I.2 Factorizations

$A = LU$ — Elimination $lu(A)$

* $A = QR$ — Gram-Schmidt

* $S = Q\Lambda Q^T = \begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} q_1^T \\ \vdots \\ q_n^T \end{bmatrix}$

orthonormal real

$A = X \Lambda X^{-1}$

* $A = U \Sigma V^T$ = (orth.)(diag.)(orth.) = SVD

$S = Q\Lambda Q^T$

$(Q\Lambda)(Q^T)$ = (cols of $Q\Lambda$) × (rows of Q^T)

S = sum of rank 1 $= \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \ldots + \lambda_n q_n q_n^T$

$\underset{n\times 1}{[\;]} \cdot \underset{1\times n}{[\quad]} = \underset{n\times n}{[\quad\;]}$

All columns are multiples of this

All rows are multiples of this

Look at $S \cdot q_1 = \lambda_1 q_1 \underbrace{q_1^T q_1}_{1} + 0 + 0 + \ldots + 0$

$\boxed{S \cdot q_1 = \lambda_1 \cdot q_1}$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ rank=1

$A = LU$

$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} \underline{2} & 3 \\ 0 & \boxed{\underline{1}} \end{bmatrix}$

$A = \underset{L}{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}}$

$A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \underset{\text{rank } 1}{\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}} + \underset{\text{rank } 1}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}$

$LU = \begin{bmatrix} l_1 \end{bmatrix} \begin{bmatrix} u_1^T \end{bmatrix} + \begin{bmatrix} l_2 \end{bmatrix} \begin{bmatrix} u_2^T \end{bmatrix}$

$A = \begin{bmatrix} \; \end{bmatrix}$ = (col. 1)(row 1) + $\begin{bmatrix} 0 & - & 0 \\ | & A_2 & \\ 0 & & \end{bmatrix}$

The Fundamental Theorem of Linear ~~Algebra~~.

4 Fundamental Subspaces.

A $m \times n$ rank r.

column space $C(A)$ dim $= r$

row space $C(A^T)$ dim $= r$

Null space $N(A)$ dim $= n - r$

Null space $N(A^T)$ dim $= m - r$

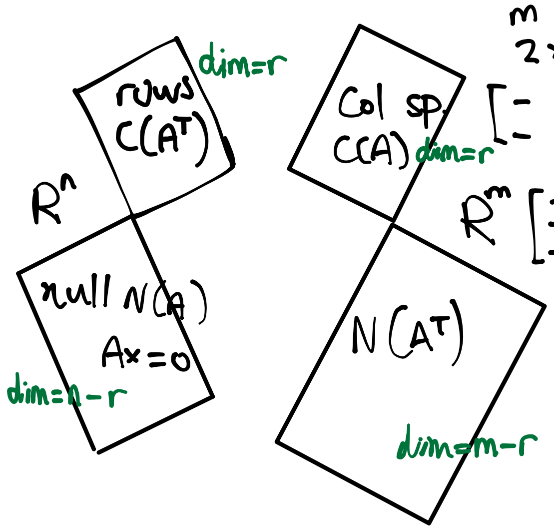
null space = kernel.

All solutions to $Ax = 0$

$A(cx) = 0$

$Ay = 0$

$A(x+y) = 0$



$\underset{m}{2} \times \underset{n}{3}$

$\begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\begin{bmatrix} - \\ - \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$

dim $\underset{\text{row}}{r}$ $\underset{\text{nullsp.}}{n-r}$ $\underset{\text{col.}}{r}$ $m-r$

$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2×3

$m=2$
$n=3$
$r=1$
$n-r=2$

$Ax = \begin{bmatrix} \equiv \end{bmatrix} \begin{bmatrix} | \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$