1) $f(x_1,x_2) = \frac{x_1}{x_2}$, $x_1 = 1$, $x_2 = 10^{-4}$

$f'(x) = (\partial_{x_1} f(x), \partial_{x_2} f(x)) = \left(\frac{1}{x_2}, -\frac{x_1}{x^2}\right)$

$\kappa_0 = \| f'(x) \|_\infty = \|(\qquad)\| = 1\,0001 \cdot 10^4$

2) $\|A\|_\infty = 1 + \beta$

$$\|A^{-1}\|_\infty = \begin{cases} \left|\left(-\frac{1}{\beta^n}\right)^{n-1}\right| \cdot \left|\sum_{i=1}^{n-1} \beta^{n-1}\right| \\ \vdots \\ \left|\left(-\frac{1}{\beta^n}\right)\right| \cdot |\beta^{n-1}| \\ \left|\left(-\frac{1}{\beta^n}\right)^n\right| \cdot \left|\sum_{i=1}^{n} -\beta^{n-1}\right| \end{cases}$$

da $\beta > 1 \Rightarrow \frac{1}{\beta^n} < 0$

$\sum \beta^{n-1}$ ist am größten in Zeile n

$\left|\left(-\frac{1}{\beta^n}\right)^z\right|$ mit $z \geq 1$ ist am kleinsten in Zeile n

$\Rightarrow$ das Produkt ist am größten in Zeile n

$$\Rightarrow \|A^{-1}\|_\infty = \left(\frac{1}{\beta^n}\right)^n \cdot \sum_{i=1}^{n} \beta^{n-1}$$

$$= \sum_{i=1}^{n} \frac{\beta^{n-1}}{\beta^n} = \sum_{i=1}^{n} \frac{1}{\beta^n}$$

$$\kappa_a = \|A\|_\infty \|A^{-1}\|_\infty = (1+\beta) \sum_{i=1}^{n} \frac{1}{\beta^n}$$

$$= \frac{(1+\beta)}{\beta^{n^2}}$$

$\Rightarrow$ Condition ist nicht unabhängig von n

3a) $\bar{r} = b - A\tilde{x} = \begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}2 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}0{,}9\\1{,}1\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}7/10\\2\end{pmatrix} = \begin{pmatrix}0{,}3\\0\end{pmatrix}$

$\bar{r} \leq B|\tilde{x}| + c = \frac{1}{10}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}0{,}9\\1{,}1\end{pmatrix} + \frac{1}{10}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2/10\\2/10\end{pmatrix} + \frac{1}{10}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}3/10\\3/10\end{pmatrix}$ ✓

b) $\bar{r} = b - A\bar{x} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}1 & -1\\-1 & 1.001\end{pmatrix}\begin{pmatrix}501\\500\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}1\\-\frac{1}{2}\end{pmatrix} = \begin{pmatrix}0\\1/2\end{pmatrix}$

$\bar{r} \leq B|\bar{x}| + c = 5 \cdot 10^{-4}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}501\\500\end{pmatrix} + 5 \cdot 10^{-4}\begin{pmatrix}1\\1\end{pmatrix} = 5 \cdot 10^{-4}\begin{pmatrix}1001\\1001\end{pmatrix} + 5 \cdot 10^{-4}\begin{pmatrix}1\\1\end{pmatrix}$

$$r \leq 5 \cdot 10^{-4} \begin{pmatrix} 1002 \\ 1002 \end{pmatrix} = \begin{pmatrix} 0{,}501 \\ 0{,}501 \end{pmatrix} \checkmark$$