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CMSC 341

Homework 2

10/2/2014

#1

#1 $2^n > n^2$ for every positive integer n when $n >= 5$

Base case: $n = 5$ $\quad 2^{(5)} > (5)^2 \quad 32 > 25$ ✓

Induction: prove: $2^{(n+1)} > (n+1)^2 \qquad 2 \cdot 2^n > n^2 + 2n + 1$

$2^{n+1} = 2 \cdot 2^n > 2 \cdot n^2 > (n+1)^2$ when $n >= 5$

$> 2n^2 = n^2 + n^2$

$n^2 + n^2 > n^2 + 2n + 1 \qquad n^2 + 2n + 1 = (n+1)^2$

$\boxed{\text{Therefore: } 2^{(n+1)} > (n+1)^2}$

Therefore: $2^n > n^2$ is true for every positive integer n that when $n >= 5$ by induction.

$1 \cdot 2 \cdot 3$

#2 $\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$

Base case: $(1)^2 = \frac{1}{6}(1)((1)+1)(2(1)+1)$

n=1

$1=1$ ✓

induction: assume: $i^2 = \frac{1}{6} i(i+1)(2i+1)$

prove: $\sum_{i=1}^{n} (i+1)^2 = \frac{1}{6}(i+1)((i+1)+1)(2(i+1)+1)$

$$\sum_{i=1}^{n} i^2 + (i+1)^2 = \frac{1}{6} i(i+1)(2i+1) + (i+1)^2$$

Left Hand side $= \frac{i(i+1)(2i+1)+6(i+1)^2}{6}$

$$= \frac{(i+1)[i(2i+1)+6(i+1)]}{6}$$

$$= \frac{(i+1)[(2i^2+i)+(6i+6)]}{6}$$

$$= \frac{(i+1)(2i^2+7i+6)}{6}$$

$$= \frac{(i+1)(i+2)(2i+3)}{6}$$ ✓

Right Hand side:

$$\frac{1}{6}(i+1)((i+1)+1)(2(i+1)+1)$$

$$\frac{(i+1)(i+2)(2i+3)}{6}$$ ✓

Therefore, $\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$ is true for every positive integer n by induction.

$$\boxed{\frac{(i+1)(i+2)(2i+3)}{6} = \frac{(i+1)(i+2)(2i+3}{6}}$$

#2

Therefore, $\cancel{\frac{1}{1*2}} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

is true for all positive n,

by mathematical induction.

#3

#3 $\frac{1}{1*2} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Base case: $n=1$ $\quad \frac{1}{(1)((1)+1)} = \frac{1}{(1)+1} \quad \frac{1}{2} = \frac{1}{2}$ ✓

assume: $\frac{1}{1*2} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

~~Left Hand side~~ Prove: $\frac{1}{1*2} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{(k+1)}{(k+1)+1}$

Left hand side:

$$\frac{1}{1*2} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$\frac{k\cdot(k+2)}{(k+1)\cdot(k+2)} + \frac{1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)\cancel{(k+1)}}{\cancel{(k+1)}(k+2)}$$

$= \frac{k+1}{k+2}$ ← left hand $\qquad$ Right Hand → $\frac{(k+1)}{(k+1)+1} = \frac{k+1}{k+2}$

$$\boxed{\frac{k+1}{k+2} = \frac{k+1}{k+2}}$$

Therefore, $\frac{1}{1*2} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ is true for all positive n, by mathematical induction.

#4

Code Complexity

~~Sum~~ #4	Cost#	# of times	Total
sum = 0;	1	1	1
for (i=0; i<n; i++)	1(int i) + 1(<) + 1(++)	$1 + 2(n+1)$	$2n+3$
for (j=0; j<n; j++)	n· (1(j=0)+1+1)	$(1 + 2(n+1))n$	$n(2n+3)$
++sum;	1	$1 \cdot n \cdot n$	n^2
Final $1+2n+3+n(2n+3)+n^2$			$3n^2+5n+4$ $O(n^2)$

#5

5

#5

sum = 0;	1	1	1
for(i = 0; i < n; i += 2)	1+1+1	$1 + n + 1 + n/2 + 1$	$\frac{n}{2} + n + 3$
for(j = 0; j < n; j++)	1+1+1	$(1 + 2(n+1)) \cdot \frac{n}{2}$	$\frac{3n + 3n}{2}$
++sum	1	$1 \cdot n \cdot n/2$	$\frac{n^2}{2}$

Final: $1 + \frac{n}{2} + n + 3 + \frac{3n + 3n}{2} + \frac{n^2}{2}$

$\frac{n^2 + 7n}{2} + n + 4$

$O(n^2)$

#6

#6

			$O(n\log(n))$
sum = 0;	1	1	1
for(i=0; i<n; i*=2)	1+1+1	$1 + n + \log(n)+1$	$n+\log(n)+3$
for(j=0; j<n; j++)	1+1+1	$(2n+3)\cdot\log(n)$	$2n\log(n)+3\log(n)$
++sum	1	$1\cdot n\cdot\log(n)$	$n\log(n)$
Final; $n\log(n)+2n\log(n)+3\log(n)+\log(n)+n+$		$3n\log(n)+\log(n)+n+4$	
			$O(n\log(n))$

#7

#7

	cost	# of times	Total
sum = 0;	1	1	1
for (i = 0; i < n; i++)	1+2+1	$2n+3$	$2n+3$
for (j = 0; j < i * i; j++)	1+1+1	$1+n^2+1+n+1$	n^2+n+3
for (k = 0; k < j; k++)	1+1+1	$n^2(1+n^2+1+n+1)$	$(n^4+n^3+3)n^2$
++sum	1	$1 \cdot n^2 \cdot n^2$	n^4

Final: $1+2n+3+n^2+n+3+n^4+n^3+3n^2+n^4$ $2n^4+n^3+4n^2+3n+7$

$O(n^4)$