Pair Trading Report - BlueCrest

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2024-09-17

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1 Introduction

Pair trading is a relative-value trading strategy that involves simultaneously going long (buying) on one security and short (selling) on another correlated security. The idea is to profit from the relative price movement between the two assets, rather than relying on the direction of the underlying assets.

The core of the strategy is based on the assumption that two historically correlated assets will converge after they diverge from their typical relationship. Traders identify pairs of securities, such as stocks from the same sector or industry, that have shown a historical price relationship. When one of the securities in the pair deviates significantly from this relationship, the trader takes a long position on the undervalued asset and a short position on the overvalued one, expecting the prices to revert to their mean over time.

An example of two correlated names is JPM and GS. We plot their normalized historical prices in 1

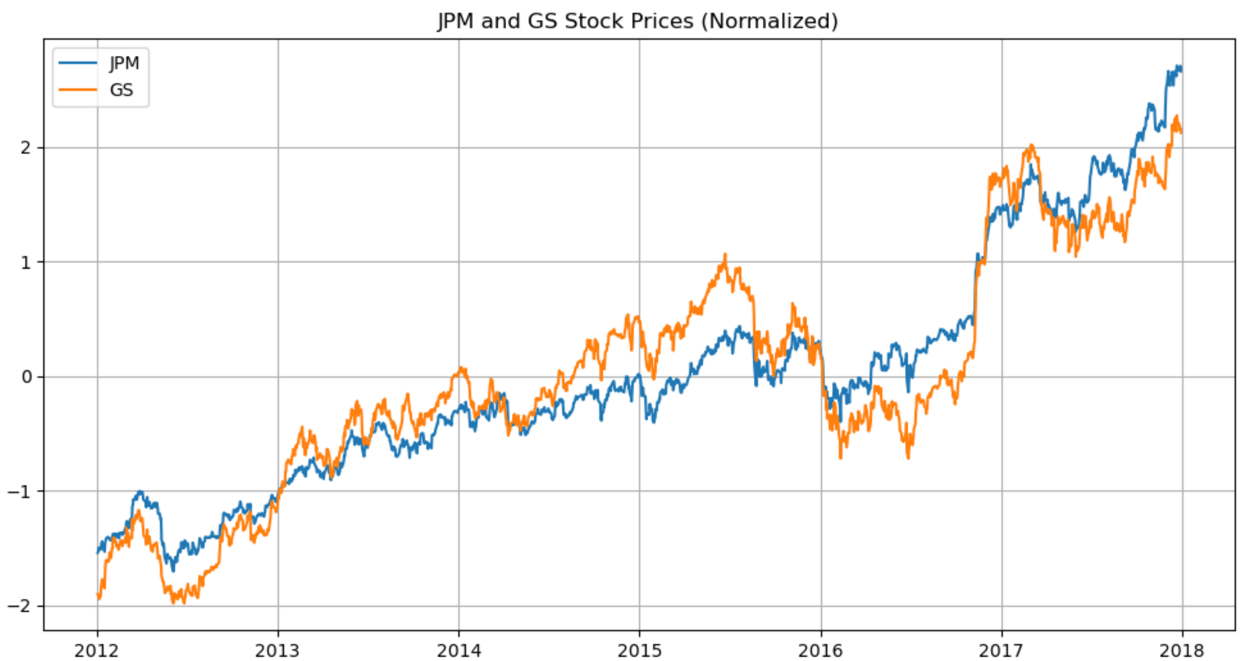

Figure 1: Normalized JPM and GS stock prices

In this report, we delve into the pair trading strategy in the U.S. equity market. Section 2 introduces the critical aspects of constructing a pair trading strategy, including pair selection and strategy construction. Section 3 presents simulations based on historical data to evaluate the strategy's performance. Finally, Section 4 outlines potential directions for future research.

2 Model and Strategy

In this section we will introduce how we model and build the pair trading strategy. There are three main components of the trading strategy - pairs selection, strategy, and hyper-parameter tuning. Then we will go through them respectively

2.1 Pairs Selection

Selecting the right asset pairs is crucial for the success of a pairs trading strategy. The objective is to find pairs of assets whose price movements are statistically related, enabling us to exploit mean-reversion strategies effectively. Below, we outline some commonly used techniques for pairs selection:

- **Correlation Analysis.** Correlation measures the degree to which two securities move in relation to each other. By calculating the correlation coefficient between asset returns, we can identify those that exhibit strong positive or negative relationships. High correlation suggests that the assets tend to move together, which is desirable for pairs trading.

- **Augmented Dickey-Fuller (ADF) Stationarity Test.** The ADF test helps identify whether a time series is stationary, meaning its statistical properties do not change over time. In the context of pairs trading, we use the ADF test to check for the ratio of two asset prices. Pairs with stationary price ratio share a long-term equilibrium relationship, making them suitable candidates for mean-reversion strategies.

- **Granger Causality Test.** The Granger Causality Test assesses whether one time series can predict another. If changes in the price of one asset consistently precede changes in another, it suggests a potential causal relationship. Identifying such pairs can enhance the effectiveness of trading strategies by exploiting predictive signals. We will perform Granger Causality test for asset returns in both directions in practice.

- **Distance Method.** This method involves selecting pairs based on the minimum sum of squared cumulative returns over time. By calculating the Euclidean distance between cumulative return series of different assets, we can identify pairs whose prices have moved closely together historically.

While ideally, we would employ all these techniques to select our trading pairs, some of them are computationally intensive, especially when dealing with a large universe of assets. To balance thoroughness with efficiency, we adopt a two-step selection process:

- **Step 1: Filtering Based on Price Correlation.** We begin by calculating the correlation coefficients for all possible pairs within our asset universe. By filtering out pairs with low correlation, we reduce the dataset to a more manageable size. This initial screening minimizes overfitting risks and decreases the computational resources required for further analysis.

- **Step 2: Selection Using Advanced Techniques.** For the subset of pairs identified in Step 1, we apply more sophisticated methods such as the ADF Stationarity Test, Granger Causality Test, and the Distance Method. This deeper analysis provides a comprehensive evaluation of each pair's statistical properties. By scoring and ranking the pairs based on these advanced metrics, we ensure that we select the most robust candidates for our trading strategy.

Following this procedure, we will ultimately choose 10 pairs to include in our trading portfolio.

2.2 Strategy

In this section we try to develop a systematic approach to trade the selected pairs, focusing on spread determination, entry and exit rules, and position sizing. We implement the strategy in the following way:

- **Spread Determination.**

  For each trading day, we retrieve the prices of two selected stocks over a predetermined look-back window (a hyper-parameter to be decided), denoted as p_1 and p_2, respectively. The steps are as follows:

  - We perform a linear regression of the logarithmic prices to establish a relationship between the two stocks:

    $$\log(p_1) = \hat{\alpha} + \hat{\beta} \log(p_2) + \epsilon$$

    Here, $\hat{\beta}$ is the estimated hedge ratio, and ϵ represents the residuals.

  - The residuals ϵ from the regression are defined as the spread S between the two stocks:

    $$S = \epsilon = \log(p_1) - \hat{\beta} \log(p_2) - \hat{\alpha}$$

- **Entry and exit rules.**

  We calculate the mean μ and standard deviation σ of the spread S over the look-back window. Then we define the entry and exit rules as follows:

  - If $S_t > \mu + \text{open score} \times \sigma$, short the spread
  - If $S_t < \mu + \text{open score} \times \sigma$, long the spread
  - If $S_t < \mu + \text{close score} \times \sigma$, close the short position
  - If $S_t > \mu + \text{close score} \times \sigma$, close the long position

  Here the open score and close score are the hyper-parameters to be optimized based on historical data to achieve the best performance.

- **Position Sizing**

  We aim to maintain a fixed Gross Market Value (GMV) for the strategy, defined as:

  $$\text{GMV} = |\text{long value}| + |\text{short value}|.$$

Using the hedge ratio $\hat{\beta}$ obtained from the regression, we calculate the number of shares to trade for each stock (long postion as an exmaple):

- Stock 1: $\frac{\text{GMV}}{\textbf{price}_1+|\hat{\beta}|\textbf{price}_2}$
- Stock 2: $\frac{-\hat{\beta}\text{GMV}}{\textbf{price}_1+|\hat{\beta}|\textbf{price}_2}$

For simplicity, we use a strategy where the position is either fully invested or not invested at all, based on the trading signals. While this may not be optimal, it provides a clear illustration of the trading concept.

Noticing that the entry and exit rules we're using are actually the simplest ones available, more complex strategies could be explored in the future.

2.3 Hyper-parameter Tuning

Selecting optimal hyper-parameters is a crucial step in maximizing the performance of a trading strategy while avoiding the risks of over-fitting to historical data. To achieve this, we employ a grid search approach, which systematically explores a predefined set of hyper-parameter combinations. Each combination is evaluated based on in-sample Sharpe Ratio. We will choose the hyper-parameter that achieves the highest Sharpe Ratio for each pair independently.

3 Empirical Results

3.1 Experimental Results

We select the S&P 500 as our trading universe to keep the number of pairs manageable. The in-sample period is 2012-2017, while 2018-2019 is used as out-of-sample data. For each pair selection method, we trade the top 10 pairs with the highest similarity scores, assuming equal capital allocation across the pairs. A summary of the strategy metrics is in 3.1 and 3.2

Table 3.1: In-sample performance of each strategy

Strategy	**Annualized Return**	**Sharpe Ratio**	**Annualized Turnover**	**Average Trading Duration**
Correlation Selection	2.72%	1.98	17.78	8.91 (d)
ADF Selection	8.06%	2.34	19.81	11.79 (d)
Granger Selection	8.78%	2.23	18.43	14.08 (d)
Distance Selection	4.21%	2.18	13.30	16.10 (d)

We observe that the distance selection method performs best on the out-of-sample data, although significant overfitting is generally observed. This suggests that we may not be identifying stable relationships among stocks. (Note that while a Sharpe ratio of 1.66 is reasonable, it may also suffer from the multiple-testing problem.) We also report the selected pairs and cumulative returns for the distance method in 3.3 and 2.

Table 3.2: Out-of-sample performance of each strategy

Strategy	Annualized Return	Sharpe Ratio	Annualized Turnover	Average Trading Duration
Correlation Selection	1.14%	0.82	15.10	9.86 (d)
ADF Selection	1.73%	0.38	17.45	13.14 (d)
Granger Selection	0.52%	0.13	14.47	15.70 (d)
Distance Selection	3.00%	1.66	12.30	15.79 (d)

Table 3.3: Selected pairs by distance method

Security 0	Security 1	Window Size	Open Score	Close Score
GOOG	GOOGL	30	2	1
AVB	EQR	30	2	-1
FRT	SPG	50	2.5	0
LNT	WEC	50	2.5	0
ES	PEP	30	2.5	0
CB	JNJ	70	2.5	1
AEP	ES	30	1.5	0
AEP	WEC	70	1.5	-1
D	UPS	50	2	1
CB	DTE	70	2.5	1

3.2 Sensitivity Test

We conduct a sensitivity test on our hyperparameters selected from the training data. For the window size, we vary it by $\pm 20\%$, and for the open/close score, we adjust it by ± 0.6. We report the resulting changes in the Sharpe ratio on both the training and test sets in 3 and 4

We observe relatively robust behavior across different parameter settings. This suggests that the overfitting may not be due to inadequate parameter selection.

Additionally, we analyzed the relationship between trading duration and profitability. Intuitively, in an efficient market, mispricings should correct themselves quickly. Therefore, we would expect profitability to be negatively correlated with trading duration. We have plotted the results in 5.

Indeed, the correlation is approximately -22%, supporting our intuition. Based on this finding, we could modify the strategy by exiting positions if they do not converge within a certain time frame.

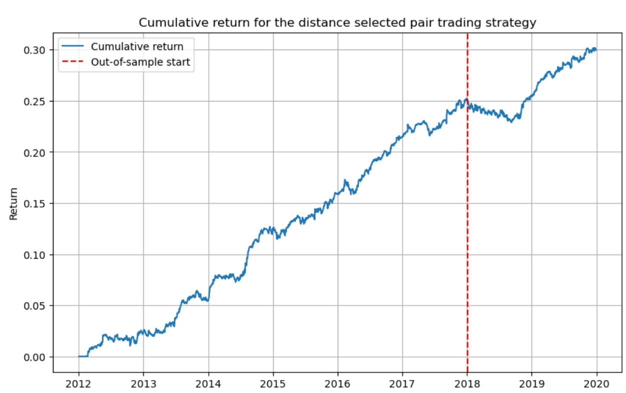

Figure 2: Cumulative return for the distance pair trading strategy

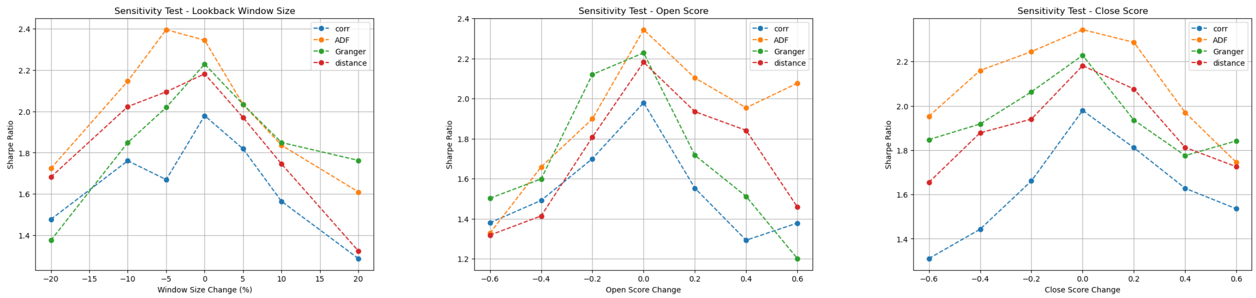

Figure 3: In-sample hyper-parameter sensitivity analysis

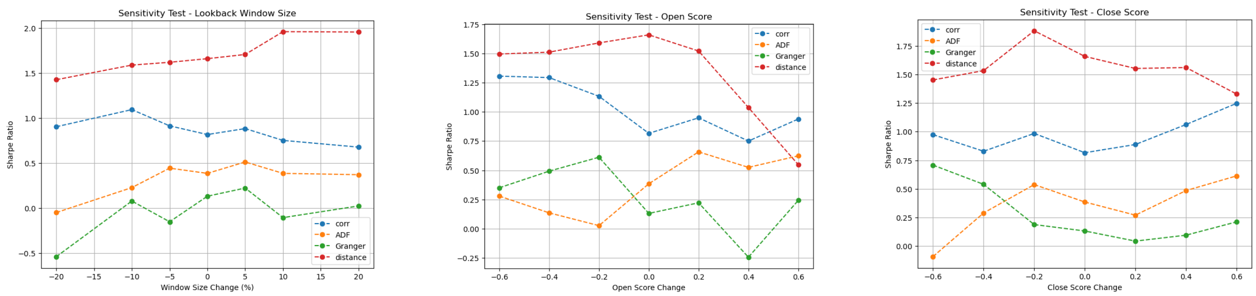

Figure 4: Out-of-sample hyper-parameter sensitivity analysis

4 Potential Issues and Future Research

4.1 Potential issues

- **Survivorship bias** In this report, we selected our trading universe based on the current constituents of the S&P 500 index, retrieved historical data, and excluded stocks with excessive missing values (NaNs). This approach may introduce survivorship bias into our analysis. By focusing only on companies that are presently in the S&P 500, we might overlook firms that were previously part of the index but have since been removed due to bankruptcy, mergers, or other factors. This could lead to an overestimation of our strategy's performance, as the dataset does not account for companies that underperformed or failed.

- **Transaction Cost** We did not include transaction costs in our previous analysis, which could significantly impact the strategy's real-world applicability. Trading incurs various costs such as commissions, bid-ask spreads, and slippage. Moreover, it's generally challenging to execute trades exactly at the closing price due to market volatility and liquidity constraints. Ignoring these factors may result in an overestimation of the strategy's profitability.

4.2 Future Research

For future research, we could explore several directions:

- **Pairs Selection**

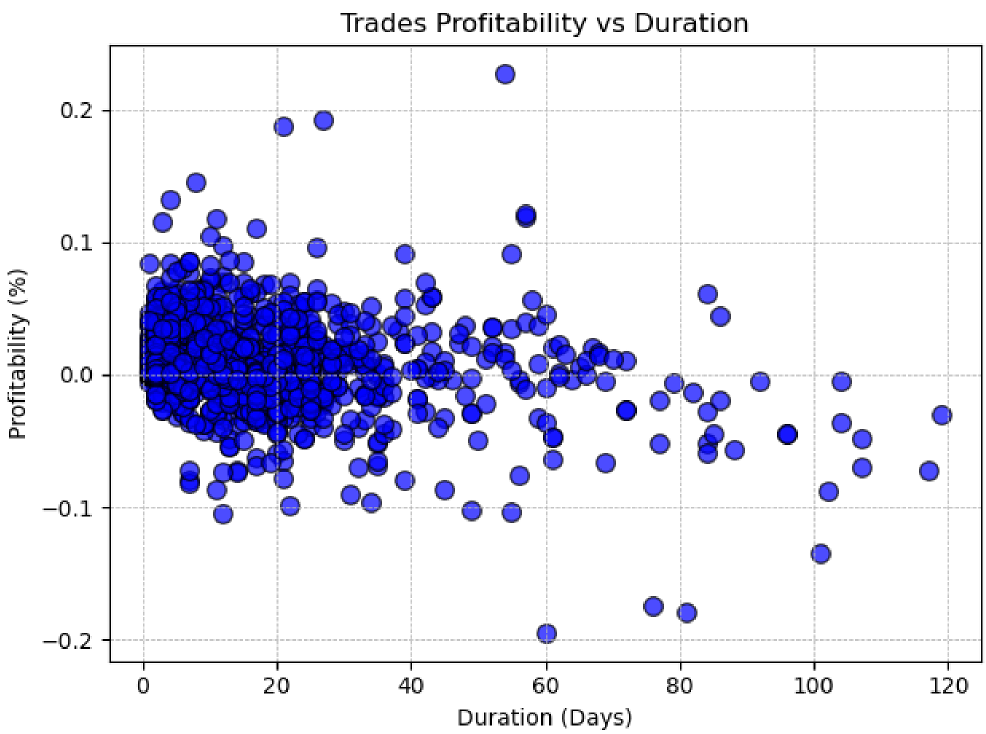

Figure 5: Relationship Between Trades Duration and Profitability

In this study, we selected pairs solely based on closing prices. To improve the robustness of our pair selection, we can incorporate additional data sources. The observed overfitting in many of our strategies suggests that relying exclusively on market price data may not be sufficient. Incorporating alternative data, such as supply chain relationships, sector classifications, or fundamental financial metrics, could provide valuable insights and enhance the effectiveness of our pair selection process.

- **Strategy**

  Previously, we utilized a simple Ordinary Least Squares (OLS) regression and z-score strategy to model the spread dynamics between pairs. To capture the complexities of spread behavior more accurately, we could employ more sophisticated models. For example, implementing a Kalman filter or modeling the spread as an Ornstein–Uhlenbeck (OU) process could improve our understanding and forecasting of spread dynamics, potentially leading to more effective trading strategies.

- **Hyper-parameter Selection**

  In this report, we selected hyperparameters purely based on the in-sample Sharpe Ratio of the strategy. However, in practice, it's crucial to consider the robustness and generalizability of the chosen hyperparameters. Future research should focus on developing methods for hyperparameter selection that balance in-sample performance with out-of-sample robustness, possibly through techniques like cross-validation or incorporating stability metrics.

- **Strategy Combination**

  The pairs selected from different methods exhibit significant variations. By combining strategies derived from multiple selection methods, we could enhance overall performance and achieve a higher Sharpe Ratio. Diversifying our approach may mitigate the risks associated with relying on a single method and improve the consistency of returns.