a) $x_0 = -1 \quad x_1 = 0 \quad x_2 = 1 \quad x_3 = 3$

$$L_0 = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \frac{x - x_3}{x_0 - x_3}$$

$$= \frac{x-0}{-1-0} \cdot \frac{x-1}{-1-1} \cdot \frac{x-3}{-1-3} = \frac{x(x-1)(x-3)}{-8}$$

$$L_1 = \frac{x-(-1)}{0-(-1)} \cdot \frac{x-1}{0-1} \cdot \frac{x-3}{0-3} = \frac{(x+1)(x-1)(x-3)}{3}$$

$$L_2 = \frac{x-(-1)}{1-(-1)} \cdot \frac{x-0}{1-0} \cdot \frac{x-3}{1-3} = \frac{(x+1)\,x\,(x-3)}{-4}$$

$$L_3 = \frac{x-(-1)}{3-(-1)} \cdot \frac{x-0}{3-0} \cdot \frac{x-1}{3-1} = \frac{x(x+1)(x-1)}{24}$$

$$\Rightarrow p(x) = y_0 \cdot L_0(x) + y_1 \cdot L_1(x) + y_2 L_2(x) + y_3 \cdot L_3(x)$$

$$= \frac{-2}{-8} x(x-1)(x-3) + \frac{4}{3}(x+1)(x-1)(x-3) + \frac{6}{-4} x(x+1)(x-3)$$
$$+ \frac{22}{24} x(x+1)(x-1)$$

$$= \frac{1}{4} x(x^2 - 4x + 3) + \frac{4}{3}(x+1)(x^2 - 4x + 3) - \frac{3}{2} x(x^2 - 2x - 3)$$
$$+ \frac{11}{12} x(x^2 - 1) = x^3 - 2x^2 + 3x + 4$$

b)

x_i	y_i			
-1	$-2 = a_0$			
0	4	$\frac{4-(-2)}{0-(-1)} = 6 = a_1$		
1	6	$\frac{6-4}{1-0} = 2$	$\frac{2-6}{1-(-1)} = -2 = a_2$	
3	22	$\frac{22-6}{3-1} = 8$	$\frac{8-2}{3-0} = 2$	$\frac{2+2}{3-(-1)} = 1 = a_3$

$$p(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$
$$+ a_3(x - x_0)(x - x_1)(x - x_2)$$

$$= -2 + 6(x+1) - 2(x+1)x + (x+1)x(x-1)$$
$$= -2 + 6x + 6 - 2x^2 - 2x + x^3 - x$$
$$= x^3 - 2x^2 + 3x + 4$$

3) $$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \cdot f(x_n)$$

$x_n = q_n(0)$ $\quad y_n = f(x_n)$

$x_2 = q_2(0)$ $\quad y_2 = f(x_2)$

für $n = 1$ ist $q(0)$ eine Gerade mit

$$x_3 = x - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_2)$$