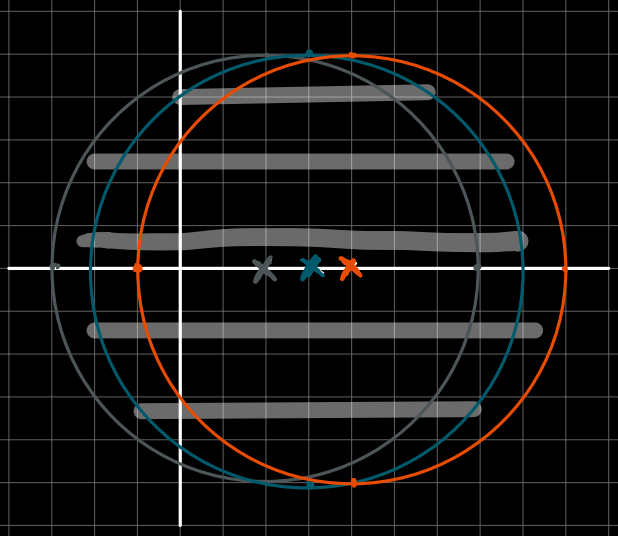
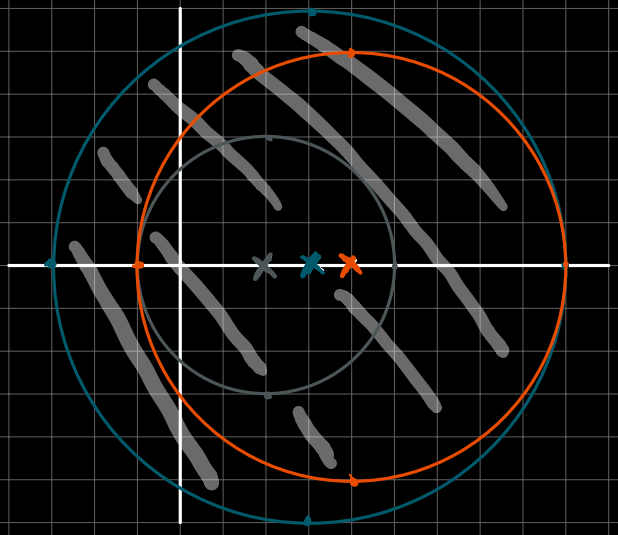
2) $A = \begin{pmatrix} 4 & -2 & 3 \\ 3 & 2 & -2 \\ 2 & -1 & 3 \end{pmatrix}$

$a_{11} = 4 \quad r_1 = 5$
$a_{22} = 2 \quad r_2 = 5$
$a_{33} = 3 \quad r_3 = 5$



$A^T = \begin{pmatrix} 4 & 3 & 2 \\ -2 & 2 & -1 \\ 3 & -2 & 3 \end{pmatrix}$

$a_{11} = 4 \quad r_1 = 5$
$a_{22} = 2 \quad r_2 = 3$
$a_{33} = 3 \quad r_3 = 6$



3 a) $\min |\lambda - \lambda_i| \leq \frac{\|d\|_2}{\|x\|_2} \qquad x = \sum x_i v_i \Rightarrow \|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$

$d = Ax - \lambda x = \sum_{i=1}^{n} \lambda_i x_i v_i - \sum_{i=1}^{n} \lambda x_i v_i \Rightarrow \sum_{i=1}^{n} (\lambda_i - \lambda) x_i v_i$

$\|d\|_2 = \sqrt{\sum (\lambda_i - \lambda)^2 x_i^2}$

$\|d\|_2^2 = \sum (\lambda_i - \lambda)^2 x_i^2 \geq \min_{i=1..n} (\lambda_i - \lambda)^2 \cdot \underbrace{\sum x_i^2}_{\|x\|_2^2 > 0}$

$\frac{\|d\|_2^2}{\|x\|_2^2} \geq \min_{i=1..n} (\lambda_i - \lambda)^2$

$$\frac{\|d\|_2^2}{\|x\|_2^2} \geq \min_{i=1\ldots n} |\lambda - \lambda_i|^2$$

$$\frac{\|d\|_2}{\|x\|_2} \geq \min_{i=1\ldots n} |\lambda_i - \lambda|$$

b) $d = \begin{pmatrix} 6 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 7 \end{pmatrix} \begin{pmatrix} 0{,}5 \\ 1 \\ 1{,}1 \end{pmatrix} - 12 \begin{pmatrix} 0{,}5 \\ 1 \\ 1{,}1 \end{pmatrix} = \begin{pmatrix} 12{,}7 \\ 12{,}9 \\ 13{,}4 \end{pmatrix} - \begin{pmatrix} 10{,}8 \\ 12 \\ 13{,}2 \end{pmatrix} = \begin{pmatrix} 1{,}9 \\ 0{,}9 \\ 0{,}2 \end{pmatrix}$

$\|d\|_2 = \frac{\sqrt{446}}{10}$ $\quad \|x\|_2 = \frac{\sqrt{302}}{10}$

$\min_{i=1,\ldots,n} |\lambda - \lambda_i| \leq \frac{\|d\|_2}{\|x\|_2} = \frac{\sqrt{446}}{\sqrt{302}}$

für $\lambda_i \leq 0$

$\lambda - \lambda_i \leq \frac{\sqrt{446}}{\sqrt{302}}$

$\lambda_i \geq 10{,}785$ ↯ $\lambda_i \leq 0$

für $\lambda_i \geq 0 \wedge \lambda_i \leq 12$

$\lambda - \lambda_i \leq \frac{\sqrt{446}}{\sqrt{302}}$

$\lambda_i \geq 10{,}785 \Rightarrow 10{,}785 \leq \lambda_i \leq 12$

für $\lambda_i \geq 0 \wedge \lambda_i > 12 \Rightarrow \lambda_i > 12$

$\lambda_i - \lambda \leq \frac{\sqrt{446}}{\sqrt{302}}$

$\lambda_i \leq 13{,}215 \Rightarrow 12 > \lambda_i \leq 13{,}215$

$\Rightarrow 10{,}785 \leq \lambda_i \leq 13{,}215$