# Introduction to Bayesian Inference: Selected Resources

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CASt Summer School

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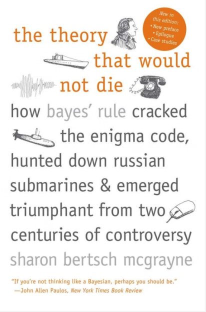
# *Non-technical/semi-technical introductions*

*Ten Great Ideas about Chance (2017)*


Semi-technical survey by a leading statistician/mathematician (Diaconis) and a leading philosopher of science (Skyrms). "This is a history book, a probability book, and a philosophy book."

"To anyone with an interest in probability or statistics, this is a book you must read. . . . [It] is far-ranging and can be read at many levels, from the novice to the expert. It is also thoroughly engaging." —David M. Bressoud, UMAP Journal

# *The Theory That Would Not Die*




While not covering the Bayesian approach from a technical perspective, McGrayne's 2011 book provides a popular-level treatment of the history of Bayesian inference and its place in modern science. This book was a New York Times "Editor's Choice" selection; see the 2011 NYT review, "The Mathematics of Changing Your Mind." For more info about the book, see McGrayne's "The Theory..." web page. The book was timed to come out just before the 250th anniversary of the publication of Bayes's paper presenting a special case of what came to be called Bayes's theorem. McGrayne was the dinner speaker at Bayes 250 Day, a meeting held by the International Society for Bayesian Analysis (ISBA) in 2013 to honor the anniversary.

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## *Stan team resources on Bayesian workflow*

- Bayesian Workflow (arXiv:2011.01808)
- Andrew Gelman's informal video overview in the *Laplace's Demon* webinar series
- Visualization in Bayesian Workflow
- Toward a principled Bayesian workflow in cognitive science
- An Introduction to Bayesian Data Analysis for Cognitive Science See Ch. 7 on Bayesian workflow

## *Astronomy-specific Bayesian workflow recommendations*

- How proper are Bayesian models in the astronomical literature? (Tak$^+$ 2018) — Guidance on priors
- Practical Guidance for Bayesian Inference in Astronomy (Eadie$^+$ 2023)
- Six Maxims of Statistical Acumen for Astronomical Data Analysis (Tak$^+$ 2024) — General astrostatistics guidance, with a healthy dose of Bayesian advice

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# *Books by physicists and astronomers*

- *Probability Theory: The Logic of Science* (PTLOS)
  Edwin T. Jaynes; ed. G. Larry Bretthorst [`http://bayes.wustl.edu/`]
  [Cambridge U. Press]
  Jaynes worked on this book for over 30 years; it was unfinished at his death in 1998, but Bretthorst thankfully assembled the book from his last draft chapters. Provides the best (and lengthiest) coverage of foundations and fundamentals for a physical scientist audience. It dates from before the development of modern computational tools, and is thus not the most practical text.
  See reviews by: Persi Diaconis (theoretical & applied statistics), Anton Garrett (physics), Terry Fine (applied math, philosophy), Will Faris (for AMS).
  Diaconis: "There are many places in which I want to yell at him. He's so full of himself. That's what makes the book so terrific. It's the real thing—the best introduction to Bayesian statistics that I know. Go take a look for yourself."
- *Bayesian Logical Data Analysis for the Physical Sciences, A Comparative Approach with Mathematica Support*
  Phil Gregory [Cambridge U. Press (2010)]
  Could be regarded as a practical companion to PTLOS; adopts similar point of view but focuses on applications, including basic coverage of MCMC. Some comparison with frequentist approaches. Two chapters based on TL's notes.
- *Data Analysis: A Bayesian Tutorial*
  Devinder Sivia, John Skilling [Oxford U. Press (2006)]
  The most accessible book on Bayesian methods by physical scientists; somewhat idiosyncratic coverage of computational methods.

- *Bayesian Probability Theory: Applications in the Physical Sciences*
  Wolfgang von der Linden, Volker Dose, Udo von Toussaint
  [Cambridge U. Press, coming July 2014]
  Authors are highly-regarded pioneers of application of Bayesian methods to problems in plasma physics and other areas. Some weaknesses on theory/fundamental topics, but numerous very good examples from physics.
- *Statistics, Data Mining, and Machine Learning in Astronomy: A Practical Python Guide for the Analysis of Survey Data*
  Zeljko Ivezić, Andrew Connolly, Jacob VanderPlas, Alexander Gray
  [Princeton U. Press]
  Balanced coverage of frequentist and Bayesian methods, mostly in the context of analyzing large survey datasets. Extensive accompanying Python software, datasets, and reproducible analyses.
- *Bayesian Methods for the Physical Sciences*
  Stefano Andreon, Brian Weaver [Springer; authors' site]
  New (2015) book by astronomers, highlighting use of the JAGS probabilistic programming language. See the somewhat mixed review by astronomer David Hogg.
- *Bayesian Models for Astrophysical Data Using R, JAGS, Python, and Stan*
  By statistician Joseph Hilbe and astronomers Rafael de Souza and Emille E. O. Ishida [Cambridge U. Press 2018]

- *Information Theory, Inference, and Learning Algorithms*
  David MacKay [Cambridge U. Press, 2003; free PDF/DJVU at MacKay's site]
  By a physicist-turned-statistician/information theorist. An extremely original and influential account of ideas underlying statistics, machine learning, signal processing, and communication, from a Bayesian viewpoint. A strong emphasis on information theory and coding problems makes it not the most straightforward introduction for a data analyst, yet it has exceptionally clear coverage of model comparison, information-based experimental design, neural networks, Gaussian processes, and Monte Carlo methods (including MCMC).
- *Practical Bayesian Inference: A Primer for Physical Scientists*
  Coryn Bailer-Jones [(CUP 2017)]
  "It describes the Bayesian approach, and explains how this can be used to fit and compare models in a range of problems." Includes R code.
- *Practical Statistics for Astronomers*
  Jasper V. Wall & C. R. Jenkins [(CUP 2012, 2nd edn.)]
  Emphasizes Bayesian methods.
- *Statistics for Astrophysics: Bayesian Methodology*
  Ed. by Didier Fraix-Burnet, Stephane Girard, Julyan Arbel, Jean-Baptiste Marquette [(EDP Sci., 2018)]
  Contains lectures given during the School of Statistics for Astrophysics that took place at Autrans, France, in October 2017.
- *Bayesian Methods in Cosmology*
  Ed. by Michael Hobson et al. [Cambridge U. Press (2010)]
  Chapters by multiple authors and thus with varying quality and notation.

## *Selected Bayesian statistics books*

- *Doing Bayesian Data Analysis*
  John K. Kruschke [author's book site]
  Known as "the dog book," for the illustration of dogs on the cover, it offers an exceptionally clear, thorough, and accessible introduction to Bayesian concepts and computational techniques. I recommend Kruschke's book (and McElreath's, below) to beginning students. Be sure to get the 2nd edn., which switches from BUGS to JAGS and Stan as computational tools. See Kruschke's BDA course web site for supplementary reading.
- *Statistical Rethinking: A Bayesian Course with Examples in R and Stan*
  Richard McElreath []
  Another accessible introduction, at a similar level to Kruschke's book. The treatment is a bit idiosyncratic. McElreath was strongly influenced by Jaynes. The accompanying web site also includes code for PyMC3 and Julia. McElreath has made available lectures based on the book.
- *Bayesian Data Analysis* (BDA)
  Andrew Gelman et al. [CRC Press (3rd edn. 2013)]
  Probably the most influential and widely-used Bayesian text by statisticians. Both broad and deep, including coverage of multilevel modeling, nonparametric Bayes, model testing, and modern computational methods.

- *Bayesian Methods for Data Analysis*
  Bradley Carlin & Thomas Louis [CRC Press (3rd edn. 2008)
  Earlier editions were titled, "Bayes and Empirical Bayes Methods for Data Analysis," reflecting the book's particularly strong coverage of empirical/hierarchical Bayesian modeling (multilevel modeling). See Gelman's comparison of BDA and Carlin & Louis.
- *Handbook of Markov Chain Monte Carlo*
  Ed. by Brooks, Gelman, Jones, Meng [CRC Press (2011)]
  Accessible, authoritative coverage of a wide range of MCMC techniques, including good coverage of output analysis. Selected chapters online, including a must-read intro to MCMC by Charlie Geyer, and an authorative overview of Hamiltonian Monte Carlo (HMC) by Radford Neal.

There are many other excellent Bayesian texts by statisticians; this brief list just scratches the surface.

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# Tools for Computational Bayes

## *Astronomer/Physicist Tools*

- **mc3** `https://github.com/pcubillos/mc3`
  Multi-Core Markov-Chain Monte Carlo: A multi-method package spearheaded by Joe Harrington, with Patricio Cubillos as main developer (TL consulting). Implments optimization, MCMC by Metropolis random walk or differential evolution, and nested sampling (using `dynesty`).
- **CosmoMC** `http://cosmologist.info/cosmomc/`
  Parameter estimation for cosmological models using CMB, etc., via MCMC
- **DNest4** `https://github.com/eggplantbren/DNest4`
  Posterior sampling and marginal likelihoods via diffusive nested sampling; see arXiv:1606.03757
- **MultiNest** `http://ccpforge.cse.rl.ac.uk/gf/project/multinest/`
  Bayesian inference via an approximate implementation of the nested sampling algorithm
- **PolyChord** `https://ccpforge.cse.rl.ac.uk/gf/project/polychord/`
  "Next generation" nested sampling
- **AstroML** `http://www.astroml.org/`
  Python package supporting machine learning and statistical inference for analyzing astronomical data. Built in part to support the book, "Statistics, Data Mining, and Machine Learning in Astronomy;" it includes modules supporting Bayesian calculations from the book. Not actively maintained.

- **Cobaya** `https://cobaya.readthedocs.io/`
  General purpose Bayesian analysis framework; includes interfaces to cosmological Boltzmann codes and likelihoods; see arXiv:2005.05290
- **dynesty** `https://github.com/joshspeagle/dynesty`
  Dynamic nested sampling; see arXiv:1704.03459
- **emcee** `http://dan.iel.fm/emcee/`
  Python implementation of an ensemble MCMC sampler (no diagnostics—be sure to find them elsewhere!)
- **extreme-deconvolution**
  `http://code.google.com/p/extreme-deconvolution/`
  Multivariate density estimation with measurement error, via a multivariate normal finite mixture model; partly Bayesian; Python & IDL wrappers
- **George, celerite2, tinygp** `https://george.readthedocs.io/`, `https://celerite2.readthedocs.io/`, `https://github.com/dfm/tinygp`
  Fast Gaussian process implementations, for nonparametric Bayesian regression.
- **ExoFit** `http://www.homepages.ucl.ac.uk/~ucapola/exofit.html`
  Adaptive MCMC for fitting exoplanet RV data
- **XSpec** `http://heasarc.nasa.gov/xanadu/xspec/`
  Includes some basic MCMC capability
- **CIAO/Sherpa** `http://cxc.harvard.edu/sherpa/`
  On/off marginal likelihood support, and Bayesian Low-Count X-ray Spectral (BLoCXS) analysis via MCMC via the **pyblocxs** extension
  `https://github.com/brefsdal/pyblocxs`
- **root/RooStats** `https://twiki.cern.ch/twiki/bin/view/RooStats/WebHome`
  Statistical tools for particle physicists; Bayesian support being incorporated

- **CDF Bayesian Limit Software**
  http://www-cdf.fnal.gov/physics/statistics/statistics_software.html
  Limits for Poisson counting processes, with background & efficiency uncertainties
- **CUBA** http://www.feynarts.de/cuba/
  Multidimensional integration via adaptive cubature, adaptive importance sampling & stratification, and QMC (C/C++, Fortran, and Mathematica; R interface also via 3rd-party R2Cuba)
- **Cubature** http://ab-initio.mit.edu/wiki/index.php/Cubature
  Subregion-adaptive cubature in C, with a 3rd-party R interface; intended for low dimensions ($< 7$)
- **APEMoST** http://apemost.sourceforge.net/doc/
  Automated Parameter Estimation and Model Selection Toolkit in C, a general-purpose MCMC environment that includes parallel computing support via MPI; motivated by asteroseismology problems
- **SuperBayeS** http://www.superbayes.org/
  Bayesian exploration of supersymmetric theories in particle physics using the MultiNest algorithm; includes a MATLAB GUI for plotting
- **Inference** Forthcoming at https://tloredo.github.io/
  Python package targeting statistical inference problems arising in the physical sciences; several self-contained Bayesian modules; Parametric Inference Engine
- **BIE** http://www.astro.umass.edu/~weinberg/BIE/
  C++ Bayesian Inference Engine (ca. 2010); earliest large-scale astro framework

## *Probabilistic programming languages & libraries*

- **Stan** `http://mc-stan.org/`
  Budding successor to BUGS/JAGS, with a similar modeling language based on describing a generative model via conditional distributions for parameters and data; compiles models to C++; uses Hamiltonian Monte Carlo for posterior sampling, supported by automatic differentiation of models
- **Turing.jl** `https://turing.ml/stable/`
  Probabilistic programming language for Julia, achieving near-C speed with a high-level programming language interface
- **JAGS** `http://www-fis.iarc.fr/~martyn/software/jags/`
  "Just Another Gibbs Sampler;" MCMC, esp. for Bayesian hierarchical models
- **BUGS/WinBUGS** `http://www.mrc-bsu.cam.ac.uk/bugs/`
  Bayesian Inference Using Gibbs Sampling: Flexible software for the Bayesian analysis of complex statistical models using MCMC
- **OpenBUGS** `http://mathstat.helsinki.fi/openbugs/`
  BUGS on Windows and Linux, and from inside the R
- **XLisp-stat** `http://www.stat.uiowa.edu/~luke/xls/xlsinfo/xlsinfo.html`
  Lisp-based data analysis environment, with an emphasis on providing a framework for exploring the use of dynamic graphical methods; apparently abandoned, but influential for R's development
- **ReBEL** `https://github.com/SaeedKeshavarzi/ReBEL`
  Library supporting recursive Bayesian estimation in Matlab (Kalman filter, particle filters, sequential Monte Carlo).

## *Python*

- **CmdStanPy** https://mc-stan.org/cmdstanpy/
  Python interface to the Stan probabilistic programming language, for partly automated posterior sampling for graphical (hierarchical) models. See also PyStan for an alternative interface to a subset of Stan's features.
- **PyMC** https://www.pymc.io/welcome.html
  A framework for MCMC via Metropolis-Hastings; also implements Kalman filters and Gaussian processes. Originally targeted biometrics, but is general. Includes output analysis tools.
- **rpy2** http://rpy2.readthedocs.io/
  Call R from Python; see the CRAN Bayesian Task View for Bayesian resources. Also see **RSPython** https://web.archive.org/web/20151130002540/http://www.omegahat.org/RSPython, with bi-directional communication between Python and R (abandoned?)
- See also machine learning libraries that support Bayesian machine learning, such as PyTorch and TensorFlow, and associated packages particularly associated packages such as NumPyro, GPyTorch (for Gaussian processes), and Keras.

# *R packages and interfaces*

- **CRAN Bayesian task view**
  `http://cran.r-project.org/web/views/Bayesian.html`
  Overview of many R packages implementing various Bayesian models and methods; pedagogical packages; packages linking R to other Bayesian software (BUGS, JAGS)
- **BOA** `http://www.public-health.uiowa.edu/boa/`
  Bayesian Output Analysis: Convergence diagnostics and statistical and graphical analysis of MCMC output; can read BUGS output files.
- **CODA** `http://www.mrc-bsu.cam.ac.uk/bugs/documentation/coda03/cdaman03.html`
  Convergence Diagnosis and Output Analysis: Menu-driven R/S plugins for analyzing BUGS output
- **LearnBayes**
  `http://cran.r-project.org/web/packages/LearnBayes/index.html`
  Companion software for the introductory book, *Bayesian Computation With R* by Jim Albert
- **R2Cuba** `http://w3.jouy.inra.fr/unites/miaj/public/logiciels/R2Cuba/welcome.html`
  R interface to Thomas Hahn's `Cuba` library (see above) for deterministic and Monte Carlo cubature
- **rpy2** `http://rpy.sourceforge.net/rpy2.html`
  Provides access to R from Python; see also **PypeR** (`http://www.webarray.org/softwares/PypeR/`) for an alternative interface relying on pipes, with simpler installation requirements but less efficiency

## *C/C++/Fortran*

- **BayeSys 3** `http://www.inference.phy.cam.ac.uk/bayesys/`
  Sophisticated suite of MCMC samplers including transdimensional capability, by the author of MemSys
- **fbm** `http://www.cs.utoronto.ca/~radford/fbm.software.html`
  Flexible Bayesian Modeling: MCMC for simple Bayes, nonparametric Bayesian regression and classification models based on neural networks and Gaussian processes, and Bayesian density estimation and clustering using mixture models and Dirichlet diffusion trees
- **BayesPack, DCUHRE**
  `http://www.sci.wsu.edu/math/faculty/genz/homepage`
  Adaptive quadrature, randomized quadrature, Monte Carlo integration
- **CUDAHM** `https://github.com/tloredo/CUDAHM-Paper1v2`
  C++ framework for accelerating hierarchical Bayesian methods (by astronomers Brandon Kelly, Tamas Budavari, TL); not actively developed; see arXiv:2105.08026
- **BIE**, **CDF Bayesian limits**, **CUBA** (see above)

## *Java*

- **Hydra** `https://www.jstatsoft.org/article/view/v007i04`
  HYDRA provides methods for implementing MCMC samplers using Metropolis, Metropolis-Hastings, Gibbs methods. In addition, it provides classes implementing several unique adaptive and multiple chain/parallel MCMC methods. (Appears abandoned.)
- **YADAS** `https://github.com/gertvv/yadas`
  Software system for statistical analysis using MCMC, based on the multi-parameter Metropolis-Hastings algorithm (rather than parameter-at-a-time Gibbs sampling); appears abandoned as of 2008
- **Omega-hat** `http://www.omegahat.net/`
  Java environment for statistical computing, being developed by XLisp-stat and R developers; appears abandoned as of 2011