1. Solutions:
   (a) Test
2. TRIBES function requires that x and y share at least one 1 on every row of the matrix. Solutions:
   (a) The most straightforward way to solve TRIBES(x, y) is for Alice to just send over her entire matrix, which requires n bits, and wait for Bob's answer, giving overall communication complexity $O(n)$. This proves the upper bound for the problem.
   TODO: Show lower bound.
   (b) By definition of the problem, to show that TRIBES$(x, y) = 1$ we only need to find one index j on every row i such that $x_{i,j} = y_{i,j} = 1$. Therefore an all-powerful prover just needs to point out these indices on each row. There are $\sqrt{n}$ rows, and each column index takes $\log_2(\sqrt{n})$ bits to represent, giving us an overall bit count of $\sqrt{n} \cdot \frac{1}{2}\log_2 n$. Alice can send over these values to Bob and wait for him to confirm that there is a match. Hence the upper bound on N^1(TRIBES) is $O(\sqrt{n} \log n)$.

   We can use the fooling set technique to show the lower bound. Consider a $1 \times \sqrt{n}$ vector e_i^T, which has zeros everywhere except the ith position. Consider a matrix x where each row is a vector e_{i_k} for some $i_k \in \{1, \ldots, \sqrt{n}\}$. Clearly, TRIBES$(x, x) = 1$ because x has at least one 1 on each row. At the same time, if we permute any of the rows of x by shifting the 1 in that row left or right and define the new matrix x^*, we'll see that TRIBES$(x, x^*) = 0$ because there is now at least one row where x and x^* do not match up. We can exploit this to generate a fooling set. Let M be the set of matrices that start off as an identity matrix, but have either 1 within some row shifted left or right, or have some rows swapped, or both. Then we can define the fooling set S as:

   $$S = \{((M^*, M^*) : \text{distinct } M^* \in M\}$$

   Clearly, this a 1-fooling set because for any two distinct pairs $(M_1, M_2), (M_1^*, M_2^*) \in S$, neither (M_1, M_2^*) nor (M_1^*, M_2) can be evaluated to 1 since they must differ in at least one place (we can show by contradiction that if this is not the case, the pairs must be identical). Now for the size of this fooling set: there are $\sqrt{n}$ rows in total, and for each row we have $\sqrt{n}$ different choices for the vector e_i^T. This means that $|S| = \sqrt{n}^{\sqrt{n}}$, and we can obtain a lower bound $N^1(\text{TRIBES}) \geq \Omega(\log \left(\sqrt{n}^{\sqrt{n}}\right))$, or, equivalently, $N^1(\text{TRIBES}) \geq \Omega(\sqrt{n} \log n)$.

   Combining the lower and upper bound, we get $N^1(\text{TRIBES}) = \Theta(\sqrt{n} \log n)$.

   (c) To show that TRIBES$(x, y) = 0$, we need to find a single row i where in each position j we have $x_{i,j} \wedge y_{i,j} = 0$. Our all-powerful prover can identify this row and show its index to Alice and Bob. Then, Alice can send Bob a bitmask of said row, taking up $\sqrt{n}$ bits in total. Including the 1 bit of Bob's reply, we get the overall complexity of $O(\sqrt{n})$, proving the upper bound for N^0(TRIBES).

   The lower bound for N^0 can be proved by showing a 0-fooling set of size $2^{\sqrt{n}}$.

3. Solutions:
   (a) A disconnected graph with a clique of size two and any amount of independent vertices. This way, whatever the size of independent set that Alice holds, she only ever needs communicate whether she holds one of the vertices in the clique. If we enumerate all nodes in G as $1, \ldots, n$, the amount of bits exchanged is $2 \log n + 2$ (checking each of the nodes in the clique and waiting for Bob's reply), which gives us complexity $O(\log n)$ overall.
   (b) ‘