ACM104 Problem Set #6 Solutions

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Problem 1

See attached `ps6problem1Kuzhagaliyev.m` for the solution.

Problem 2

See attached `ps6problem2Kuzhagaliyev.m` for the solution. The ID numbers of the top 10 airports for different α values can be seen below, in the order of descending importance.

- $\alpha = 0.10$: 6 1 7 3 2 21 11 8 18 10
- $\alpha = 0.15$: 6 7 3 1 2 21 11 8 10 18
- $\alpha = 0.20$: 6 7 3 1 2 21 11 10 8 18

Problem 3

$$F = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

The eigenvalues of F are the solutions of the following equation:

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = 0$$

$$\Downarrow$$

$$\begin{aligned} -\lambda(1-\lambda) - 1 &= 0 \\ \lambda^2 - \lambda - 1 &= 0 \end{aligned}$$

Using the quadratic formula we can find two solutions, which are the eigenvalues of F:

$$\lambda = \frac{1 \pm \sqrt{5}}{2}$$

To find the corresponding eigenvectors we need to solve the equation

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Downarrow$$

$$\begin{aligned} x + y &= \lambda x \\ x &= \lambda y \end{aligned} \quad \Rightarrow \quad x = \frac{1 \pm \sqrt{5}}{2} y$$

Therefore we can pick eigenvectors v_1 and v_2 (corresponding to eigenvalues λ_1 and λ_2) to be:

$$v_1 = \begin{bmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{bmatrix} \quad \text{and} \quad v_2 = \begin{bmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{bmatrix}$$

Finally, we can diagonalise F as follows:

$$F = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{bmatrix}^{-1}$$

Or equivalently:

$$F = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{1-\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{1-\sqrt{5}}{2\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & \frac{1+\sqrt{5}}{2\sqrt{5}} \end{bmatrix}$$

Indeed, when we evaluate this expression we receive the original matrix F.

Problem 4

See attached `ps6problem4Kuzhagaliyev.m` and the relevant plot for the solution.

Problem 5

See attached `ps6problem5Kuzhagaliyev.m` and the relevant plot for the solution.