Project Report
Optimization of Airport Waiting Time

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April 2019

1 Introduction

When traveling from one country to another by airplane, people usually need to pass for the custom clearance through service booths at the airport. This procedure is crucial in legalizing their arrival in the foreign country or their reentering their home country. Therefore, optimizing the service booth allocation is essential to improve overall well-being of travelers.

However, crossing the border is not always an enjoyable experience because people may wait in queues for a long time. To solve this problem, the U.S. Customs and Border Protection(CBP) needs to consider the trade-off between booth operating cost and passenger waiting time, which is illustrated in below graph.

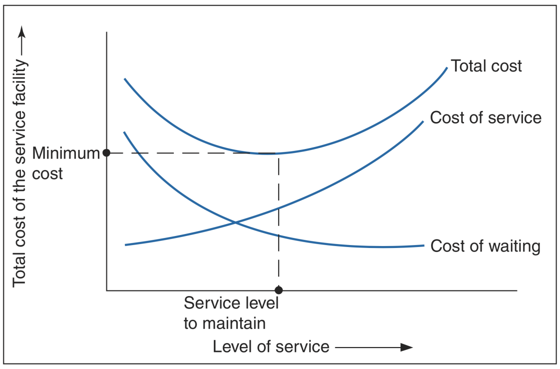


Figure 1: Cost trade-off for service levels. (Taylor, 2007)

We therefore seek to construct a model that efficiently allocates the number of service booths at the custom to best accommodate incoming passengers at every time interval. To this end, we want to minimize the number of customers waiting longer than 30 minutes for their clearance at the custom.

2 Methodology

The notion of waiting line is essential in analyzing and optimizing airport operations. A waiting line can be defined in terms of the arrival rate of customers, the rate at which customers are served by service providers, and the number of service providers. We study the optimization problem under the framework of M/M/c queueing model(See Appendix A). Hence we follow the commonly-used assumptions in queueing theory, which are:

1. Arrival of customers follows the law of Poisson process.
2. Service time follows an exponential distribution.

We will input the waiting time of respective intervals to obtain approximations of the parameter for the Poisson process. With the number of booths as the decision variable, we intend to minimize the average waiting time for incoming customers. Main constraints include a maximum capacity of operating booths. A mathematical expression for the optimization statement is as follows:

In this model, L denotes the number of average customers in the system, λ denotes the average number of customers arrived per hour, and W denotes the average waiting time customer spends in the system. c denotes the serves provided by booths in use and c_m denotes the total capacity of all booths.

$$\min U = (\sum_{t=1}^{t=24} U_t)/24$$

$$\text{subject to} \;\; c_t \leq c_m$$

$$U_t = u_1 * \frac{L_t}{\lambda_t} + u_2 * c_t$$

$$P_{0t} = \frac{1}{\left[\sum_{n=0}^{n=c-1} \frac{1}{n!}\left(\frac{\lambda_t}{\mu_t}\right)^n\right] + \frac{1}{c_t!}\left(\frac{\lambda_t}{\mu_t}\right)^{c_t}\left(\frac{c_t\mu_t}{c_t\mu_t-\lambda_t}\right)} \tag{1}$$

$$L_t = \frac{\lambda_t \mu (\lambda_t/\mu)^{c_t}}{(c_t-1)!(c_t\mu-\lambda_t)^2} P_{0t} + \frac{\lambda_t}{\mu} \tag{2}$$

$$c_t \geq 0$$

3 Data Source and Description

We used open data from the U.S. Customs and Border Protection (CBP), one of the largest law enforcement organization. CBP closely monitors the flight processing times, commonly referred to as wait times, for arriving flights at the busiest international airports. The data provided by CBP shows the number of passengers processed on flights arriving in each hour based on how long it took for those passengers to clear Passport Control. Moreover, they also displays their average compensation for employers on their websites, which serves as a budget benchmark in our model

Considering our computational constraint, we only focused on John F. Kennedy International Airport since it is the most crowded international airport in the world. Moreover, to probe into the general working pattern of airports, we set the duration of our data time span to three month, namely from January 2019 to April 2019.

In terms of the operating budget, we only considered the salary for the entry-level workers since they are those officers working at the booth. According to CBP's open information, the average base income for Grade G5 (the most junior position) is $29350. Assuming that they work for 260 days per year and 8 hours per day, their hourly wage would be $14.11. In our model, we rounded it as approximation. Furthermore, based on the border security's "Report to Congressional Requesters" in 2011, CBP air and marine operations has $814.5 million budget per year to cover their 300 entry ports in the US. Roughly, the daily budget for one entry port is $7,600. To simplify our calculation, we estimated its daily budget to be $10,000 since JFK is such a busy and important station.

4 Data Preprocessing and Solution Algorithm

To get feasible values of average customer arrival (which is dependent on the timing of the day), average service time, we run several statistics of the CBP airport waiting time dataset over a month and get the a few representative graphs:

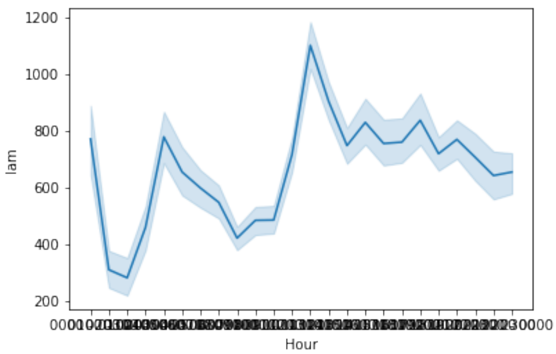


Figure 2: Arrival distribution of passengers in one day period.

The above graph delineates the arrival characteristics of number of customers arriving at the airport, from which there is rush hours between $5-6am$ and one around $12pm$. We then get an approximate arrival rate $\lambda(t)$ which depends on the hour of the day. Note that for simplicity reasons, $\lambda(t)$ is a step function whose value in each hour interval is constant.

Now we need to specify the average service rate μ (the number of customers a booth process in one hour) over the whole day period, which is 47.9047. We also obtained the average number of operating booths c_t graph, as follows.

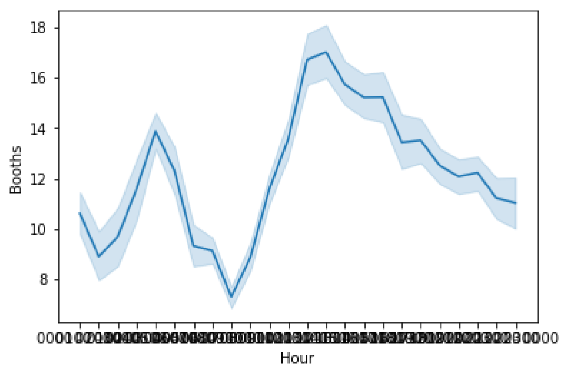


Figure 3: Average number of operating booths in one day period.

After getting preliminary values for λ and μ, we then went on to figure out the relationship between our objective function (i.e., the utility function which consists of average waiting time and booth operating cost) and number of opening booths. To do this, we ran several plots with given λ and μ in Mathematica, with the equation specified in optimization statement. We noticed that even with different λ, μ value pairs, the structure of the graph of utility function against number of opening booths is very similar, and only the values of vertical asymptotes differ.

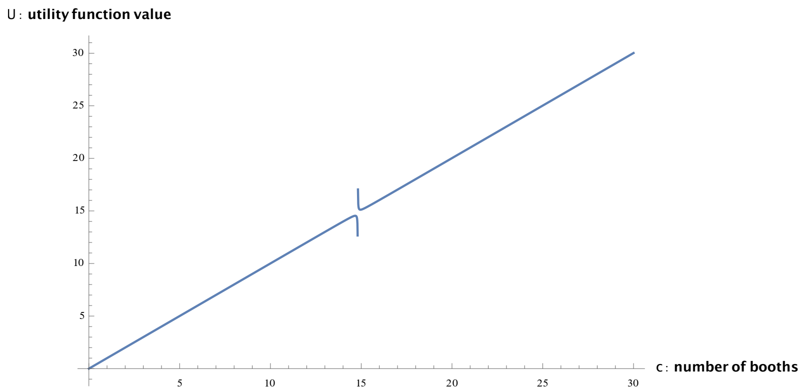


Figure 4: Graph of the utility function value versus number of booth, taking λ and μ to be their mean, which are 711 and 48, respectively.

We also plotted the difference between the objective function and our proposed simplified function, which indeed verified our speculation.

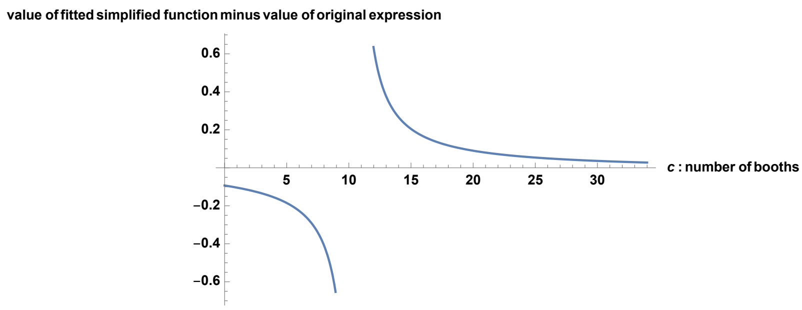


Figure 5: Graph of the difference between the original utility function expression and simplified expression, taking λ and μ to be their mean, which are 711 and 48, respectively.

Given the above information about the graph, we decided to regress U_t on c_t and $\frac{1}{c_t}$ to get best approximation results for every operating hour, which is summarized in the following table.

time	Booths_params	booth_in_params	R_square	max_c
1	2.38452	110.294	0.921876	16
2	2.54965	88.8538	0.962271	17
3	2.77804	88.5761	0.942388	14
4	2.65307	53.4147	0.95566	17
5	1.91944	93.5562	0.981156	21
6	1.98047	105.11	0.953806	20
7	2.6546	63.9199	0.964709	15
8	2.96579	77.4236	0.927836	18
9	3.13254	88.9573	0.928013	14
10	2.87069	101.269	0.919259	16
11	2.73734	99.9679	0.933754	19
12	2.41814	99.377	0.966189	24
13	2.28977	105.725	0.963803	31
14	2.37313	134.09	0.942585	34
15	2.12654	175.246	0.941537	32
16	2.10257	188.151	0.941453	32
17	2.24244	214.409	0.93327	32
18	1.71045	219.73	0.913348	30
19	2.36134	107.525	0.942327	24
20	2.48574	107.436	0.902682	24
21	2.47652	114.653	0.908519	20
22	2.23202	178	0.921931	18
23	2.64025	147.771	0.933482	17
24	1.89465	135.824	0.950789	22

Figure 6: Summary of regression models for all time intervals.

It can be observed from the table above that our proposed regression model is an excellent approximation (with average explanation power 93.9%, see Appendix B for details) of the complicated expression we had in the optimization formulation. After simplification, our final model became as follows.

$$\min U = (\sum_{t=1}^{t=24} U_t)/24$$

$$\text{s.t. } c_t \leq c_m$$

$$U_t = u_1 * \frac{1}{c_t} + u_2 * c_t$$

$$c_t \geq 0$$

Based on the simplified regression model of U_t on c_t and $\frac{1}{c_t}$, the convexity of the objective function becomes apparent. In particular, U_t is convex on the positive real line. We can thus input the coefficients as parameters and combining these information with the previous budget to formulate a complete optimization model that can be comprehended by algebraic modeling language (e.g. `GAMS`).

5 Results and Discussion

We input our convex optimization model into `GAMS` to get final results. In particular, we used `SCIP` mixed-integer non-linear solver to solve our problem.

```
----     106 VARIABLE c.L  number of booths to open at hour t

1   7.000,    2   6.000,    3   6.000,    4   4.000,    5   7.000,    6   7.000
7   5.000,    8   5.000,    9   5.000,    10  6.000,    11  6.000,    12  3.000
13  7.000,    14  8.000,    15  9.000,    16  9.000,    17 10.000,    18 11.000
19  7.000,    20  3.000,    21  7.000,    22  9.000,    23  8.000,    24  8.000


----     106 VARIABLE u.L                  =       34.144  utility function
```

Figure 7: `GAMS` output.

The above output indicates the optimal strategy to arrange the number of opening booths at each hour interval throughout the day, based on the criterion that minimize the utility function (cost). The results verifies the hypothesis that it's generally more efficient to operate more booths during 14pm-24pm as opposed to operate more booths in the morning.

6 Concluding Remarks

In this project, we provided an model assessing the overall operating efficiency of airport customs, based on the data in all terminals of New York JFK airport, using M/M/c queueing framework. We simplified the objective function by speculating the graph of the function and tried regression upon the observation. We obtained a transformed approximation model which has a high explanation power of over 93%. After that, we input the simplified expression into `GAMS` and applied `SCIP` nonlinear mixed integer solver, and obtained our optimized strategy to a list of opening booths according to the hour of the day.

Space for future improvement include more rigorous proof of the mathematical approximation, maybe introducing methods of spline approximation when doing the simplification of the objective function. Moreover, more budget specifications should be obtained to improve accuracy of constraints calculation.

Appendix A M/M/c queueing model

M/M/c is a multi-server system with customers arrival follows a poisson process and exponential service time. When a customer enters an Empty system, he gets the service at once. If the system is non-empty the incoming customer joins the Queue (Sundari and Srinivasan, 2011). In this system, λ denotes the average number of customers arrived per hour, μ denotes the average number of customers served per hour, and c denotes the serves provided by booths in use.

Here we list a few formulas that would be useful in characterizing our optimization model: First, P_0 denotes the probability that there are no customers in system:

$$P_0 = \frac{1}{\sum_{n=0}^{n=c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)} \tag{3}$$

L denotes the number of average customers in the system:

$$L = \frac{\lambda\mu(\lambda/\mu)^c}{(c-1)!(c\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu} \tag{4}$$

Finally, our objective function, W, which denotes the average waiting time customer spends in the system, is expressed as

$$W = \frac{L}{\lambda} \tag{5}$$

$$W = \frac{\mu\left(\frac{\lambda}{\mu}\right)^c}{\frac{\left(\frac{e}{c-1}\right)^{c-1}(c\mu-\lambda)^2\left(\sum_{n=1}^{c-1}\frac{\left(\frac{e}{n}\right)^n\left(\frac{\lambda}{\mu}\right)^n}{\sqrt{2\pi n}}+\frac{\left(\frac{e}{c}\right)^c c\mu\left(\frac{\lambda}{\mu}\right)^c}{\sqrt{2\pi c}(c\mu-\lambda)}+1\right)}{\sqrt{2\pi(c-1)}}} \tag{6}$$

Appendix B Ordinary Least Squares Regression Results

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.922
Model:                            OLS   Adj. R-squared:                  0.918
Method:                 Least Squares   F-statistic:                     218.3
Date:                Wed, 08 May 2019   Prob (F-statistic):           3.28e-21
Time:                        11:14:59   Log-Likelihood:                -147.70
No. Observations:                  39   AIC:                             299.4
Df Residuals:                      37   BIC:                             302.7
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.3845      0.314      7.606      0.000       1.749       3.020
booth_in     110.2937     32.334      3.411      0.002      44.779     175.808
==============================================================================
Omnibus:                        9.459   Durbin-Watson:                   1.903
Prob(Omnibus):                  0.009   Jarque-Bera (JB):                8.537
Skew:                           1.082   Prob(JB):                       0.0140
Kurtosis:                       3.758   Cond. No.                         202.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 8: Summary of regression model for 0:00-1:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.962
Model:                            OLS   Adj. R-squared:                  0.960
Method:                 Least Squares   F-statistic:                     382.6
Date:                Wed, 08 May 2019   Prob (F-statistic):           4.47e-22
Time:                        11:15:01   Log-Likelihood:                -106.40
No. Observations:                  32   AIC:                             216.8
Df Residuals:                      30   BIC:                             219.7
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.5497      0.230     11.103      0.000       2.081       3.019
booth_in      88.8538     16.361      5.431      0.000      55.440     122.267
==============================================================================
Omnibus:                        8.281   Durbin-Watson:                   1.921
Prob(Omnibus):                  0.016   Jarque-Bera (JB):                6.884
Skew:                           1.080   Prob(JB):                       0.0320
Kurtosis:                       3.703   Cond. No.                         124.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 9: Summary of regression model for 1:00-2:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.942
Model:                            OLS   Adj. R-squared:                  0.936
Method:                 Least Squares   F-statistic:                     139.0
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.91e-11
Time:                        11:13:55   Log-Likelihood:                -68.997
No. Observations:                  19   AIC:                             142.0
Df Residuals:                      17   BIC:                             143.9
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.7780      0.442      6.279      0.000       1.845       3.712
booth_in      88.5761     38.107      2.324      0.033       8.177     168.975
==============================================================================
Omnibus:                        7.517   Durbin-Watson:                   1.161
Prob(Omnibus):                  0.023   Jarque-Bera (JB):                5.051
Skew:                           1.207   Prob(JB):                       0.0800
Kurtosis:                       3.743   Cond. No.                         172.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 10: Summary of regression model for 2:00-3:00.

```
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                utility   R-squared:                       0.956
Model:                            OLS   Adj. R-squared:                  0.950
Method:                 Least Squares   F-statistic:                     161.6
Date:                Wed, 08 May 2019   Prob (F-statistic):           7.10e-11
Time:                        11:13:41   Log-Likelihood:                -59.004
No. Observations:                  17   AIC:                             122.0
Df Residuals:                      15   BIC:                             123.7
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.6531      0.370      7.168      0.000       1.864       3.442
booth_in      53.4147     46.633      1.145      0.270     -45.980     152.810
==============================================================================
Omnibus:                        2.702   Durbin-Watson:                   1.096
Prob(Omnibus):                  0.259   Jarque-Bera (JB):                1.481
Skew:                           0.722   Prob(JB):                        0.477
Kurtosis:                       3.074   Cond. No.                         276.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 11: Summary of regression model for 3:00-4:00.

```
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                utility   R-squared:                       0.981
Model:                            OLS   Adj. R-squared:                  0.980
Method:                 Least Squares   F-statistic:                     1041.
Date:                Wed, 08 May 2019   Prob (F-statistic):           3.19e-35
Time:                        11:13:58   Log-Likelihood:                -124.46
No. Observations:                  42   AIC:                             252.9
Df Residuals:                      40   BIC:                             256.4
Df Model:                           2                                         
Covariance Type:            nonrobust                                         
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         1.9194      0.147     13.072      0.000       1.623       2.216
booth_in      93.5562     27.291      3.428      0.001      38.400     148.713
==============================================================================
Omnibus:                       10.664   Durbin-Watson:                   2.035
Prob(Omnibus):                  0.005   Jarque-Bera (JB):               10.166
Skew:                           1.017   Prob(JB):                      0.00620
Kurtosis:                       4.293   Cond. No.                         520.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 12: Summary of regression model for 4:00-5:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.954
Model:                            OLS   Adj. R-squared:                  0.952
Method:                 Least Squares   F-statistic:                     526.5
Date:                Wed, 08 May 2019   Prob (F-statistic):           8.85e-35
Time:                        11:14:02   Log-Likelihood:                -182.15
No. Observations:                  53   AIC:                             368.3
Df Residuals:                      51   BIC:                             372.2
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         1.9805      0.134     14.812      0.000       1.712       2.249
booth_in     105.1105     17.305      6.074      0.000      70.370     139.851
==============================================================================
Omnibus:                        6.948   Durbin-Watson:                   2.159
Prob(Omnibus):                  0.031   Jarque-Bera (JB):                6.757
Skew:                           0.875   Prob(JB):                       0.0341
Kurtosis:                       3.016   Cond. No.                         210.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 13: Summary of regression model for 5:00-6:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.965
Model:                            OLS   Adj. R-squared:                  0.964
Method:                 Least Squares   F-statistic:                     888.4
Date:                Wed, 08 May 2019   Prob (F-statistic):           6.30e-48
Time:                        11:14:05   Log-Likelihood:                -219.27
No. Observations:                  67   AIC:                             442.5
Df Residuals:                      65   BIC:                             446.9
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.6546      0.114     23.197      0.000       2.426       2.883
booth_in      63.9199      8.077      7.914      0.000      47.789      80.051
==============================================================================
Omnibus:                        0.549   Durbin-Watson:                   2.196
Prob(Omnibus):                  0.760   Jarque-Bera (JB):                0.472
Skew:                           0.199   Prob(JB):                        0.790
Kurtosis:                       2.896   Cond. No.                         101.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 14: Summary of regression model for 6:00-7:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.928
Model:                            OLS   Adj. R-squared:                  0.926
Method:                 Least Squares   F-statistic:                     649.3
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.21e-58
Time:                        11:14:08   Log-Likelihood:                -386.09
No. Observations:                 103   AIC:                             776.2
Df Residuals:                     101   BIC:                             781.5
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.9658      0.181     16.372      0.000       2.606       3.325
booth_in      77.4236     13.183      5.873      0.000      51.272     103.575
==============================================================================
Omnibus:                       15.449   Durbin-Watson:                   1.812
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               18.114
Skew:                           0.839   Prob(JB):                     0.000117
Kurtosis:                       4.185   Cond. No.                         123.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 15: Summary of regression model for 7:00-8:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.928
Model:                            OLS   Adj. R-squared:                  0.927
Method:                 Least Squares   F-statistic:                     818.6
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.73e-73
Time:                        11:14:11   Log-Likelihood:                -482.80
No. Observations:                 129   AIC:                             969.6
Df Residuals:                     127   BIC:                             975.3
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         3.1325      0.198     15.861      0.000       2.742       3.523
booth_in      88.9573      9.341      9.523      0.000      70.472     107.442
==============================================================================
Omnibus:                       14.752   Durbin-Watson:                   2.182
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               16.097
Skew:                           0.804   Prob(JB):                     0.000320
Kurtosis:                       3.637   Cond. No.                         79.2
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 16: Summary of regression model for 8:00-9:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.919
Model:                            OLS   Adj. R-squared:                  0.917
Method:                 Least Squares   F-statistic:                     461.1
Date:                Wed, 08 May 2019   Prob (F-statistic):           5.46e-45
Time:                        11:14:15   Log-Likelihood:                -319.41
No. Observations:                  83   AIC:                             642.8
Df Residuals:                      81   BIC:                             647.7
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.8707      0.236     12.147      0.000       2.400       3.341
booth_in     101.2686     16.560      6.115      0.000      68.320     134.217
==============================================================================
Omnibus:                       39.697   Durbin-Watson:                   1.945
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              114.019
Skew:                           1.582   Prob(JB):                     1.74e-25
Kurtosis:                       7.792   Cond. No.                         122.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 17: Summary of regression model for 9:00-10:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.934
Model:                            OLS   Adj. R-squared:                  0.932
Method:                 Least Squares   F-statistic:                     641.3
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.30e-54
Time:                        11:14:18   Log-Likelihood:                -355.09
No. Observations:                  93   AIC:                             714.2
Df Residuals:                      91   BIC:                             719.2
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.7373      0.188     14.560      0.000       2.364       3.111
booth_in      99.9679     22.966      4.353      0.000      54.349     145.587
==============================================================================
Omnibus:                       14.127   Durbin-Watson:                   2.283
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               16.449
Skew:                           1.029   Prob(JB):                     0.000268
Kurtosis:                       3.101   Cond. No.                         237.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 18: Summary of regression model for 10:00-11:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.966
Model:                            OLS   Adj. R-squared:                  0.965
Method:                 Least Squares   F-statistic:                     1372.
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.48e-71
Time:                        11:14:20   Log-Likelihood:                -339.19
No. Observations:                  98   AIC:                             682.4
Df Residuals:                      96   BIC:                             687.6
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.4181      0.113     21.358      0.000       2.193       2.643
booth_in      99.3770     19.353      5.135      0.000      60.962     137.792
==============================================================================
Omnibus:                       23.114   Durbin-Watson:                   1.432
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               33.874
Skew:                           1.077   Prob(JB):                     4.41e-08
Kurtosis:                       4.912   Cond. No.                         345.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 19: Summary of regression model for 11:00-12:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.964
Model:                            OLS   Adj. R-squared:                  0.963
Method:                 Least Squares   F-statistic:                     1771.
Date:                Wed, 08 May 2019   Prob (F-statistic):           1.42e-96
Time:                        11:14:23   Log-Likelihood:                -489.29
No. Observations:                 135   AIC:                             982.6
Df Residuals:                     133   BIC:                             988.4
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.2898      0.067     34.205      0.000       2.157       2.422
booth_in     105.7251     15.706      6.731      0.000      74.659     136.792
==============================================================================
Omnibus:                       22.342   Durbin-Watson:                   1.585
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               29.706
Skew:                           0.916   Prob(JB):                     3.54e-07
Kurtosis:                       4.387   Cond. No.                         354.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 20: Summary of regression model for 12:00-13:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.943
Model:                            OLS   Adj. R-squared:                  0.942
Method:                 Least Squares   F-statistic:                     1330.
Date:                Wed, 08 May 2019   Prob (F-statistic):          3.03e-101
Time:                        11:14:26   Log-Likelihood:                -650.32
No. Observations:                 164   AIC:                             1305.
Df Residuals:                     162   BIC:                             1311.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.3731      0.072     32.872      0.000       2.231       2.516
booth_in     134.0902     16.149      8.303      0.000     102.201     165.980
==============================================================================
Omnibus:                       25.558   Durbin-Watson:                   2.015
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               35.767
Skew:                           0.882   Prob(JB):                     1.71e-08
Kurtosis:                       4.457   Cond. No.                         297.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 21: Summary of regression model for 13:00-14:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.942
Model:                            OLS   Adj. R-squared:                  0.941
Method:                 Least Squares   F-statistic:                     1433.
Date:                Wed, 08 May 2019   Prob (F-statistic):          1.79e-110
Time:                        11:14:28   Log-Likelihood:                -699.01
No. Observations:                 180   AIC:                             1402.
Df Residuals:                     178   BIC:                             1408.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.1265      0.078     27.207      0.000       1.972       2.281
booth_in     175.2464     16.516     10.611      0.000     142.654     207.839
==============================================================================
Omnibus:                       69.516   Durbin-Watson:                   2.241
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              264.318
Skew:                           1.479   Prob(JB):                     4.02e-58
Kurtosis:                       8.147   Cond. No.                         316.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 22: Summary of regression model for 14:00-15:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.941
Model:                            OLS   Adj. R-squared:                  0.941
Method:                 Least Squares   F-statistic:                     1198.
Date:                Wed, 08 May 2019   Prob (F-statistic):           1.51e-92
Time:                        11:14:31   Log-Likelihood:                -584.65
No. Observations:                 151   AIC:                             1173.
Df Residuals:                     149   BIC:                             1179.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.1026      0.092     22.737      0.000       1.920       2.285
booth_in     188.1508     18.555     10.140      0.000     151.485     224.817
==============================================================================
Omnibus:                       11.894   Durbin-Watson:                   1.934
Prob(Omnibus):                  0.003   Jarque-Bera (JB):               12.861
Skew:                           0.714   Prob(JB):                      0.00161
Kurtosis:                       3.076   Cond. No.                         315.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 23: Summary of regression model for 15:00-16:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.933
Model:                            OLS   Adj. R-squared:                  0.932
Method:                 Least Squares   F-statistic:                     769.2
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.18e-65
Time:                        11:14:34   Log-Likelihood:                -450.80
No. Observations:                 112   AIC:                             905.6
Df Residuals:                     110   BIC:                             911.0
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.2424      0.127     17.639      0.000       1.991       2.494
booth_in     214.4093     25.833      8.300      0.000     163.214     265.605
==============================================================================
Omnibus:                       21.286   Durbin-Watson:                   1.920
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               27.404
Skew:                           1.013   Prob(JB):                     1.12e-06
Kurtosis:                       4.328   Cond. No.                         323.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 24: Summary of regression model for 16:00-17:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.913
Model:                            OLS   Adj. R-squared:                  0.911
Method:                 Least Squares   F-statistic:                     358.4
Date:                Wed, 08 May 2019   Prob (F-statistic):           7.67e-37
Time:                        11:14:37   Log-Likelihood:                -277.55
No. Observations:                  70   AIC:                             559.1
Df Residuals:                      68   BIC:                             563.6
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         1.7104      0.182      9.406      0.000       1.348       2.073
booth_in     219.7296     29.377      7.480      0.000     161.108     278.351
==============================================================================
Omnibus:                       28.723   Durbin-Watson:                   2.859
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               51.030
Skew:                           1.500   Prob(JB):                     8.30e-12
Kurtosis:                       5.915   Cond. No.                         270.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 25: Summary of regression model for 17:00-18:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.942
Model:                            OLS   Adj. R-squared:                  0.941
Method:                 Least Squares   F-statistic:                     555.5
Date:                Wed, 08 May 2019   Prob (F-statistic):           7.47e-43
Time:                        11:14:41   Log-Likelihood:                -261.56
No. Observations:                  70   AIC:                             527.1
Df Residuals:                      68   BIC:                             531.6
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.3613      0.162     14.606      0.000       2.039       2.684
booth_in     107.5252     26.563      4.048      0.000      54.520     160.530
==============================================================================
Omnibus:                        6.561   Durbin-Watson:                   1.680
Prob(Omnibus):                  0.038   Jarque-Bera (JB):                5.750
Skew:                           0.635   Prob(JB):                       0.0564
Kurtosis:                       3.601   Cond. No.                         302.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 26: Summary of regression model for 18:00-19:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.903
Model:                            OLS   Adj. R-squared:                  0.901
Method:                 Least Squares   F-statistic:                     533.3
Date:                Wed, 08 May 2019   Prob (F-statistic):           6.62e-59
Time:                        11:14:43   Log-Likelihood:                -471.08
No. Observations:                 117   AIC:                             946.2
Df Residuals:                     115   BIC:                             951.7
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.4857      0.159     15.642      0.000       2.171       2.801
booth_in     107.4356     21.829      4.922      0.000      64.197     150.674
==============================================================================
Omnibus:                       32.224   Durbin-Watson:                   1.714
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               52.340
Skew:                           1.274   Prob(JB):                     4.31e-12
Kurtosis:                       5.061   Cond. No.                         227.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 27: Summary of regression model for 19:00-20:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.909
Model:                            OLS   Adj. R-squared:                  0.907
Method:                 Least Squares   F-statistic:                     506.5
Date:                Wed, 08 May 2019   Prob (F-statistic):           1.07e-53
Time:                        11:14:47   Log-Likelihood:                -413.74
No. Observations:                 104   AIC:                             831.5
Df Residuals:                     102   BIC:                             836.8
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.4765      0.180     13.754      0.000       2.119       2.834
booth_in     114.6532     23.474      4.884      0.000      68.092     161.215
==============================================================================
Omnibus:                       50.244   Durbin-Watson:                   1.793
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              135.219
Skew:                           1.812   Prob(JB):                     4.34e-30
Kurtosis:                       7.252   Cond. No.                         231.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 28: Summary of regression model for 20:00-21:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.922
Model:                            OLS   Adj. R-squared:                  0.920
Method:                 Least Squares   F-statistic:                     401.5
Date:                Wed, 08 May 2019   Prob (F-statistic):           2.21e-38
Time:                        11:14:49   Log-Likelihood:                -275.96
No. Observations:                  70   AIC:                             555.9
Df Residuals:                      68   BIC:                             560.4
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.2320      0.257      8.686      0.000       1.719       2.745
booth_in     178.0004     35.699      4.986      0.000     106.765     249.236
==============================================================================
Omnibus:                       42.319   Durbin-Watson:                   1.858
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              159.307
Skew:                           1.764   Prob(JB):                     2.55e-35
Kurtosis:                       9.494   Cond. No.                         296.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 29: Summary of regression model for 21:00-22:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.933
Model:                            OLS   Adj. R-squared:                  0.931
Method:                 Least Squares   F-statistic:                     336.8
Date:                Wed, 08 May 2019   Prob (F-statistic):           5.63e-29
Time:                        11:14:52   Log-Likelihood:                -194.15
No. Observations:                  50   AIC:                             392.3
Df Residuals:                      48   BIC:                             396.1
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.6403      0.282      9.355      0.000       2.073       3.208
booth_in     147.7707     32.374      4.565      0.000      82.679     212.862
==============================================================================
Omnibus:                        2.280   Durbin-Watson:                   2.081
Prob(Omnibus):                  0.320   Jarque-Bera (JB):                2.144
Skew:                           0.432   Prob(JB):                        0.342
Kurtosis:                       2.468   Cond. No.                         221.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 30: Summary of regression model for 22:00-23:00.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.951
Model:                            OLS   Adj. R-squared:                  0.949
Method:                 Least Squares   F-statistic:                     560.3
Date:                Wed, 08 May 2019   Prob (F-statistic):           1.17e-38
Time:                        11:14:55   Log-Likelihood:                -210.04
No. Observations:                  60   AIC:                             424.1
Df Residuals:                      58   BIC:                             428.3
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         1.8946      0.138     13.771      0.000       1.619       2.170
booth_in     135.8240     14.362      9.457      0.000     107.075     164.573
==============================================================================
Omnibus:                        9.871   Durbin-Watson:                   1.322
Prob(Omnibus):                  0.007   Jarque-Bera (JB):                9.515
Skew:                           0.913   Prob(JB):                      0.00859
Kurtosis:                       3.688   Cond. No.                         160.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure 31: Summary of regression model for 23:00-24:00.

Appendix C Codes

C.1 Mathematica

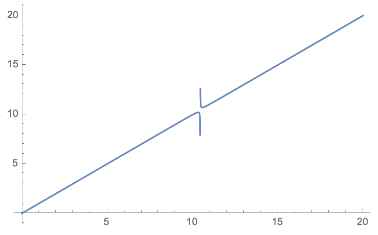
```
In[•]:= W2 = (μ (λ / μ) ^ c) / ((c - 1) ! * (c * μ - λ) ^ 2 * (1 + Sum[(n !) ^ (-1) * ((λ / μ) ^ n), {n, 1, c - 1}] + ((c !) ^ (-1) * (λ / μ) ^ c) * c * μ / ((c * μ - λ)))) + 1 / μ + c
```

Out[•]= $c + \frac{1}{\mu} + \frac{\left(\frac{\lambda}{\mu}\right)^c \mu}{(-\lambda + c\,\mu)^2\,(-1+c)!\,\left(\frac{c\left(\frac{\lambda}{\mu}\right)^c \mu}{(-\lambda + c\,\mu)\,c!} + \frac{e^{\lambda/\mu}\,\text{Gamma}\left[c, \frac{\lambda}{\mu}\right]}{\text{Gamma}[c]}\right)}$

```
In[•]:= p2 = W2 /. λ → 711
        m2 = p2 /. μ → 68
        Plot[m2, {c, 0, 20}]
```

Out[•]= $c + \frac{1}{\mu} + \frac{711^c \left(\frac{1}{\mu}\right)^{-1+c}}{(-711 + c\,\mu)^2\,(-1+c)!\,\left(\frac{711^c\, c\left(\frac{1}{\mu}\right)^{-1+c}}{(-711 + c\,\mu)\,c!} + \frac{e^{711/\mu}\,\text{Gamma}\left[c, \frac{711}{\mu}\right]}{\text{Gamma}[c]}\right)}$

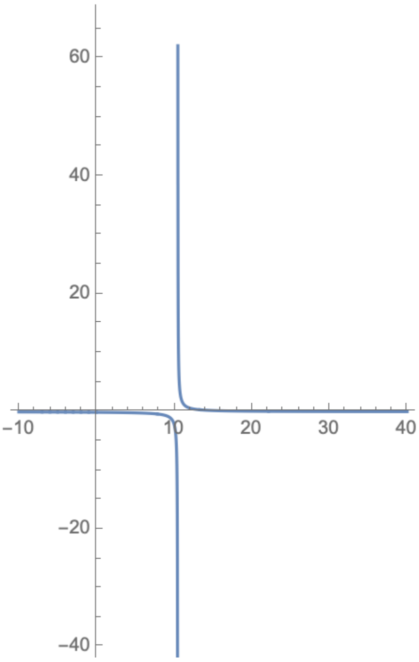
Out[•]= $\frac{1}{68} + c + \frac{68^{1-c} \times 711^c}{(-711 + 68\,c)^2\,(-1+c)!\,\left(\frac{68^{1-c} \times 711^c\, c}{(-711 + 68\,c)\,c!} + \frac{e^{711/68}\,\text{Gamma}\left[c, \frac{711}{68}\right]}{\text{Gamma}[c]}\right)}$

Out[•]=


```
p1 = c - 711 / 68 + 1 / (c - 711 / 68) + 711 / 68
Plot[p1 - m2, {c, -10, 40}, AspectRatio → 2, PlotRange → All]
```

Out[ ]= $\frac{1}{-\frac{711}{68} + c} + c$

Out[ ]=



C.2 Calculate parameters for regression (Jupyter Notebook)

Calculate Parameters

May 10, 2019

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import statsmodels.api as sm
        import seaborn as sns
        %matplotlib inline
```

```
In [2]: jfk_3_4 = pd.read_excel("JFK3-4.xlsx")
        jfk_2_3 = pd.read_excel("JFK2-3.xlsx")
        jfk_1_2 = pd.read_excel("JFK1-2.xlsx")
        frames = [jfk_1_2, jfk_2_3, jfk_3_4]
```

```
In [3]: jfk_raw = pd.concat(frames)
        print(jfk_raw.shape)
        jfk_raw.head()
```

```
(7282, 21)
```

```
Out[3]:   Airport    Terminal        Date         Hour  US_Avg_Wait_t  US_Max_Wait_t  \
        0     JFK  Terminal 1 2019-01-14  0100 - 0200              0              0
        1     JFK  Terminal 1 2019-01-14  0800 - 0900             15             28
        2     JFK  Terminal 1 2019-01-14  0900 - 1000              7             13
        3     JFK  Terminal 1 2019-01-14  1000 - 1100              9             29
        4     JFK  Terminal 1 2019-01-14  1100 - 1200              9             21

           NonUS_Avg_Wait_t  NonUS_Max_Wait_t  Avg_Wait_t  Max_Wait_t   ...    16-30  \
        0                 8                12           8          12   ...        0
        1                19                31          16          31   ...      138
        2                11                22          10          22   ...       63
        3                14                33          12          33   ...      159
        4                12                25          10          25   ...       84

           31-45  46-60  61-90  91-120  120 plus  Excluded  Total  Flights  Booths
        0      0      0      0       0         0         1     15        1       1
        1      2      0      0       0         0         8    249        1       8
        2      0      0      0       0         0        11    321        2      12
        3     19      0      0       0         0        19    673        2      12
```

```
4       0       0       0       0         0         21     573         3       15

[5 rows x 21 columns]
```

```
In [4]: jfk = jfk_raw[["Terminal","Date","Hour", "Avg_Wait_t","Booths"]]
        jfk["passed_in_one_hour"] = jfk_raw["0-15"]+jfk_raw["16-30"]+jfk_raw["31-45"]+jfk_raw["4
        jfk["total_arrival"] = jfk_raw.Total - jfk_raw.Excluded
```

```
/Users/mac/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:2: SettingWithCopyWarning
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stable/indexing.html#

/Users/mac/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:3: SettingWithCopyWarning
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stable/indexing.html#
  This is separate from the ipykernel package so we can avoid doing imports until
```

```
In [5]: def Cal_P(lam,c,mu):
            denominator = 0
            #error_term = 0.0001
            for i in range(c):
                denominator += (1/np.math.factorial(i))*((lam/mu)**i)
            denominator += (1/np.math.factorial(c))*((lam/mu)**c)*(c*mu/(c*mu-lam))
            #print(denominator)
            result = 1/denominator
            return result

        def Cal_L(lam,c,mu,P):
            result = (lam*mu*((lam/mu)**c)*P)/(np.math.factorial(c-1)*((c*mu-lam)**2)) + lam
            return result
```

```
In [6]: jfk = jfk[jfk.passed_in_one_hour != jfk.total_arrival]
        print(jfk.shape)
        jfk.head()
```

```
(2058, 7)
```

```
Out[6]:             Terminal        Date         Hour  Avg_Wait_t  Booths  \
        6         Terminal 1 2019-01-14  1300 - 1400          29      25
        11        Terminal 1 2019-01-14  1800 - 1900          28      15
        12        Terminal 1 2019-01-14  1900 - 2000          34      15
        13        Terminal 1 2019-01-14  2000 - 2100          22      15
        28  Terminal 4 (IAT) 2019-01-14  1100 - 1200          19      19
```

```
    passed_in_one_hour  total_arrival
6                 1426           1466
11                1326           1486
12                 280            352
13                 977           1020
28                 780            793
```

In [7]:
```
jfk["mu"] = (jfk.passed_in_one_hour/jfk.Booths).astype(int)
jfk["lam"] = jfk.total_arrival
jfk["P_0"] = jfk.apply(lambda x: Cal_P(x.lam,x.Booths,x.mu), axis=1)
jfk["L"] = jfk.apply(lambda x: Cal_L(x.lam,x.Booths,x.mu,x.P_0), axis=1)

jfk["W"] = (jfk.L/jfk.total_arrival) * 60
jfk.head()
```

Out[7]:
```
            Terminal        Date         Hour  Avg_Wait_t  Booths  \
6         Terminal 1 2019-01-14  1300 - 1400          29      25
11        Terminal 1 2019-01-14  1800 - 1900          28      15
12        Terminal 1 2019-01-14  1900 - 2000          34      15
13        Terminal 1 2019-01-14  2000 - 2100          22      15
28  Terminal 4 (IAT) 2019-01-14  1100 - 1200          19      19

    passed_in_one_hour  total_arrival  mu   lam           P_0            L  \
6                 1426           1466  57  1466 -2.917354e-12  1423.810694
11                1326           1486  88  1486 -1.054026e-07  1471.148500
12                 280            352  18   352 -4.864973e-08   339.699808
13                 977           1020  65  1020 -8.558506e-08   992.306341
28                 780            793  41   793 -8.626067e-10   730.974530

            W
6   58.273289
11  59.400343
12  57.903376
13  58.370961
28  55.307026
```

In [8]:
```
np.mean(jfk.mu)
```

Out[8]: 47.904761904761905

In [9]:
```
np.mean(jfk.lam)
```

Out[9]: 711.3381924198251

In [10]:
```
dummy_terminal = pd.get_dummies(jfk['Terminal'] )
dummy_terminal.columns = ['Terminal1','Terminal4','Terminal5','Terminal7','Terminal8']
jfk = pd.concat([jfk, dummy_terminal], 1)
```

3

```
In [11]: def replace_time(x):
             switcher = {'0000 - 0100': 0 , '0100 - 0200':1 , '0200 - 0300':2,
                '0300 - 0400':3, '0400 - 0500':4, '0500 - 0600':5, '0600 - 0700':6,
                '0700 - 0800':7, '0800 - 0900':8, '0900 - 1000':9, '1000 - 1100':10,
                '1100 - 1200':11, '1200 - 1300':12, '1300 - 1400':13, '1400 - 1500':14,
                '1500 - 1600':15, '1600 - 1700':16, '1700 - 1800':17, '1800 - 1900':18,
                '1900 - 2000':19, '2000 - 2100':20, '2100 - 2200':21, '2200 - 2300':22,
                '2300 - 0000':23
             }
             return switcher[x]

In [12]: jfk["time"] = jfk.Hour.apply(lambda x: replace_time(x))

In [13]: jfk_clean = jfk.drop(columns = ["Terminal","Hour","Date"]).reset_index()
         jfk_clean = jfk_clean.drop(columns = "index")
         print(jfk_clean.shape)
         jfk_clean.head()

(2058, 15)


Out[13]:    Avg_Wait_t  Booths  passed_in_one_hour  total_arrival  mu   lam  \
         0          29      25                1426           1466  57  1466
         1          28      15                1326           1486  88  1486
         2          34      15                 280            352  18   352
         3          22      15                 977           1020  65  1020
         4          19      19                 780            793  41   793

                     P_0            L          W  Terminal1  Terminal4  Terminal5  \
         0 -2.917354e-12  1423.810694  58.273289          1          0          0
         1 -1.054026e-07  1471.148500  59.400343          1          0          0
         2 -4.864973e-08   339.699808  57.903376          1          0          0
         3 -8.558506e-08   992.306341  58.370961          1          0          0
         4 -8.626067e-10   730.974530  55.307026          0          1          0

            Terminal7  Terminal8  time
         0          0          0    13
         1          0          0    18
         2          0          0    19
         3          0          0    20
         4          0          0    11

In [14]: jfk_clean["booth_in"] = 1/(jfk_clean.Booths )

In [15]: jfk_clean["utility"] = jfk_clean.Avg_Wait_t + jfk_clean.Booths

In [16]: #X = sm.add_constant(jfk_clean[["booth_in"]])
         time = 1
         group =  jfk_clean[jfk_clean.time == time]
```

4

```
est = sm.OLS(group.utility, group[["Booths","booth_in"]])
est2 = est.fit()
print(est2.summary())

# plt.rc('figure', dpi= 200)
# #plt.text(0.01, 0.05, str(model.summary()), {'fontsize': 12}) old approach
# plt.text(0.01, 0.05, str(est2.summary()), {'fontsize': 12}, fontproperties = 'monospa
# plt.axis('off')
# plt.tight_layout()
# plt.savefig('OLS%s.png'%(time))
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                utility   R-squared:                       0.962
Model:                            OLS   Adj. R-squared:                  0.960
Method:                 Least Squares   F-statistic:                     382.6
Date:                Wed, 08 May 2019   Prob (F-statistic):           4.47e-22
Time:                        13:50:53   Log-Likelihood:                -106.40
No. Observations:                  32   AIC:                             216.8
Df Residuals:                      30   BIC:                             219.7
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Booths         2.5497      0.230     11.103      0.000       2.081       3.019
booth_in      88.8538     16.361      5.431      0.000      55.440     122.267
==============================================================================
Omnibus:                        8.281   Durbin-Watson:                   1.921
Prob(Omnibus):                  0.016   Jarque-Bera (JB):                6.884
Skew:                           1.080   Prob(JB):                       0.0320
Kurtosis:                       3.703   Cond. No.                         124.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
In [17]: #est2.summary().as_latex()
```

```
In [18]: df_groups = []
         summaries = []
         results = pd.DataFrame(columns = ["Booths_params","booth_in_params", "R_square", "time"
         rsquares = []
         for i in range(24):
             group =  jfk_clean[jfk_clean.time == i]
             group =  group[["utility","Booths","booth_in", "time"]]
             df_groups.append(group)
```

```
    #X = sm.add_constant(group[["booth_in"]])
    est = sm.OLS(group.utility, group[["Booths","booth_in"]])
    est2 = est.fit()
    Booths_param = est2.params[0]
    booth_in_param = est2.params[1]

    rsquare = est2.rsquared
#     results.iloc[i,"Booths"] = Booths_param
#     results.iloc[i,"booth_in"] = booth_in_param
#     results.iloc[i,"R_square"] = rsquare
    results["Booths_params"][i] = Booths_param
    results["booth_in_params"][i] = booth_in_param

    results["R_square"][i] = rsquare
    results["time"][i] = i


    summary = est2.summary()
    summaries.append(summary)

    rsquares.append(rsquare)

print(len(df_groups))
```

```
24
```

```
/Users/mac/anaconda3/lib/python3.7/site-packages/scipy/stats/stats.py:1416: UserWarning: kurtosi
  "anyway, n=%i" % int(n))
/Users/mac/anaconda3/lib/python3.7/site-packages/scipy/stats/stats.py:1416: UserWarning: kurtosi
  "anyway, n=%i" % int(n))
```

In [19]:
```
results["max_c"] = jfk_clean.groupby(by = "time").max()["Booths"]
results["time"] = results["time"] + 1
results
```

Out[19]:

	Booths_params	booth_in_params	R_square	time	max_c
0	2.38452	110.294	0.921876	1	16
1	2.54965	88.8538	0.962271	2	17
2	2.77804	88.5761	0.942388	3	14
3	2.65307	53.4147	0.95566	4	17
4	1.91944	93.5562	0.981156	5	21
5	1.98047	105.11	0.953806	6	20
6	2.6546	63.9199	0.964709	7	15
7	2.96579	77.4236	0.927836	8	18
8	3.13254	88.9573	0.928013	9	14
9	2.87069	101.269	0.919259	10	16

10	2.73734	99.9679	0.933754	11	19
11	2.41814	99.377	0.966189	12	24
12	2.28977	105.725	0.963803	13	31
13	2.37313	134.09	0.942585	14	34
14	2.12654	175.246	0.941537	15	32
15	2.10257	188.151	0.941453	16	32
16	2.24244	214.409	0.93327	17	32
17	1.71045	219.73	0.913348	18	30
18	2.36134	107.525	0.942327	19	24
19	2.48574	107.436	0.902682	20	24
20	2.47652	114.653	0.908519	21	20
21	2.23202	178	0.921931	22	18
22	2.64025	147.771	0.933482	23	17
23	1.89465	135.824	0.950789	24	22

In [20]:
```
from tabulate import tabulate
results = results.set_index('time')
print(tabulate(results, headers='keys', tablefmt='psql'))
```

```
+--------+-----------------+-------------------+------------+---------+
|   time |   Booths_params |   booth_in_params |   R_square |   max_c |
|--------+-----------------+-------------------+------------+---------|
|      1 |         2.38452 |          110.294  |   0.921876 |      16 |
|      2 |         2.54965 |           88.8538 |   0.962271 |      17 |
|      3 |         2.77804 |           88.5761 |   0.942388 |      14 |
|      4 |         2.65307 |           53.4147 |   0.95566  |      17 |
|      5 |         1.91944 |           93.5562 |   0.981156 |      21 |
|      6 |         1.98047 |          105.11   |   0.953806 |      20 |
|      7 |         2.6546  |           63.9199 |   0.964709 |      15 |
|      8 |         2.96579 |           77.4236 |   0.927836 |      18 |
|      9 |         3.13254 |           88.9573 |   0.928013 |      14 |
|     10 |         2.87069 |          101.269  |   0.919259 |      16 |
|     11 |         2.73734 |           99.9679 |   0.933754 |      19 |
|     12 |         2.41814 |           99.377  |   0.966189 |      24 |
|     13 |         2.28977 |          105.725  |   0.963803 |      31 |
|     14 |         2.37313 |          134.09   |   0.942585 |      34 |
|     15 |         2.12654 |          175.246  |   0.941537 |      32 |
|     16 |         2.10257 |          188.151  |   0.941453 |      32 |
|     17 |         2.24244 |          214.409  |   0.93327  |      32 |
|     18 |         1.71045 |          219.73   |   0.913348 |      30 |
|     19 |         2.36134 |          107.525  |   0.942327 |      24 |
|     20 |         2.48574 |          107.436  |   0.902682 |      24 |
|     21 |         2.47652 |          114.653  |   0.908519 |      20 |
|     22 |         2.23202 |          178      |   0.921931 |      18 |
|     23 |         2.64025 |          147.771  |   0.933482 |      17 |
|     24 |         1.89465 |          135.824  |   0.950789 |      22 |
+--------+-----------------+-------------------+------------+---------+
```

7

```
In [21]: results[:12].to_latex(index=False)

Out[21]: '\\begin{tabular}{lllr}\n\\toprule\nBooths\\_params & booth\\_in\\_params &  R\\_square

In [22]: results.to_csv("parameters2.csv")

In [23]: np.mean(rsquares)

Out[23]: 0.9396934401078557

In [ ]:
```

C.3 GAMS

```
Sets
    t time /1*24/;

Scalar
    B     "Budget per day"                                   / 10000 /
    S     "personnel salary per day"                         / 14  /;

Parameter
    cmax(t)   maximum number of booth opening each hour
    /        1        16
             2        17
             3        14
             4        17
             5        21
             6        20
             7        15
             8        18
             9        14
             10       16
             11       19
             12       24
             13       31
             14       34
             15       32
             16       32
             17       32
             18       30
             19       24
             20       24
             21       20
             22       18
             23       17
             24       22   /

    a1(t) coeff for lc-lam
```

```
a1(t) coeff for 1c-lam
/   1         110.2937285
    2         88.85381539
    3         88.57607534
    4         53.41473556
    5         93.55620583
    6         105.110452
    7         63.9199022
    8         77.42358587
    9         88.95733983
    10         101.2685795
    11         99.96787133
    12         99.37699035
    13         105.7251411
    14         134.090214
    15         175.2464158
    16         188.1508381
    17         214.4093222
    18         219.7296484
    19         107.5251829
    20         107.435589
    21         114.653227
    22         178.0004176
    23         147.7706578
    24         135.8240409   /

a2(t) coeff for c
/   1         2.384518075
    2         2.549650703
    3         2.778041527
    4         2.653074157
    5         1.919444833
    6         1.980472068
    7         2.654595506
    8         2.965786605
```

```
a1(t) coeff for lc-lam
/   1        110.2937285
    2        88.85381539
    3        88.57607534
    4        53.41473556
    5        93.55620583
    6        105.110452
    7        63.9199022
    8        77.42358587
    9        88.95733983
    10        101.2685795
    11        99.96787133
    12        99.37699035
    13        105.7251411
    14        134.090214
    15        175.2464158
    16        188.1508381
    17        214.4093222
    18        219.7296484
    19        107.5251829
    20        107.435589
    21        114.653227
    22        178.0004176
    23        147.7706578
    24        135.8240409  /

a2(t) coeff for c
/   1        2.384518075
    2        2.549650703
    3        2.778041527
    4        2.653074157
    5        1.919444833
    6        1.980472068
    7        2.654595506
    8        2.965786605
```

```
        9         3.132540828
        10         2.870691008
        11         2.737337759
        12         2.41814318
        13         2.289772009
        14         2.373127565
        15         2.126542186
        16         2.10256522
        17         2.242444069
        18         1.710446439
        19         2.36133916
        20         2.485742949
        21         2.476521742
        22         2.232016961
        23         2.640254451
        24         1.894645067   /;
Variables
    u  utility function
    c(t)  number of booths to open at hour t;

Integer variable c(t);
c.lo(t) = 1;

Equations
    Utility     objective function
    con(t)      constraints for number of opening booths each hour
    budget      total budget;

Utility ..    u =e= sum(t,a1(t)/(c(t))+a2(t)*c(t))/24;
con(t) ..      c(t) =l= cmax(t);
budget ..      sum(t,c(t))*2*S =l= B;

Model portfolio /all/;

Option solver=scip;
Solve portfolio using MINLP minimizing u;
Display c.l, u.l;
```

References

Sundari, S. M. and Srinivasan, S. (2011). M/m/c queueing model for waiting time of customers in bank sectors. *Int. J. of Mathematical Sciences and Applications*, 1(3).

Taylor, III, B. W. (2007). *Introduction to Management Science*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA.