$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix}$$

red.

A ① dot product of (Row)·x

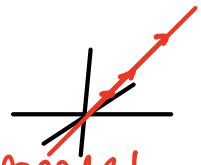
all Ax = column space of $A = C(A)$

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 8 \\ 1 & 3 & 8 \end{bmatrix}$$

$C(A)$ = line

rank = 1

independent columns.



$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 8 \end{bmatrix} = uv^T$$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

basis for row space

$A_{3\times 3} = C_{3\times 2} R_{2\times 3}$

Column rank $r = 2$ = row rank

First Great Thm.

Row space of A

all combinations of the rows.

$= C(A^T)$

column space of A^T

Ax $x = \text{rand}(m, 1)$

in column space of A

$A = CR$ or $A = C\tilde{U}\tilde{R}$ ($\tilde{U}$ is 2×2)

$A(BCx)$

$$AB = \begin{bmatrix} \text{col} \\ k \end{bmatrix} \begin{bmatrix} \text{row } k \end{bmatrix}$$

$$= \text{sum of } (\text{col } k \text{ of } A)(\text{row } k \text{ of } B)$$

outer product

$$= (\text{col } 1)(\text{row } 1) + \dots + (\text{col } n \text{ of } A)(\text{row } n \text{ of } B)$$

$m\times 1 \quad 1\times p$

$(m\times n)(n\times p) = (m\times p)$

$n \to$ multiplications (dot product)

$mp \to$ dot products

mnp mult

$n \times (mp)$