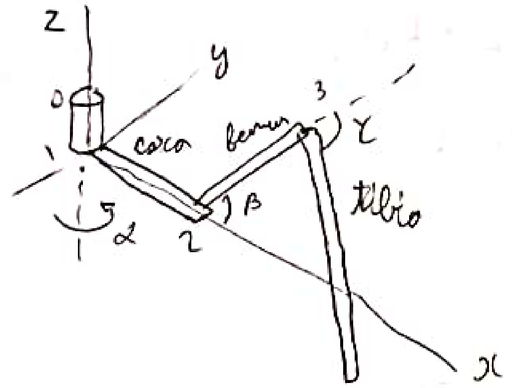


$$T_{01} = \begin{bmatrix} C\alpha & -S\alpha & 0 & 0 \\ S\alpha & C\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{23} = \begin{bmatrix} C\gamma & 0 & -S\gamma & L_2 \\ 0 & 1 & 0 & 0 \\ S\gamma & 0 & C\gamma & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} C_\beta & 0 & -S_\beta & L_1 \\ 0 & 1 & 0 & 0 \\ S_\beta & 0 & C_\beta & b \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{3\text{-}ee} = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{0\text{-}ee} = T_{01}\, T_{12}\, T_{23}\, T_{3ee}$$

This will be of the form $\begin{bmatrix} - & - & - & x \\ - & - & - & y \\ - & - & - & z \\ - & - & - & - \end{bmatrix}$

On Multiplying, we get

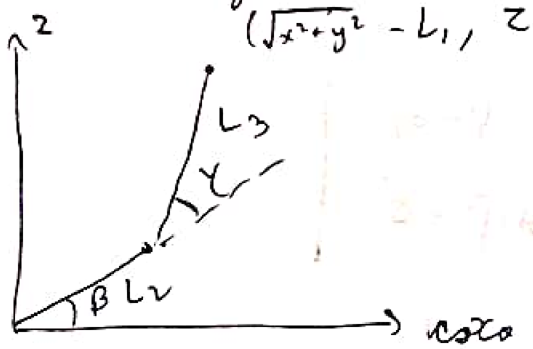
$$x = C\alpha\,[L_3 \cos(\beta+\gamma) + L_2 C_\beta + \cancel{b}\,L_1]$$

$$y = S\alpha\,[L_3 \cos(\beta+\gamma) + L_2 C_\beta + \cancel{b}\,L_1]$$

$$z = L_3 \sin(\beta+\gamma) + \cancel{b}\,L_2 S_\beta$$

We get $\alpha = \tan^{-1}\left(\frac{y}{x}\right)$ easily.

Then taking the plane containing z-axis and coxa, $(\sqrt{x^2+y^2} - L_1,\ z)$



Let $r = \sqrt{x^2+y^2} - L_1$

Using vector addition,

$$L_2^2 + L_3^2 + 2L_2L_3\cos\gamma = r^2 + z^2$$

$$\therefore \gamma = \cos^{-1}\left(\frac{r^2+z^2-L_2^2-L_3^2}{2L_2L_3}\right)$$

Also from the graph,

$$L_2\cos\beta + L_3\cos(\beta+\gamma) = r$$

$$L_2\sin\beta + L_3\sin(\beta+\gamma) = z$$

On expanding and solving, we get

$$\beta = \sin^{-1}\left(\frac{[(L_2 + L_3 \cos \gamma) \times z] - [L_3 \times \sin \gamma \times (\sqrt{x^2+y^2} - L_1)]}{(L_2 + L_3 \cos \gamma)^2 + (L_3 \sin \gamma)^2}\right)$$

Hence, we get all the joint angles.