 & 4 \end{pmatrix}$

$A^T b = \begin{pmatrix} 4 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$A^T A x = A^T b \Rightarrow \begin{pmatrix} 33 & 9 \\ 9 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\Rightarrow x_1 = -\frac{5}{17}, \quad x_2 = \frac{8}{17}$

$\Rightarrow p(x) = -\frac{5}{17}x^2 + \frac{8}{17}$