

# Choice Models Overview

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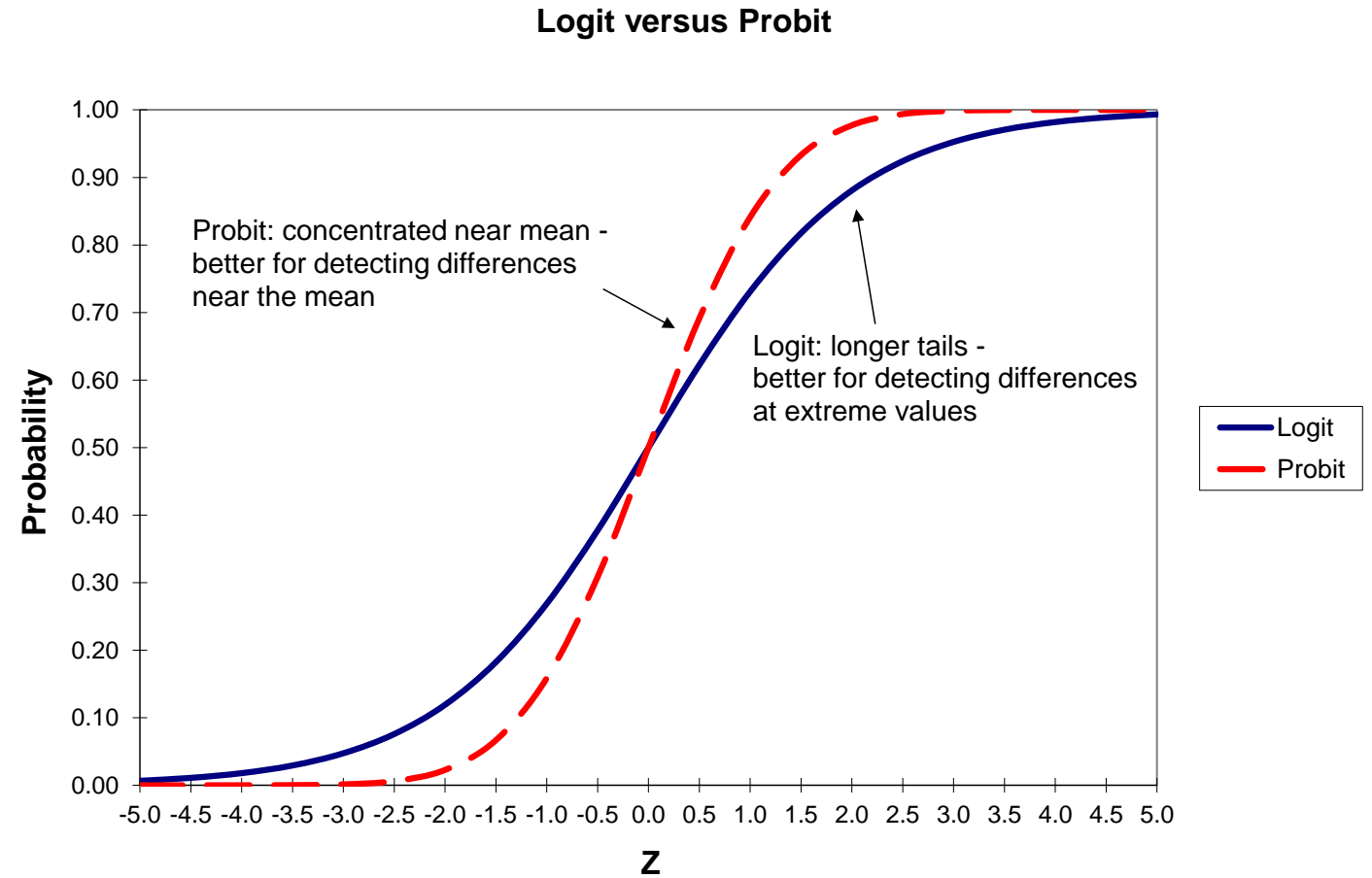
# Choice Models Overview

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- Choice models have Y-variables that are zero or one, e.g., when a customer enters a store, do they buy a TV or not?
- Some techniques to represent choice models include logit, probit, and neural networks.
- Logit and probit have Y-variables which are binary (zero or one) and predict probabilities.
- Logit and probit have s-shaped curves (monotonic): always increasing or always decreasing.
- Neural networks are not limited to monotonic behavior or binary Y-variables.

# Choice Models Overview (cont.)

- Logit uses the logistic distribution and is more sensitive to extreme values of  $X$ .
- Probit uses the normal distribution and is more sensitive near the mean of  $X$ .



Choice Models Overview

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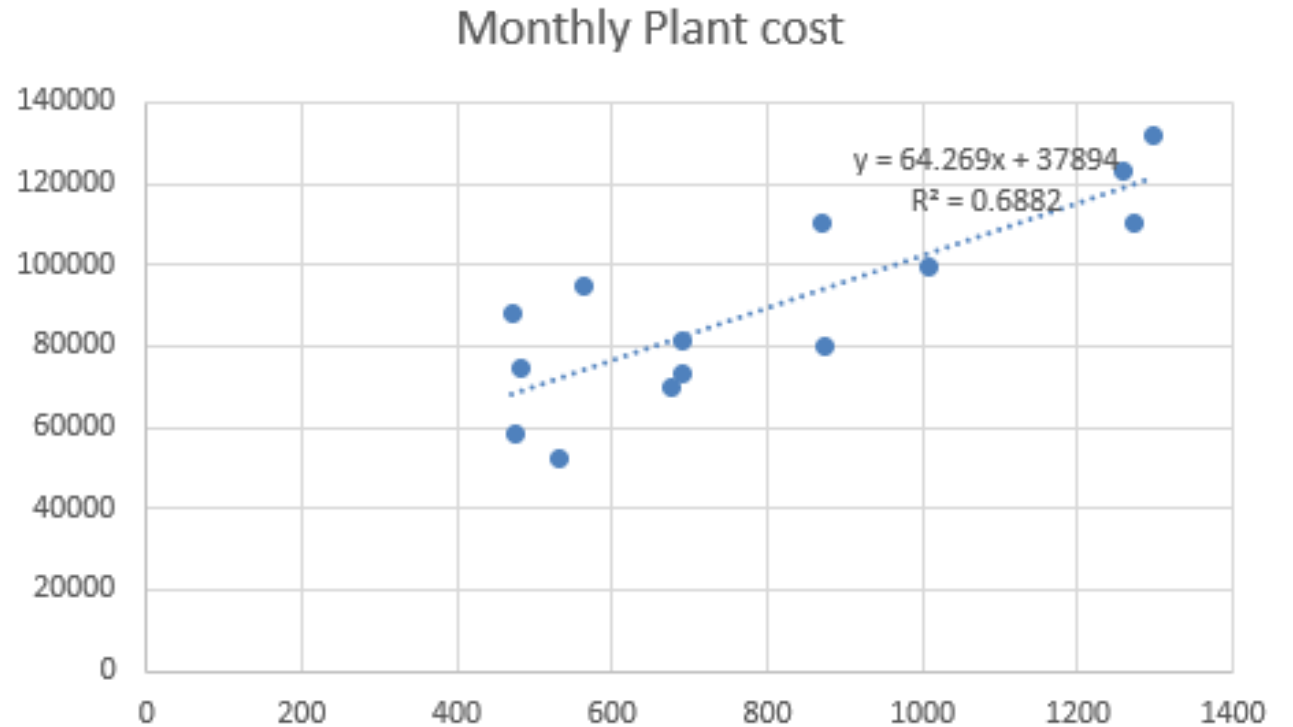
# The End

# Review of Linear Regression

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# Review of Linear Regression

Linear regression finds the line that best fits the data by minimizing the square of the error terms (residuals)

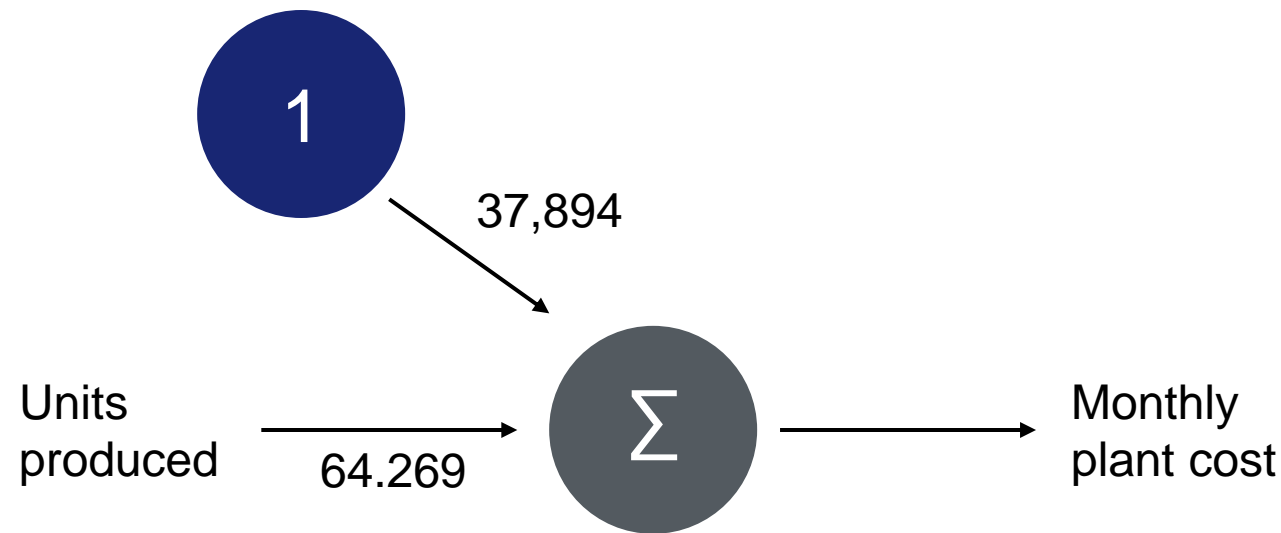


# Review of Linear Regression (cont.)

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The linear regression equation can be represented as:

$$\text{Monthly plant cost} = 37,894 + 64.269 * \text{Units produced}$$



Review of Linear Regression

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# The End

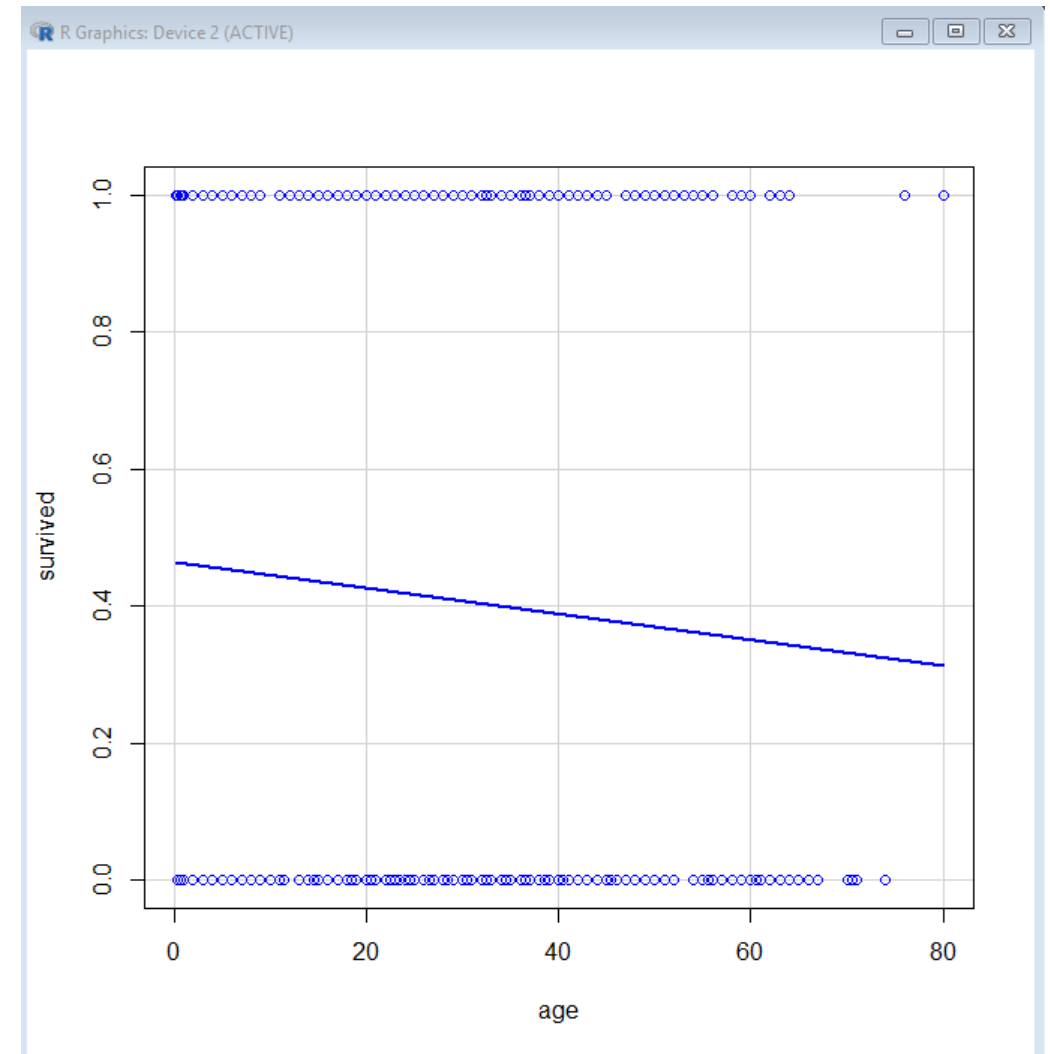


# Review of Logit and the Logistic Function

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# Review of Logit and the Logistic Function, Part I

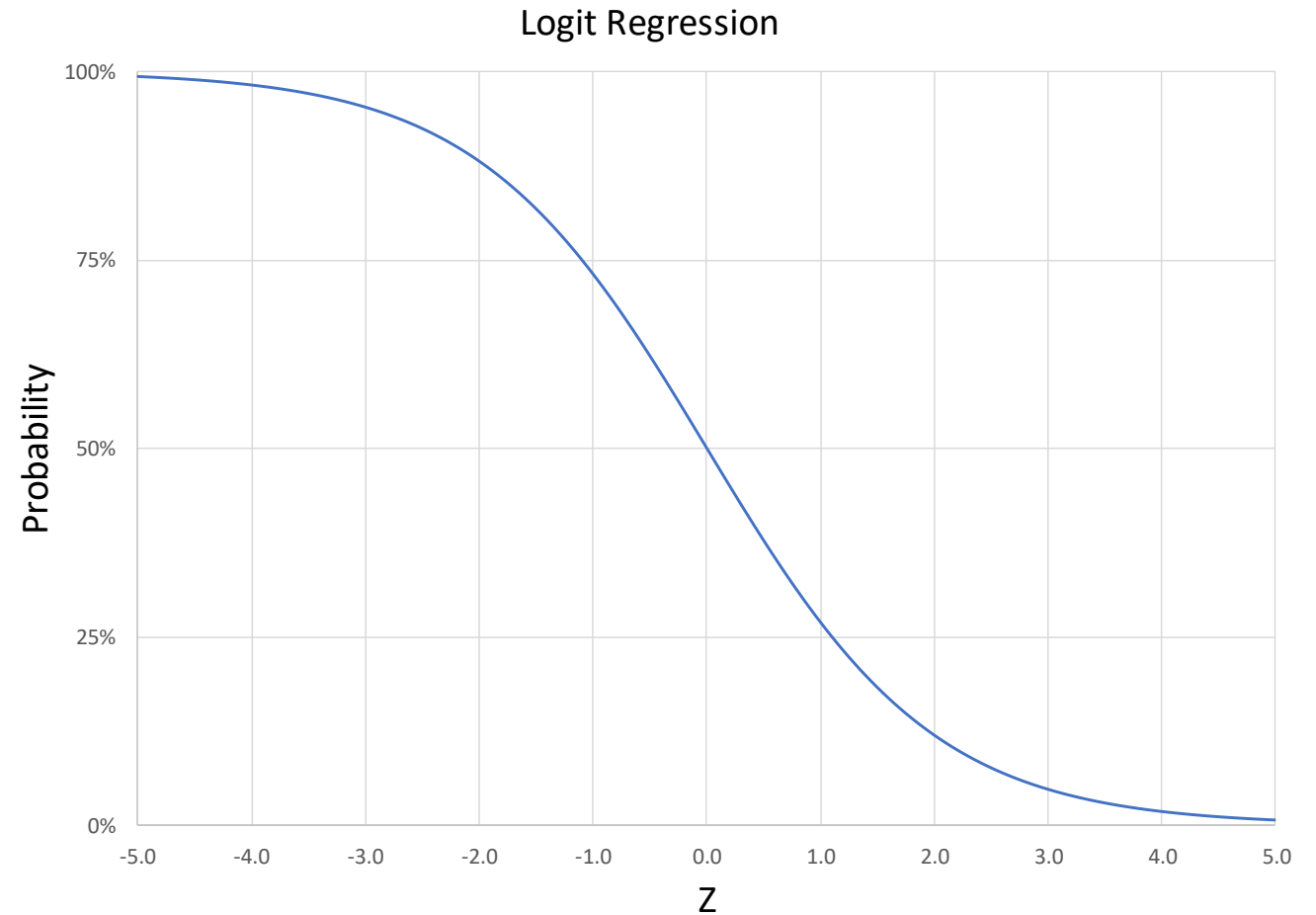
Linear regression does not work when the Y-variable is binary (zero or one)



# Review of Logit and the Logistic Function, Part II

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For binary Y-variables, logit creates an s-shaped curve using the logistic function



# Review of Logit and the Logistic Function, Part III

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The logistic function is

$$f(X) = \exp(\sum \beta_i X_i) / (1 + \exp(\sum \beta_i X_i))$$

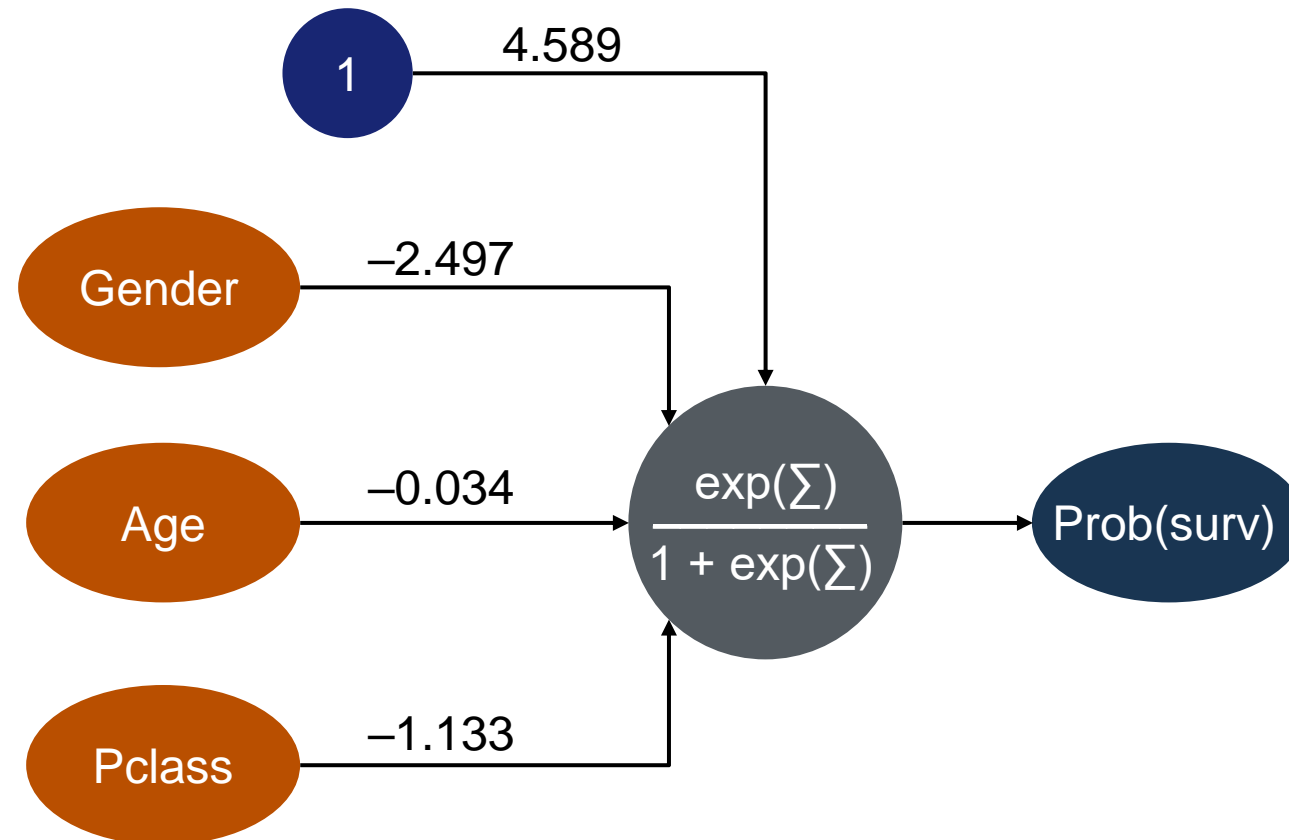
where:

- $\beta_i$  = the coefficients in a logit regression
- $X_i$  = the variables in a logit regression

# Review of Logit and the Logistic Function, Part IV

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Pictorially, this looks like:



Review of Logit and the Logistic Function

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# The End

# Perceptrons

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# Perceptrons, Part I

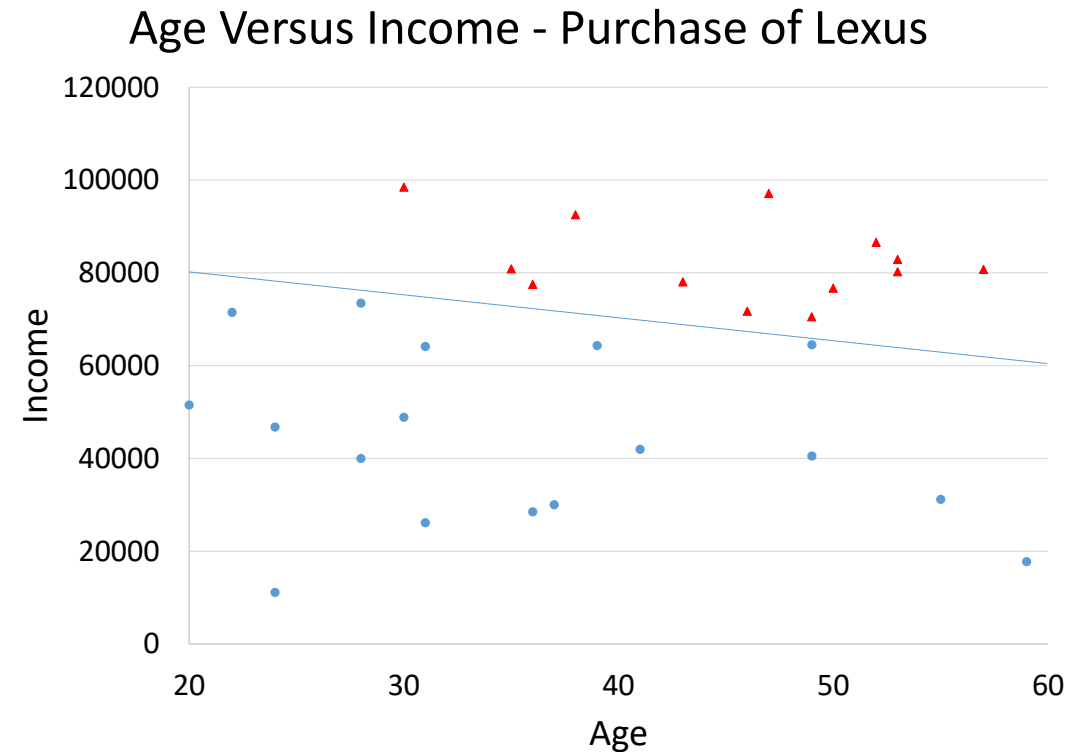
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- In 1957, Frank Rosenblatt created the perceptron learning algorithm.
- Perceptrons were linear classifiers, i.e., they separated groups which had Y-variables which were zero or one.



# Perceptrons, Part II

- In this example, datapoints above the line represent people who bought a Lexus, those below the line did not.
- Buyers tend to have higher income or are slightly older.
- The perceptron identifies the straight line which separates the two groups.



# Perceptrons, Part III

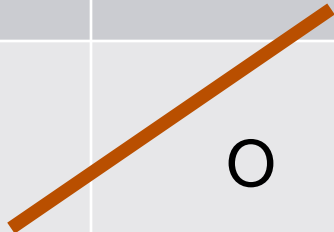
- In 1969, Minsky and Papert analyzed the effectiveness of perceptrons.
- Perceptrons worked on AND conditions; in this example, X identifies those who are both rich and young.
- The perceptron separates the rich and young from the others.

	Rich	Not rich
Young	X	O
Not young	O	O

# Perceptrons, Part IV

- Perceptrons worked on OR conditions; in this example, X identifies those who are rich or young.
- The perceptron separates the rich or young from the others.

	Rich	Not rich
Young	X	X
Not young	X	O



# Perceptrons, Part V

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- Perceptrons did not work on the exclusive OR (XOR).
- Exclusive OR means rich or young, but not both.
- In the 1970s, interest in perceptrons died because of the XOR problem.
- No straight line could be drawn to separate the groups.

	Rich	Not rich
Young	O	X
Not young	X	O

Perceptrons

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# The End

# Neural Networks

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# Neural Networks, Part I

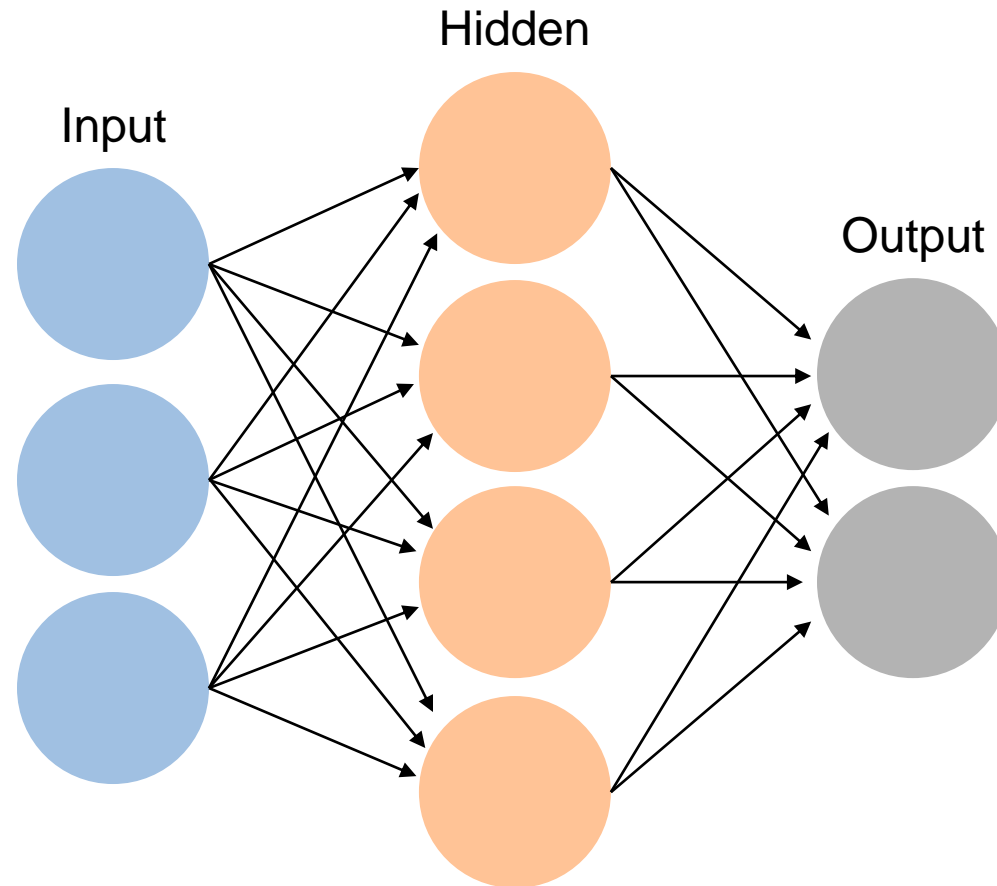
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- In the 1980s, researchers revisited perceptrons, with a twist
- Instead of using linear separators (straight line), they changed to curves using the logistic function
- In addition, instead of having inputs determine outputs, they introduced a middle layer called the hidden layer
- Inputs determined the values of nodes in the hidden layer; nodes in the hidden layer determined the values of the output
- The hidden nodes and the output use the logistic function

# Neural Networks, Part II

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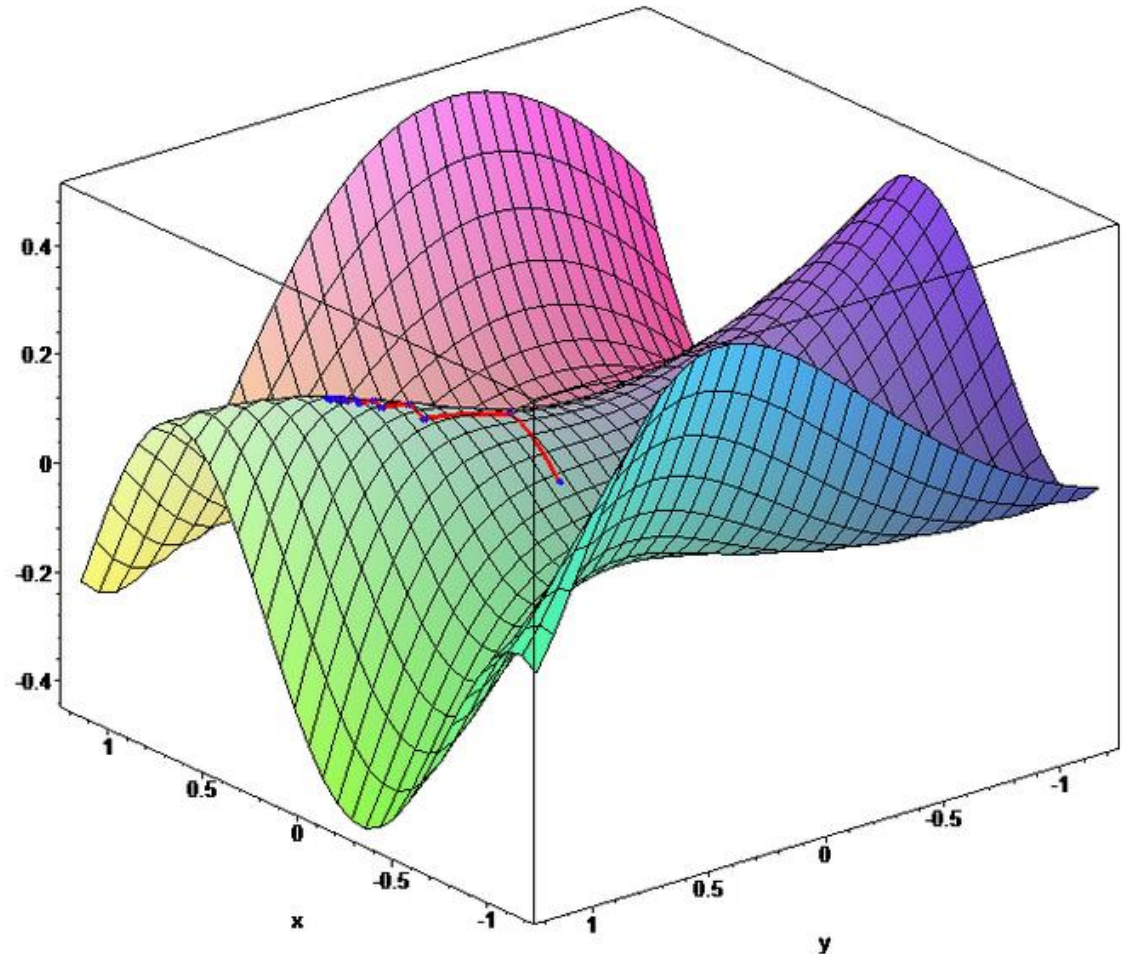
- Each hidden node and each output uses a logistic function





# Neural Networks, Part III

- The back propagation algorithm uses gradient search (hill climbing) to find the best solution
- It randomly selects a starting point, then calculates the slope and climbs the hill
- When there are multiple hills, it sometimes get stuck in a local optimum



# Neural Networks, Part IV

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- Advantages
  - Can have binary or linear outputs
  - Can have one or more outputs
  - Can represent any mathematical relationship
  - Have been used in voice recognition and speech generation (SIRI), photo recognition (law enforcement), and self-driving cars (Tesla)
  - Fast to make decisions after they have learned
- Disadvantages
  - Can get stuck in local optima; need to run several times
  - Slow to learn

Neural Networks

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# The End

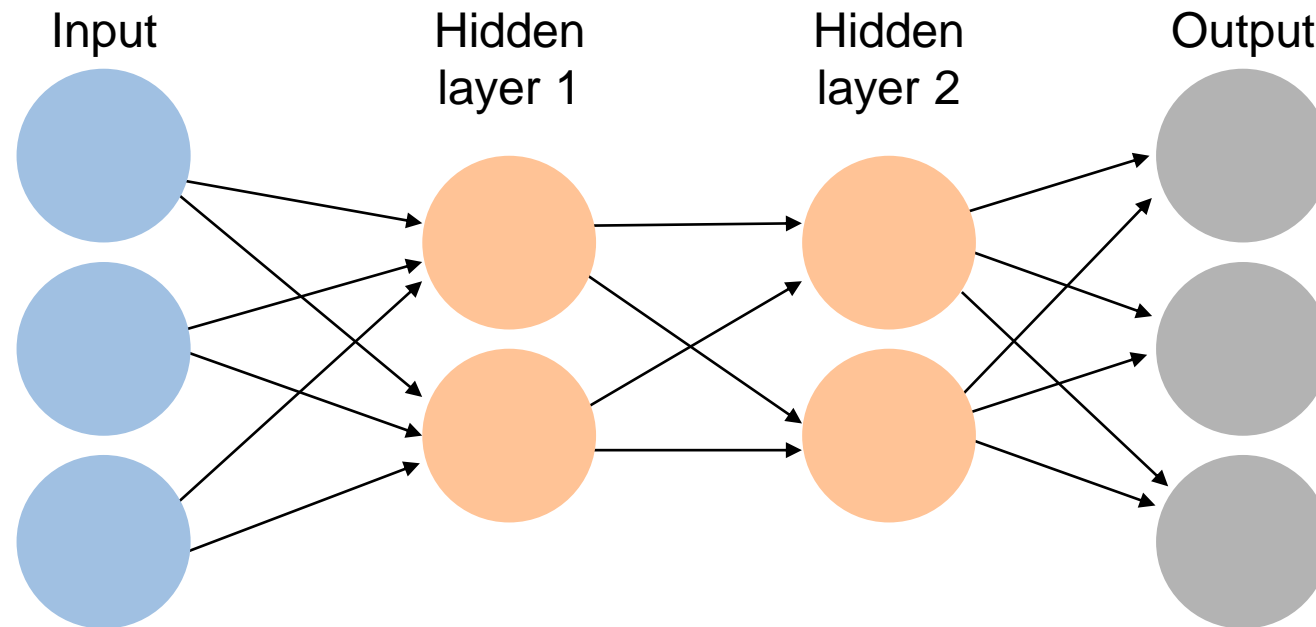
# Deep Learning

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# Deep Learning

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- Deep learning neural networks have two or more hidden layers.
- Deep learning has been used in sophisticated game analysis, such as the game of Go.



Deep Learning

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# The End