Trajectory control for a robot manipulator

General Instructions

This assignment contains two parts. Part one is theoretic with some applied calculations. In order to do this part, you may need access to material on manipulator mechanics, which you can find via textbook. For this part, there are explicit questions with space to write in the answer. All questions and tasks are signified with a bullet.

The second part of the assignment is an applied problem to be solved in matlab.

This assignment will be orally presented. You will **not** use a computer during the presentation, so make sure that you have all papers that you need ready beforehand. For part 1, you can write all answers in the space provided after each question. For part 2, you should prepare plot printouts and whatever else you need to explain your solution.

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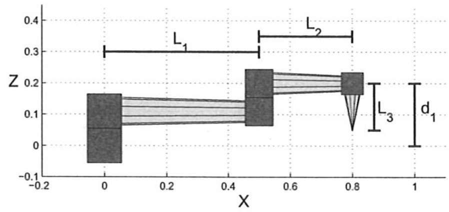
1 Robot specifications and parameters

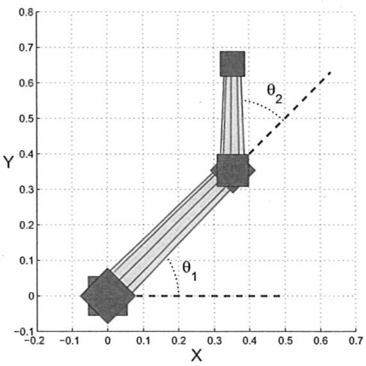
In this exercise we will be working with a SCARA type robot. SCARAs are used to solve some simpler industrial tasks, like picking objects off a conveyor belt and placing in a box (pick-and-place). Thanks to their simplicity, they are fast and robust. The robot is shown in Figures 1(a) and 1(b). This robot has 3 degrees of freedom (DoF), the two angles θ_1 and θ_2, and the variable length of the last link, L_3. The value of d_1 is 0.2 m. The limits on motion are given in Table 1.

Table 1: Motion limits for the SCARA robot.

	min	max
θ_1	0^o	360^o
θ_2	-45^o	180^o
L_3	0.035 m	0.2 m
$\dot{\theta}_1$	-50^o/s	50^o/s
$\dot{\theta}_2$	-50^o/s	50^o/s
$\dot{L}_3$	-0.1 m/s	0.1 m/s



(a) Side view



(b) Top view

Figure 1: Schematic drawing of the SCARA robot.

1.1 Workspace

Assume the following robots, with structure as in Figures 1(a) and 1(b):

Robot A: L_1 = 0.4m, L_2 = 0.4m
Robot B: L_1 = 0.5m, L_2 = 0.3m
Robot C: L_1 = 0.3m, L_2= 0.5m

For each robot A-C, answer the following questions:

- What is the workspace for the robot?

(1). $\begin{cases} x^2+y^2 \le 0.64 \\ 0 \le z \le 0.165 \end{cases}$

(2). $\begin{cases} 0.04 \le x^2+y^2 \le 0.64 \\ 0 \le z \le 0.165 \end{cases}$.

(3) $\begin{cases} 0.04 \le x^2+y^2 \le 0.64 \\ 0 \le z \le 0.165 \end{cases}$

- Are there any kinematic singularities? If so, where?

在奇异点处 ~~末~~末端执行器会失去一个或多个自由度
而这发生于 L_1 与 L_2 共线时，故此时 $\sin\theta_2=0$
$\theta_2 \in [-45°, 180°]$
∴ 奇异点 $\theta_2=0$ 或 $180°$.

1.2 Kinematics

The position of the robot can be given in two different coordinate spaces, *cartesian space* or *joint space*. In cartesian space we will here call the position X_{cart}, which is the vector $[x\ y\ z]^T$. In joint space we will call the position Θ, which is the vector $[\theta_1\ \theta_2\ L_3]^T$. The forward kinematics, K_f, is the function that relates cartesian and joint space:

$$X_{cart} = K_f(\Theta) \tag{1}$$

- Write down the function K_f explicitly.

	θ	a	b	α
1	θ_1	0	0	0
2	0	L_1	0	0
3	θ_2	L_2	$0.2-L_3$	0

$$T_n = T_{1,2}T_{2,3}T_{3,4} = \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & L_2\cos(\theta_1+\theta_2)+L_1\cos\theta_1 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & L_2\sin(\theta_1+\theta_2)+L_1\sin\theta_1 \\ 0 & 0 & 0 & 0.2-L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_f = \begin{pmatrix} L_2\cos(\theta_1+\theta_2)+L_1\cos\theta_1 \\ L_2\sin(\theta_1+\theta_2)+L_1\sin\theta_1 \\ 0.2-L_3 \end{pmatrix}$$

1.3 Inverese Kinematics

The inverese kinematics K_i is the inverse function of K_f, that is,

$$\Theta = K_i(X_{cart}), \tag{2}$$

- How many different joint space solutions can be found for each set of cartesian coordinates?

$x^2+y^2 = L_1^2+L_2^2+2L_1L_2\cos\theta_2$.

$$\cos\theta_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}$$

而 $\theta_2 \in [-45°, 180°]$

当 $\frac{\sqrt{2}}{2} \le \cos\theta_2 \le 1$ 时 θ_2存在两解.
否则 $\cos\theta_2$ 仅有一解.
又一个θ_2对应一个θ_1, z 对应与 L_3 仅一解.
故 当 $\frac{\sqrt{2}}{2} \le \cos\theta_2 < 1$ 时 有两个解.
否则 仅一个解.

- Write down the function K_i, making sure you get all possible solutions.

$$\begin{cases} \cos\theta_2 = \dfrac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}. \\ \sin\theta_1 = \dfrac{(L_1+L_2\cos\theta_2)y-L_2\sin\theta_2\cdot x}{x^2+y^2} \\ \cos\theta_1 = \dfrac{(L_1+L_2\cos\theta_2)x+L_2\sin\theta_2\cdot y}{x^2+y^2}. \\ L_3 = 0.2-z \end{cases}$$

- What happens when the cartesian coordinates are a singularity point?

奇异点 $\sin\theta_2 = 0$.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (L_1+L_2)\cos\theta_1 \\ (L_1+L_2)\sin\theta_1 \\ 0.2-L_3 \end{pmatrix} \quad 或 \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (L_1-L_2)\cos\theta_1 \\ (L_1-L_2)\sin\theta_1 \\ 0.2-L_3 \end{pmatrix}$$

$$\begin{cases} x^2+y^2 = (L_1+L_2)^2. \\ z = 0.2-L_3 \end{cases} \qquad \begin{cases} x^2+y^2 = (L_1-L_2)^2 \\ z = 0.2-L_3 \end{cases}$$

1.4 Velocity Jacobian

The velocity Jacobian J is the function that relates the velocities in cartesian and joint space:

$$J_{ij} = \frac{\delta K_{f,i}}{\delta \Theta_j} \tag{3}$$

Note that since K_f is non-linear, J will be a function of Θ, that can be written $J(\Theta)$, and we will have that

$$\dot{X}_{cart} = J(\Theta)\dot{\Theta} \tag{4}$$

- Calculate the Jacobian J.

$$J = \begin{pmatrix} \frac{\partial x}{\partial \theta_1}, & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1}, & \frac{\partial y}{\partial \theta_2} \end{pmatrix} = \begin{pmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1+\theta_2) & -L_2 \sin(\theta_1+\theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) & L_2 \cos(\theta_1+\theta_2) \end{pmatrix}$$

- What happens to J when the arm is in a singularity?

$\because |J| = L_1 L_2 \sin\theta_2$. 奇异点处 $\sin\theta_2 = 0$.

$\therefore |J| = 0$.

J also has an inverse function, J_{inv}, such that

$$\dot{\Theta} = J_{inv}(X_{cart})\dot{X}_{cart} \tag{5}$$

- How can we calculate J_{inv} using only the expressions previously derived in (1.3) and (1.4)?

$$J^{-1} = \begin{pmatrix} \frac{\cos(\theta_1+\theta_2)}{L_1 \sin\theta_2} & \frac{\sin(\theta_1+\theta_2)}{L_1 \sin\theta_2} \\ -\frac{L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2)}{L_1 L_2 \sin\theta_2} & -\frac{L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2)}{L_1 L_2 \sin\theta_2} \end{pmatrix} \quad (\sin\theta_2 \neq 0).$$

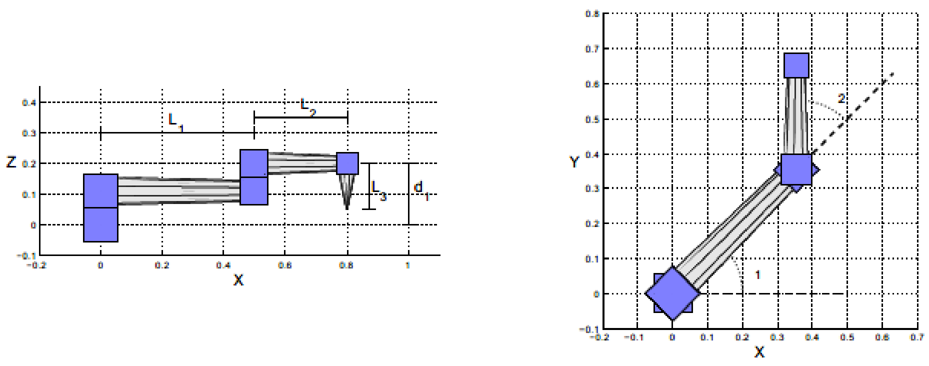
- What happens to J_{inv} when the arm is in a singularity?

奇异点处 $|J| = 0$ 故不存在逆矩阵 即 J_{inv} 不存在.(为无穷大

- Which DoFs are coupled, and how does this affect complexity and the structure of J?

$\theta_1 \theta_2$ 相互耦合, J 的 4 个元素相互关联.

几何法求解



(a) Side view　　　　(b) Top view

一、正运动学求解

根据几何关系，可以很容易得出以下关系：

$$\begin{cases} x = L_1 \cdot \cos(\theta_1) + L_2 \cdot \cos(\theta_1 + \theta_2) \\ y = L_1 \cdot \sin(\theta_1) + L_2 \cdot \sin(\theta_1 + \theta_2) \\ z = d_1 - L_3 \end{cases}$$

二、逆运动学求解

在 MATLAB 中构建上述正运动学方程，用 solve 函数求解$\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix}$，结果如下：

$\theta_1 =$

2*atan((2*L1*y + (L1^2*((- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2)*(L1^2 + 2*L1*L2 + L2^2 - x^2 - y^2))^(1/2))/(- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2) + (L2^2*((- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2)*(L1^2 + 2*L1*L2 + L2^2 - x^2 - y^2))^(1/2))/(- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2) - (x^2*((- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2)*(L1^2 + 2*L1*L2 + L2^2 - x^2 - y^2))^(1/2))/(- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2) - (y^2*((- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2)*(L1^2 + 2*L1*L2 + L2^2 - x^2 - y^2))^(1/2))/(- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2) - (2*L1*L2*((- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2)*(L1^2 + 2*L1*L2 + L2^2 - x^2 - y^2))^(1/2))/(- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2))/(L1^2 + 2*L1*x - L2^2 + x^2 + y^2))

$\theta_2 =$

2*atan(((- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2)*(L1^2 + 2*L1*L2 + L2^2 - x^2 - y^2))^(1/2)/(- L1^2 + 2*L1*L2 - L2^2 + x^2 + y^2))

$L_3 = d_1 - z$

为了方便计算，化简如下：

令 $c_2 = \dfrac{(x^2 + y^2 - {L_1}^2 - L_2^2)}{2 \cdot L_1 \cdot L_2}$ $s_2 = (1 - {c_2}^2)^{\frac{1}{2}}$

则 $\theta_2 = \operatorname{atan}\left(\dfrac{s_2}{c_2}\right)$

令 $c_1 = \dfrac{((L_1 + L_2 \cdot c_2) \cdot x + L_2 \cdot s_2 \cdot y)}{x^2 + y^2}$ $s_1 = \dfrac{(y \cdot (L_1 + L_2 \cdot c_2) - L_2 \cdot s_2 \cdot x)}{x^2 + y^2}$

则 $\theta_1 = \operatorname{atan}\left(\dfrac{s_1}{c_1}\right)$

至此，根据给定的 $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ 就可以反算出需要的关节转角和焊头下降距离 $\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix}$

三、进行第一个实验，焊一条直线

1. 确定机械臂初始位置

   根据初始条件：$\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 60^o \\ -40^o \\ 0.1 \end{bmatrix}$，可以根据正运动学方程，计算此时焊头位置：

   $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.5759 \\ 0.4832 \\ 0.1 \end{bmatrix}$

2. 确定焊接初始点，需要的关节转角

   焊接初始点信息为：$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.3 \\ 0 \end{bmatrix}$，根据逆运动学方程，求解出需要的关节信息为：

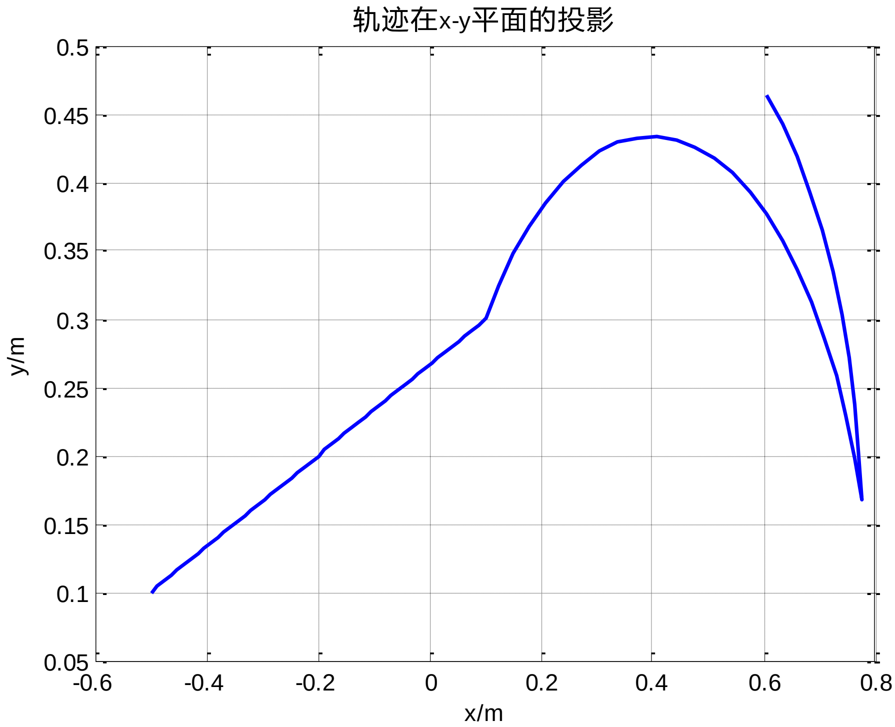
   $\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 4.8311^o \\ 133.4212^o \\ 0.2 \end{bmatrix}$

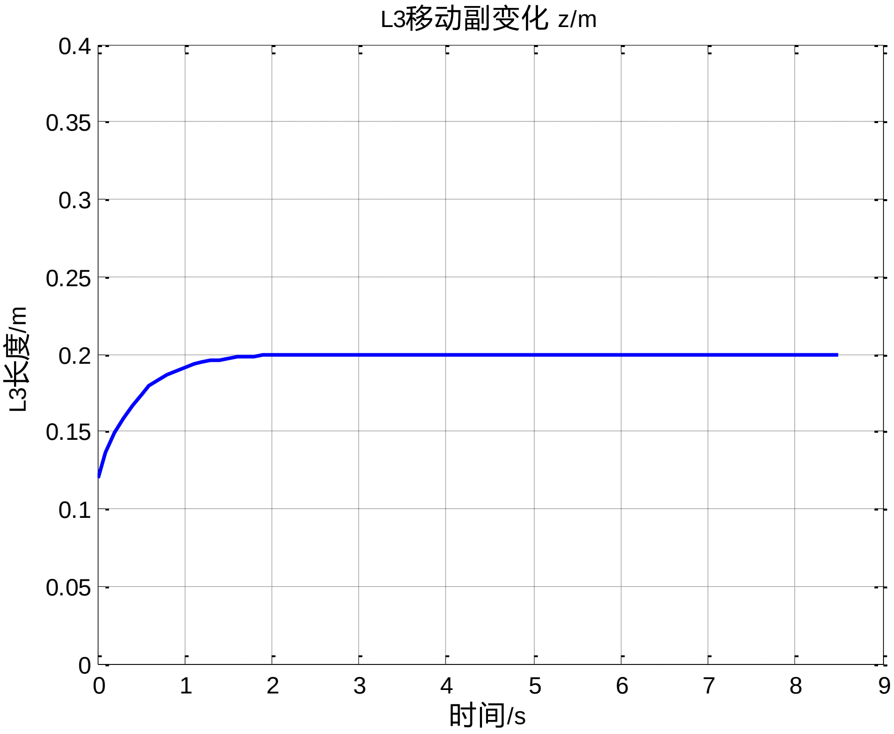
3. 确定焊接线的 x,y 信息，推算出需要的关节转角

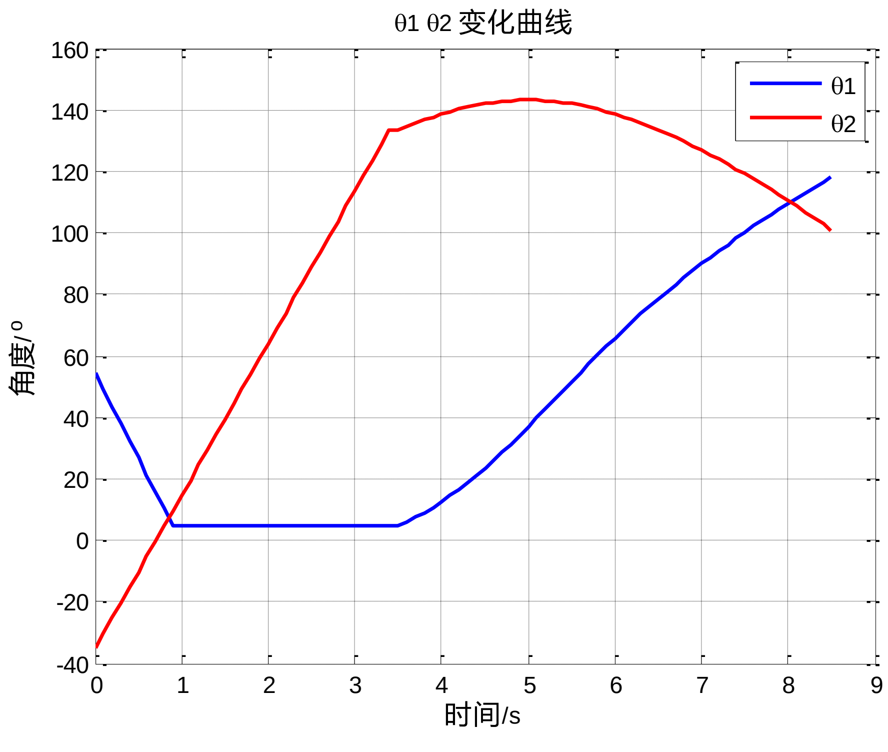
   焊头的行进轨迹为：$\begin{bmatrix} 0.1 \\ 0.3 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -0.5 \\ 0.1 \\ 0 \end{bmatrix}$

假设需要$N=50$个步长焊完全程。则将 x,y 均分为 50 等分，用每一步长的$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$信息去反算$\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix}$信息。即可得到关节转动轨迹。

4. 实验结果如下：




L3移动副变化 z/m
L3长度/m
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9
时间/s


θ1 θ2 变化曲线
θ1
θ2
角度/o
160
140
120
100
80
60
40
20
0
-20
-40
0 1 2 3 4 5 6 7 8 9
时间/s

轨迹符合要求

四、第二个实验，要求耗时最短

1. 确定移动到哪个焊接初始点，耗时最短

目前有两种方案，移动到$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.3 \\ 0 \end{bmatrix}$ 或$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.1 \\ 0 \end{bmatrix}$

那么对应的关节转动的路程为:

$\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix}$: $\begin{bmatrix} 4.8311^o - 60^o \\ 133.4212^o + 40^o \\ 0.1 \end{bmatrix}$ 或者 $\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix}$: $\begin{bmatrix} 118.2835^o - 60^o \\ 100.8098 + 40^o \\ 0.1 \end{bmatrix}$

可以发现，第一种方案，θ_2需要转动 $133.4212^o + 40^o = 173.4212^o$

第二种方案，θ_2只需要转动 $100.8098^o + 40^o = 140.8098^o$

所以第二种方案，耗时最短。$t_1 = \dfrac{140.8098}{50} = 2.8162$

考虑到 0.1s 的仿真步长，取$t_1 = 2.9\ s$

2. 确定焊接时的最大速度

根据书上的公式

$$J\dot{\theta} = t$$

$$\Rightarrow \begin{bmatrix} A \\ B \end{bmatrix} \cdot \dot{\theta} = \begin{bmatrix} \omega \\ \dot{p} \end{bmatrix}$$

只考虑焊头线速度:

$$B \cdot \dot{\theta} = \dot{p}$$

$$\Rightarrow \begin{bmatrix} -L_1 \cdot \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

将 B 取逆，可得:

$$\dot{\theta} = B^{-1} \cdot \dot{p}$$

令$B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$，并考虑到线速度方向$\dot{y} = \dfrac{\dot{x}}{3}$，可以求解出:

$$\dot{\theta}_1 = \dot{x} \cdot \frac{3b_4 - b_2}{3(b_1 b_4 - b_2 b_3)} \qquad \dot{\theta}_2 = \dot{x} \cdot \frac{b_1 - 3b_3}{3(b_1 b_4 - b_2 b_3)}$$

（其中 $-\dfrac{50 \cdot \pi}{180} < \dot{\theta}_1 < \dfrac{50 \cdot \pi}{180}$，$-\dfrac{50 \cdot \pi}{180} < \dot{\theta}_2 < \dfrac{50 \cdot \pi}{180}$）

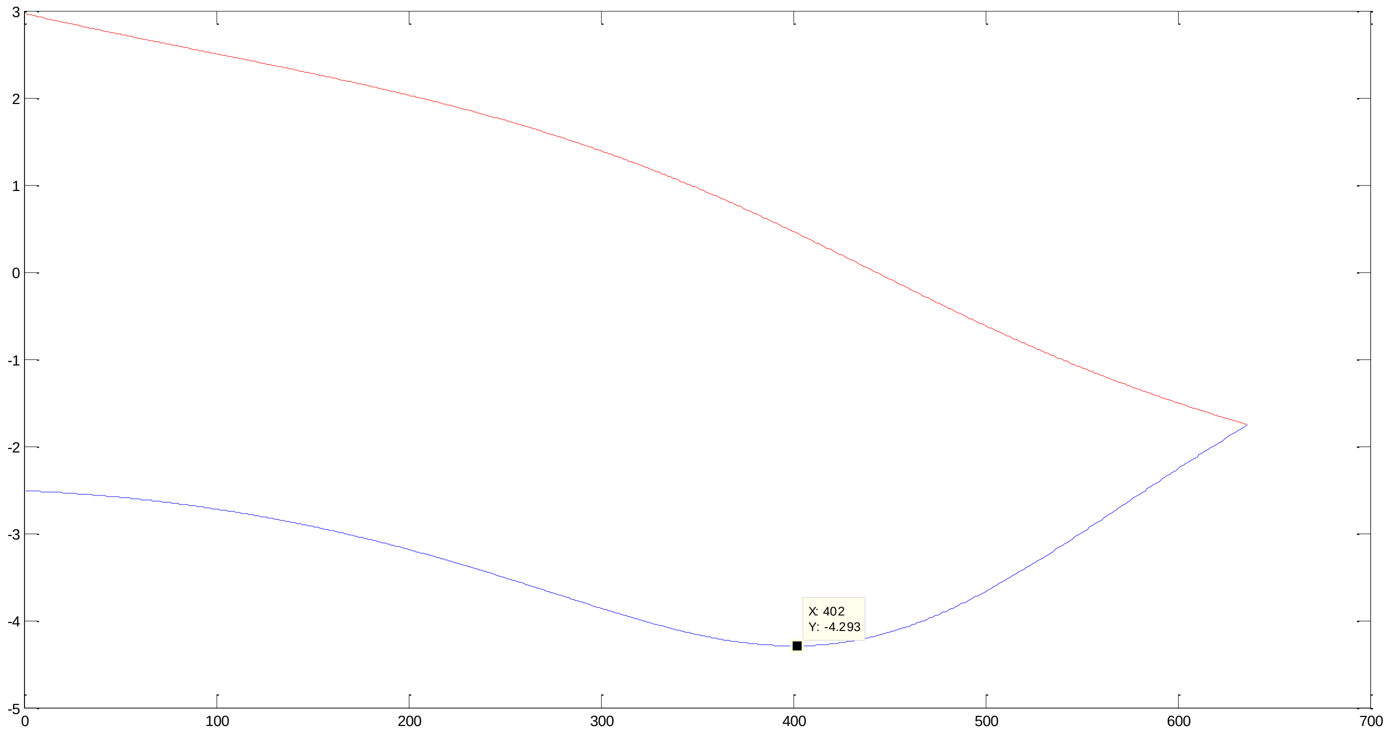
注意到 $\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$ 中带有随时间变化的 $\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$

当 $\begin{bmatrix} x \\ y \end{bmatrix}$ 沿轨迹变化时，$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ 也跟着变化。对应范围如下：

(x,y)：$(-0.5,0.1) \rightarrow (0.1,0.3)$

$\rightleftharpoons (\theta_1,\theta_2)$:$(118.2835^o, 100.8098^o) \rightarrow (4.8311^o, 133.4212^o)$

将 θ_1,θ_2 平均分割，计算对应的 $\dfrac{\dfrac{3b_4-b_2}{3(b_1b_4-b_2b_3)}}{\dfrac{b_1-3b_3}{3(b_1b_4-b_2b_3)}}$ 的值，如下图所示：（蓝色为 $\dfrac{3b_4-b_2}{3(b_1b_4-b_2b_3)}$ 的值）



发现最大处，为-4.293

所以：$\dfrac{-50\cdot\pi}{180} = -0.872665 < \dot{x} \cdot -4.293 < 0.872665$

推出来：$\dot{x} < 0.2033$

用焊接路程除以速度：$\dfrac{0.6}{0.2033} = 2.9513\,\mathrm{s}$

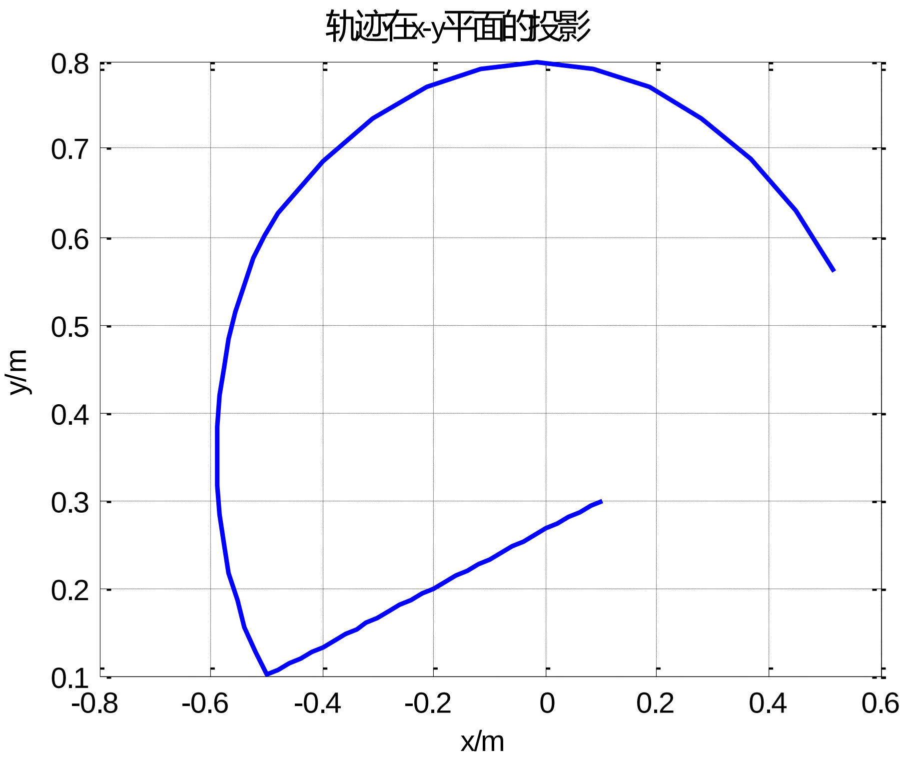
推出来最短的焊接时间为 2.9513s 共 30 个仿真步数

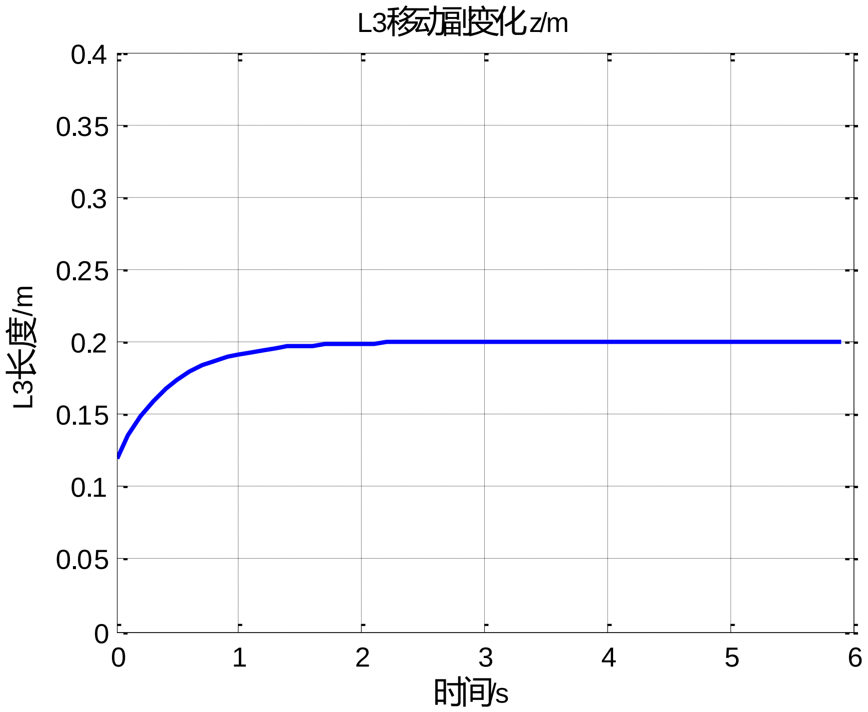
3. 开始第二个实验

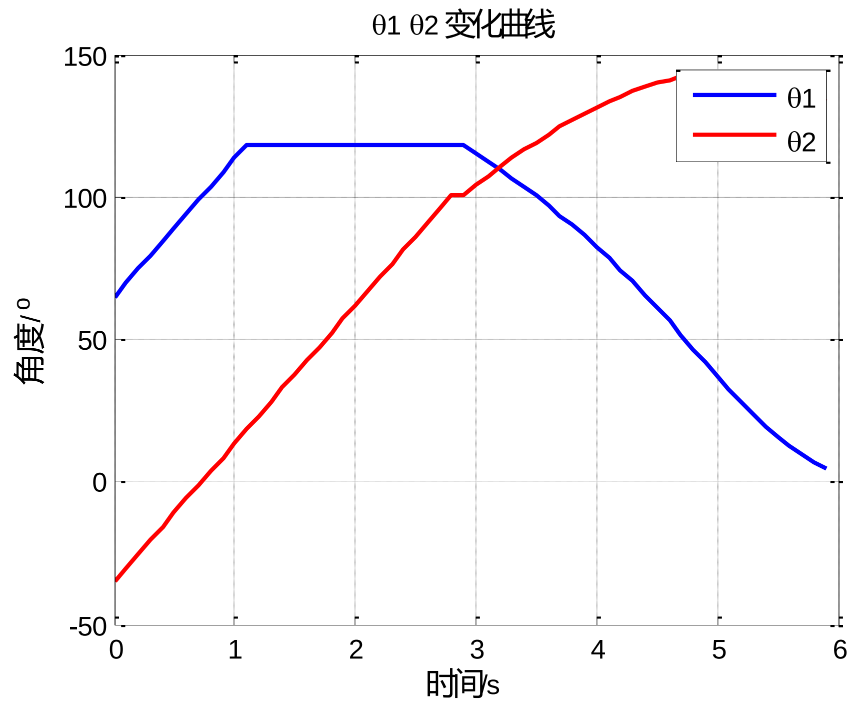
机械臂首先移动到$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.1 \\ 0 \end{bmatrix}$，开始焊接

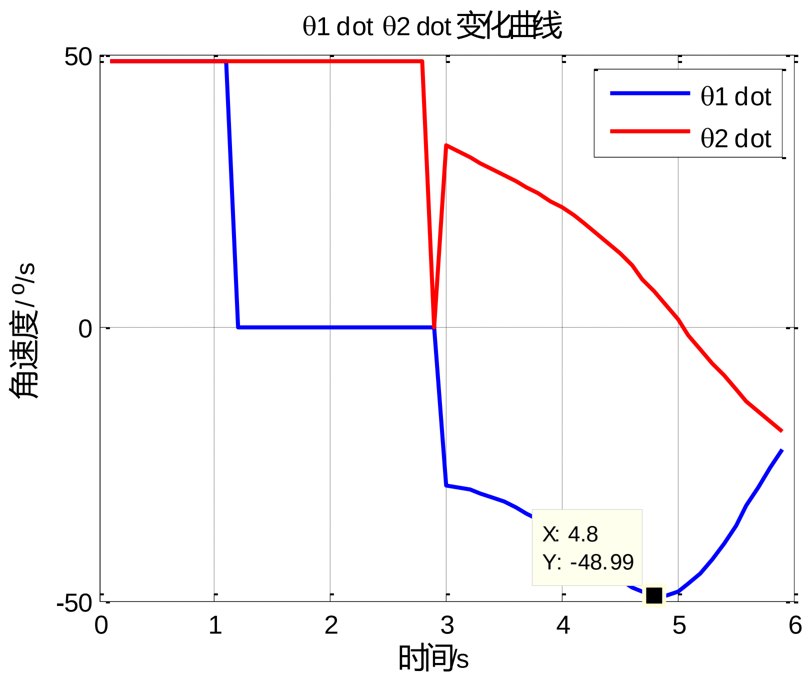
然后以$\dot{x} = 0.2033$，仿真步数为 30 步，进行焊接，直至$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.3 \\ 0 \end{bmatrix}$

实验结果如下



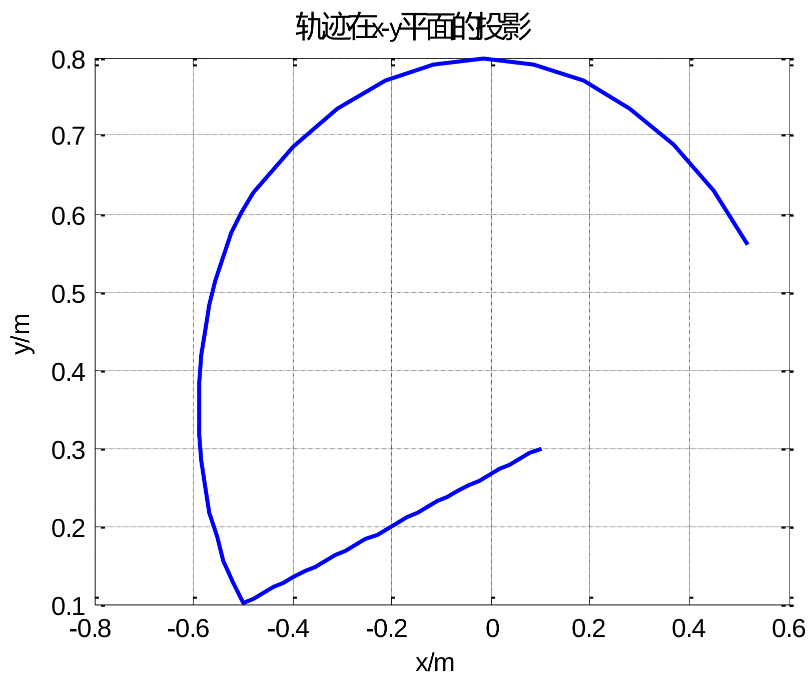

L3移动副变化z/m
L3长度/m
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
时间/s

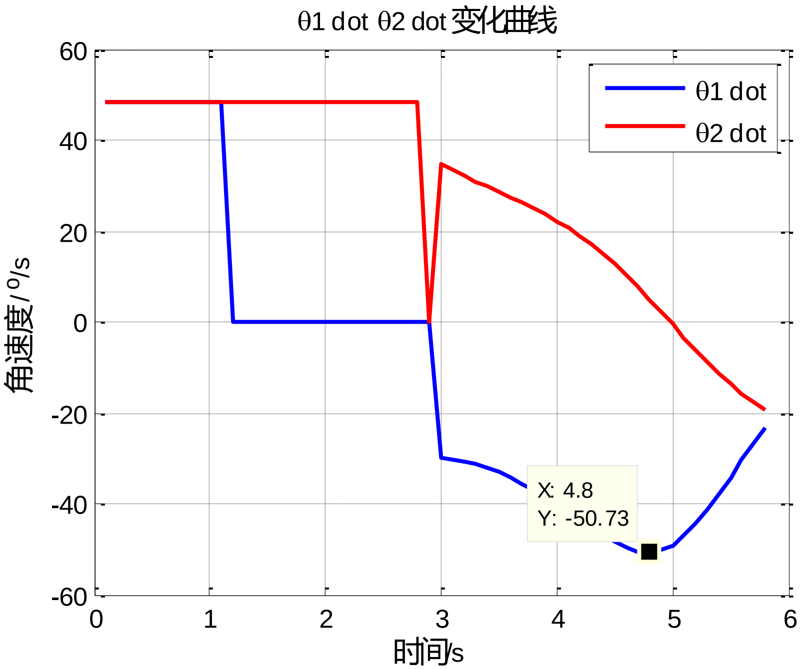

θ1 θ2 变化曲线
角度/°
150
100
50
0
-50
0
1
2
3
4
5
6
时间/s
θ1
θ2



可以看到 $\dot{\theta}_1 = -48.99\ ^o/s$ 快要达到转角速度极限。但也说明此速度并不是最大值。

再次进行试验：取仿真步数为 29 步





可以看到，此时关节转速已经超过了极限，焊接速度不能再大了。所以焊接过程的最短时间为 2.9s

整个过程的最短时间为$t = t_1 + t_2 = 2.9 + 2.9 = 5.8\ s$

用雅各布矩阵求解

一、基本概念

根据书上的公式

$$J\dot{\theta}=t$$

$$\Rightarrow \begin{bmatrix} A \\ B \end{bmatrix}\cdot\dot{\theta}=\begin{bmatrix} \omega \\ \dot{p} \end{bmatrix}$$

只考虑焊头线速度:

$$B\cdot\dot{\theta}=\dot{p}$$

$$\Rightarrow \begin{bmatrix} -L_1\cdot\sin(\theta_1)-L_2\sin(\theta_1+\theta_2) & -L_2\sin(\theta_1+\theta_2) \\ L_1\cos(\theta_1)+L_2\cos(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2) \end{bmatrix}\cdot\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}=\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

将 B 取逆，可得:

$$\dot{\theta}=B^{-1}\cdot\dot{p}$$

那么设置期望的$\dot{p}=\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$就可以反算出对应的关节转速$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$

二、实验一，焊接一条直线

1. 转动到焊接初始位置$\begin{bmatrix} \theta_1 \\ \theta_2 \\ L_3 \end{bmatrix}=\begin{bmatrix} 118.2835^o \\ 100.8098^o \\ 0.2 \end{bmatrix}\Leftrightarrow\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -0.5 \\ 0.1 \\ 0 \end{bmatrix}$

   在上文中，已经有描述，不再赘述

2. 开始沿着既定路线焊接

   首先计算期望的焊头线速度$\dot{p}=\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$

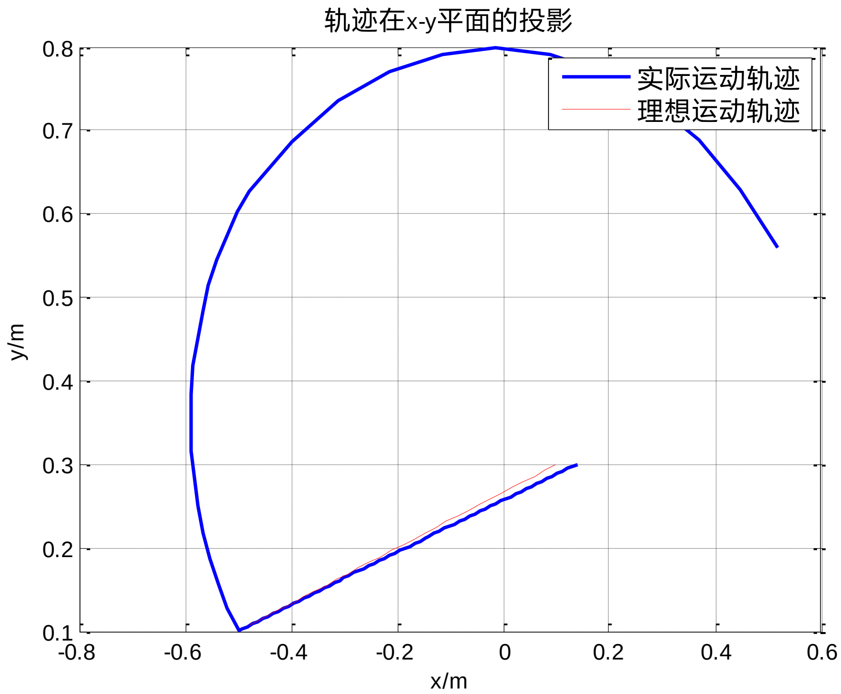
   设焊头线速度$\dot{p}=v$

   则$\begin{cases} \dot{x}=\dfrac{3v}{\sqrt{10}} \\ \dot{y}=\dfrac{v}{\sqrt{10}} \end{cases}$

按照此速度，就可以求解出每时每刻应有的$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$。

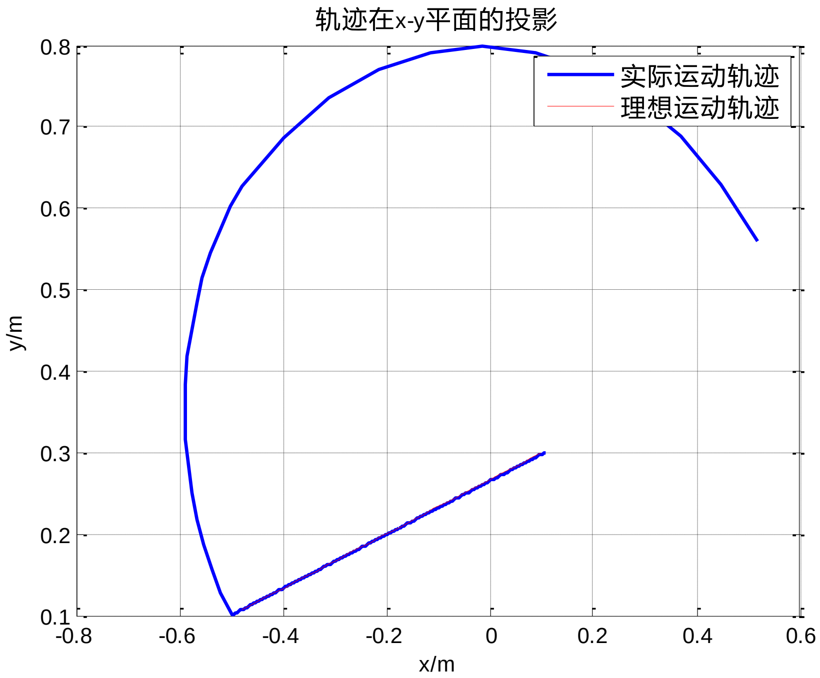
3. 实验结果

设置$\dot{p} = v = 0.1\ m/s^2$



发现路径偏差较大。

如果设置$\dot{p} = v = 0.02\ m/s^2$，实验结果如下：



其路径符合要求。

分析：当焊头线速度较大时，计算出来的关节角速度会保持 0.1s 不变，但在这个过程中，雅各布矩阵已经变化，所以造成了误差。所以只有在低速度情况下，雅各布矩阵计算方法才能实现。

优化：想要实现利用雅各布矩阵控制，那么仿真间隔（控制器周期）就必须尽可能小。我们设置仿真步长为 0.01s，提取计算结果中的每 0.1s 的数据再进行试验

发现相同的速度下$\dot{p}=v=0.1\ m/s^2$时，以 0.01s 的仿真步长计算，轨迹基本没有误差