České vysoké učení technické v Praze

Fakulta informačních technologií

Katedra teoretické informatiky

Evropský sociální fond
Praha & EU: Investujeme do vaší budoucnosti

MI-PDD – Data Preprocessing module (2011/2012)

Lecture 8: Feature extraction from time series

Pavel Kordík, FIT, Czech Technical University in Prague

Feature Extraction from signals

- Time series
  - Industry, signals from sensors
  - Biological data
  - Financial time series
  - Speech
  - Music
  - ...
- Why we need to describe time series by features?

Examples of time series

A. Far-infrared laser excitation
B. Sleep Apnea
C. Currency exchange rates
D. Particle driven in nonlinear multiple well potentials
E. Variable star data
F. J. S. Bach fugue notes

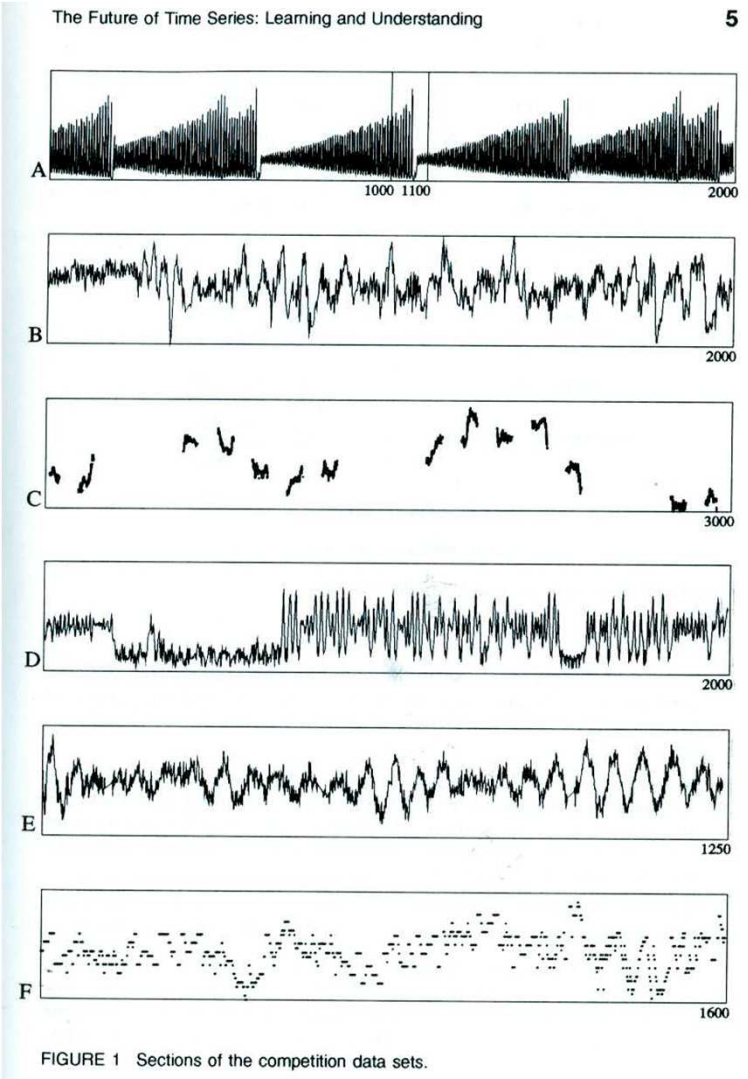
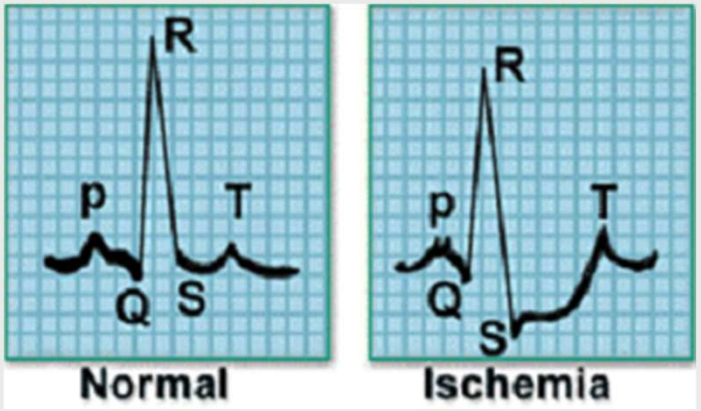


FIGURE 1 Sections of the competition data sets.

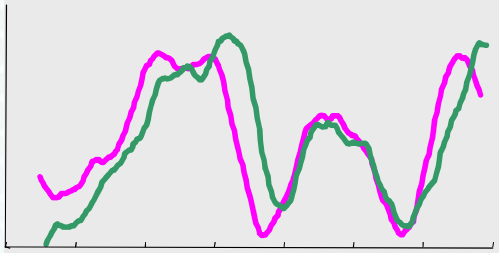
Time series data mining tasks

Clustering

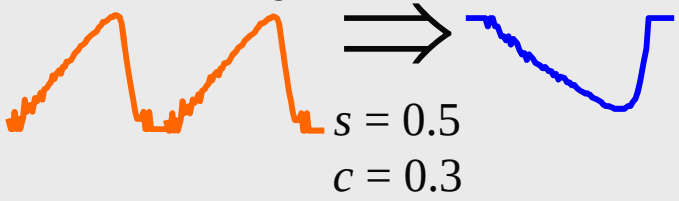
Classification



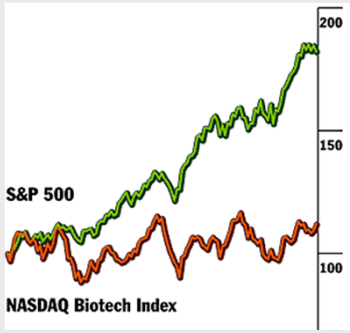
Motif Discovery



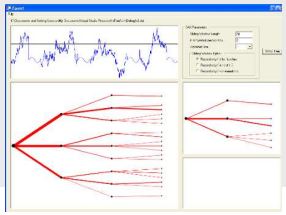
Rule Discovery



Query by Content

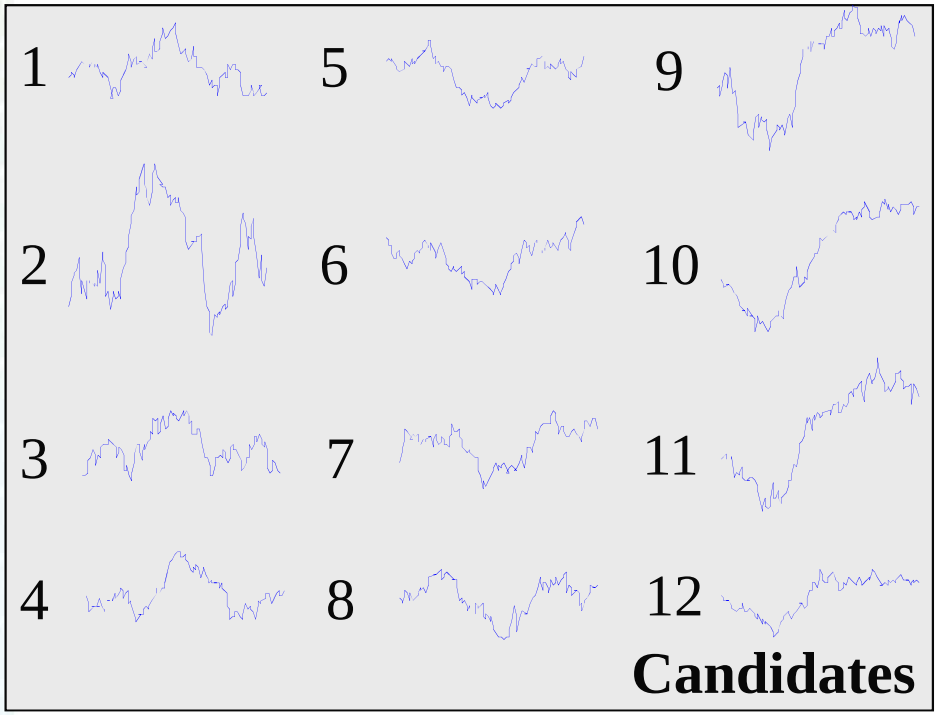


Visualization



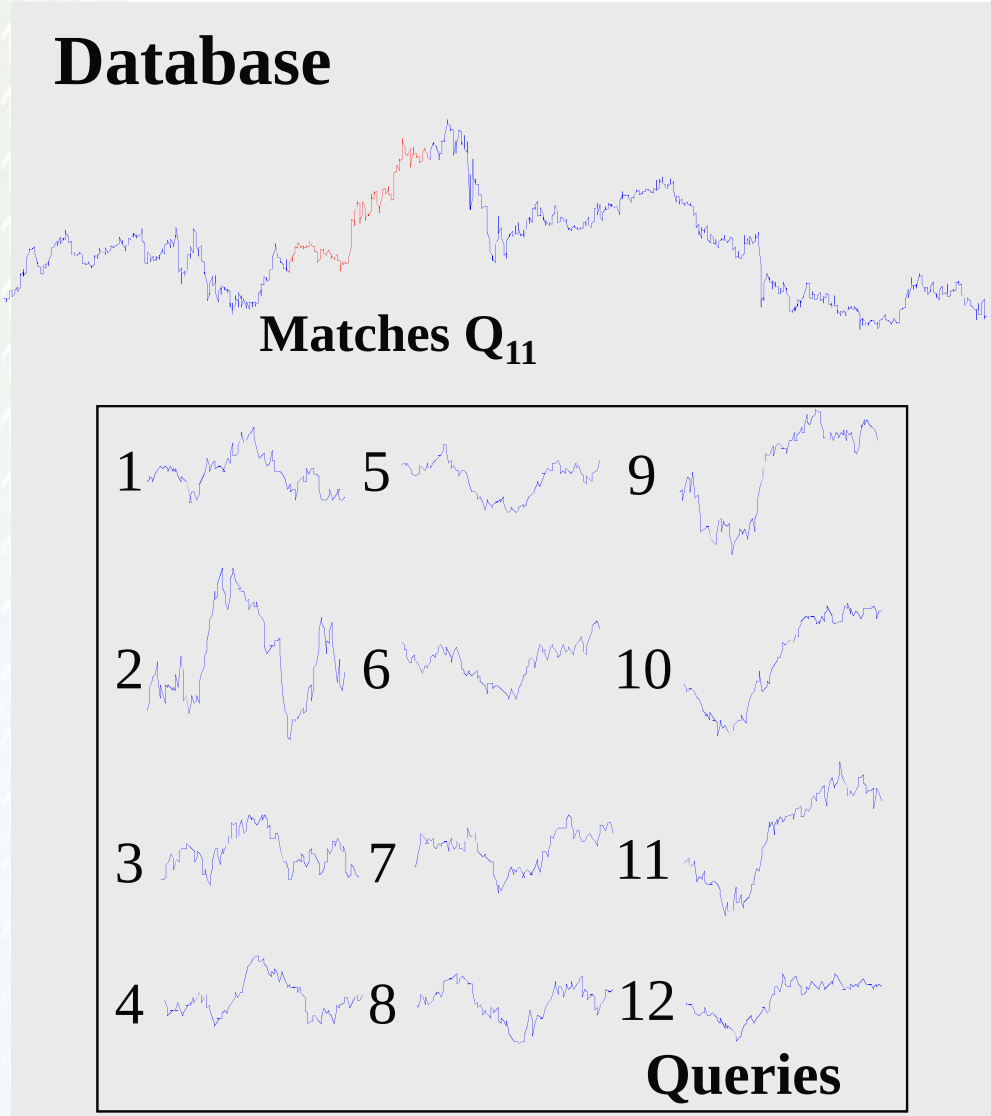
Novelty Detection

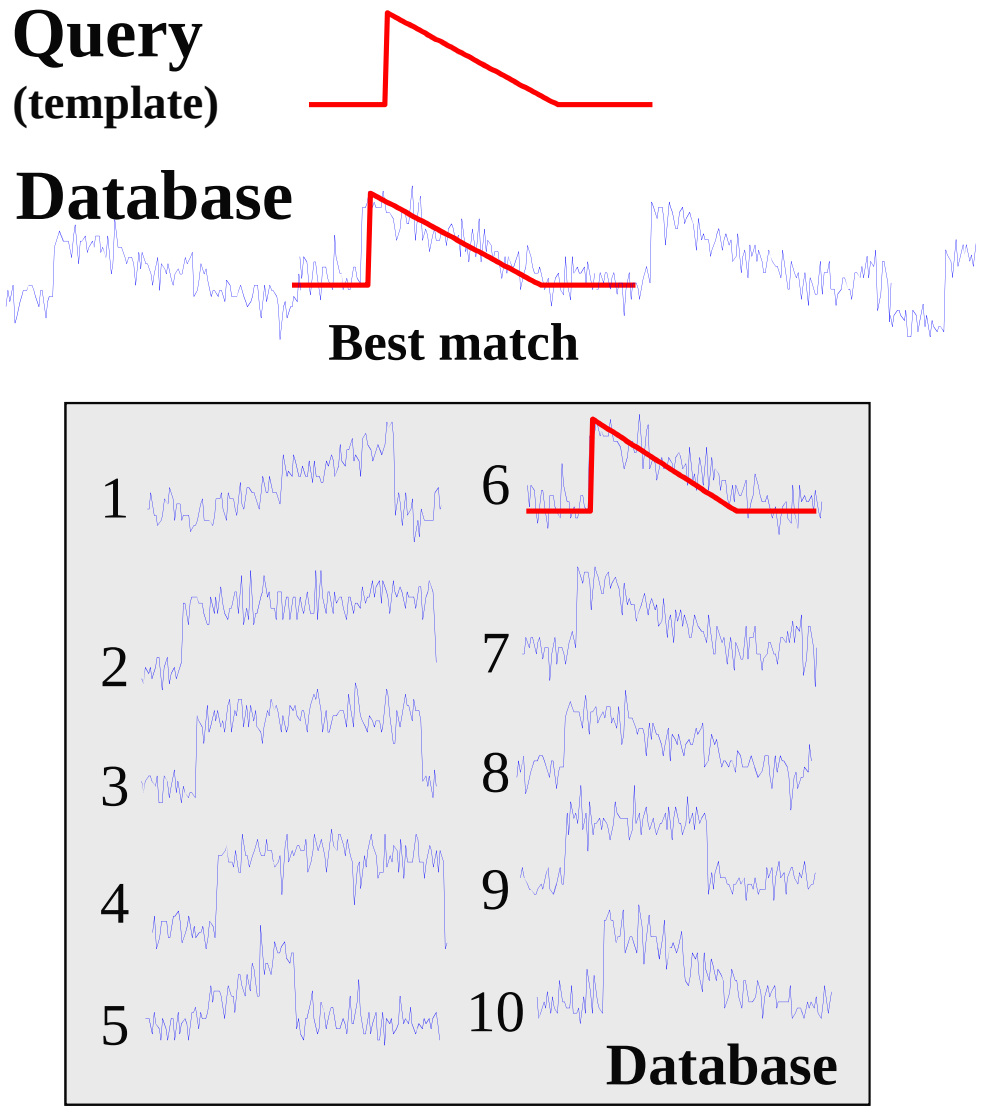
Time Series Filtering



Given a Time Series ***T***, a set of Candidates ***C*** and a distance threshold ***r***, find all subsequences in ***T*** that are within ***r*** distance to any of the candidates in ***C***.

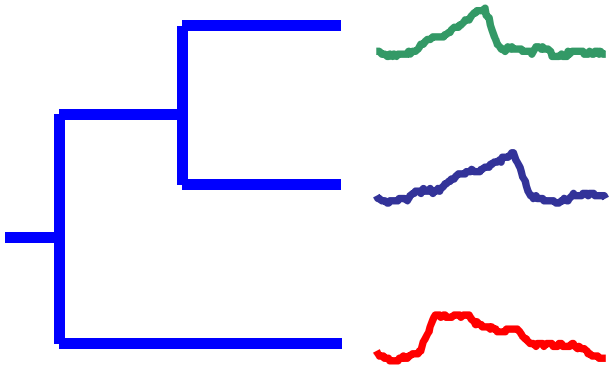
Filtering vs. Querying



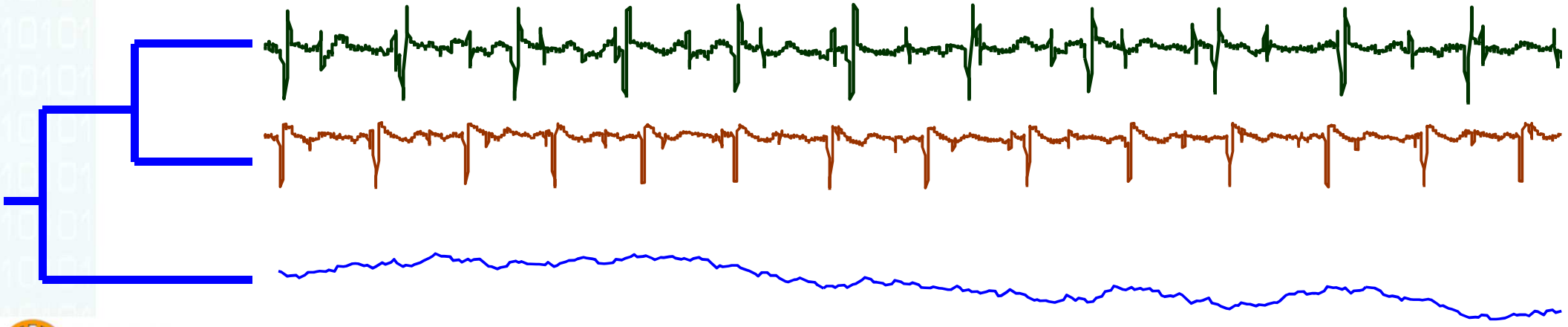


Time series similarity

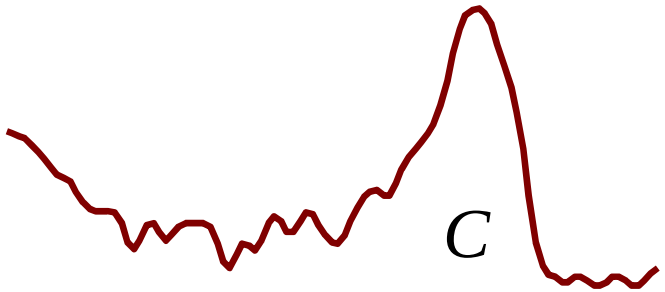
Similarity at the level of *shape*

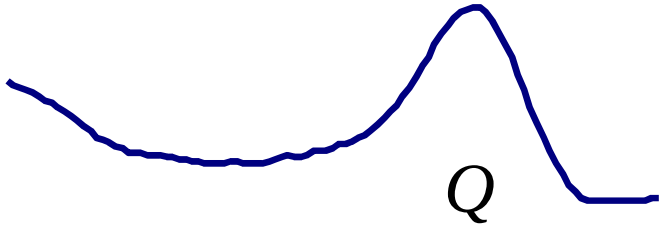


Similarity at the *structural* level



Distance of two signals



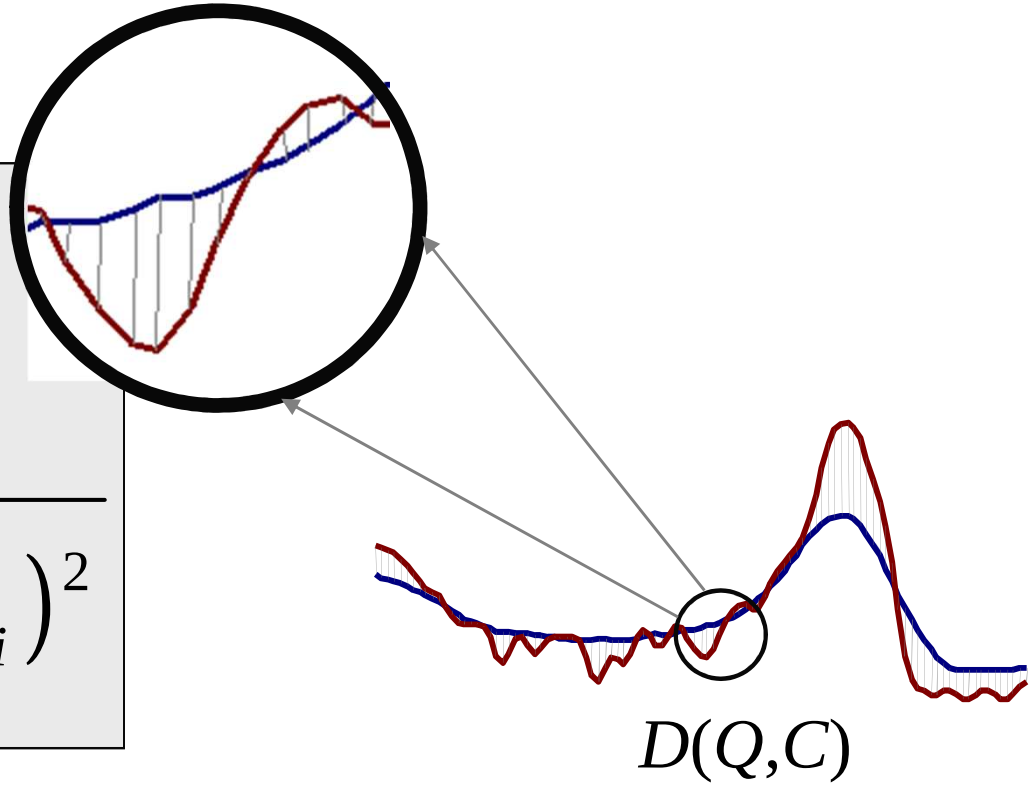


Given two time series:

$$Q = q_1 \ldots q_n$$

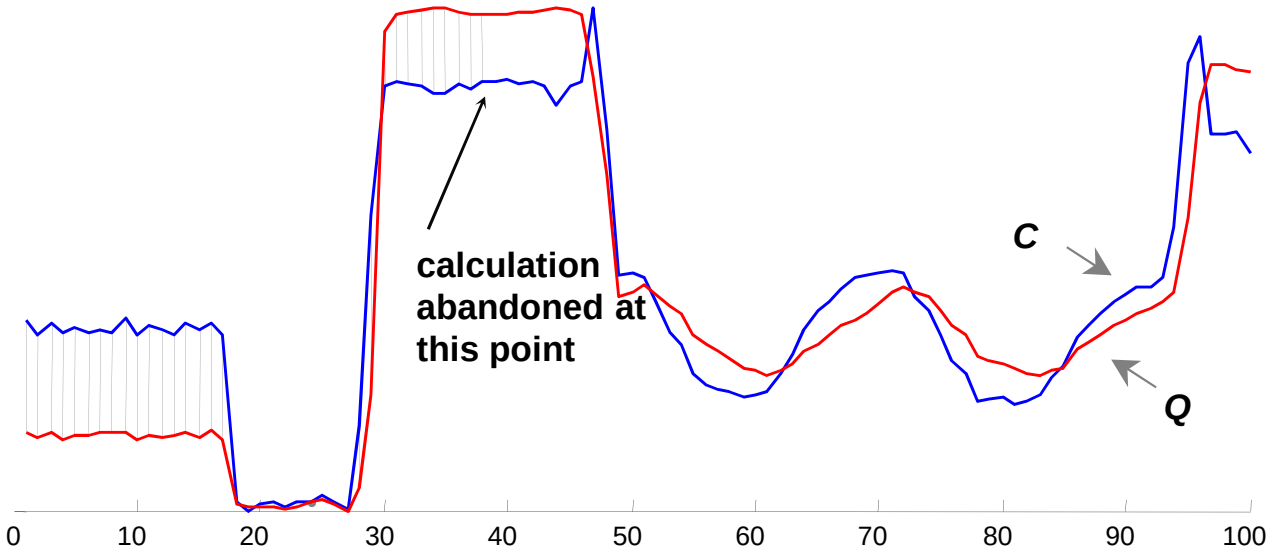
$$C = c_1 \ldots c_n$$

$$D(Q,C) \equiv \sqrt{\sum_{i=1}^{n} (q_i - c_i)^2}$$

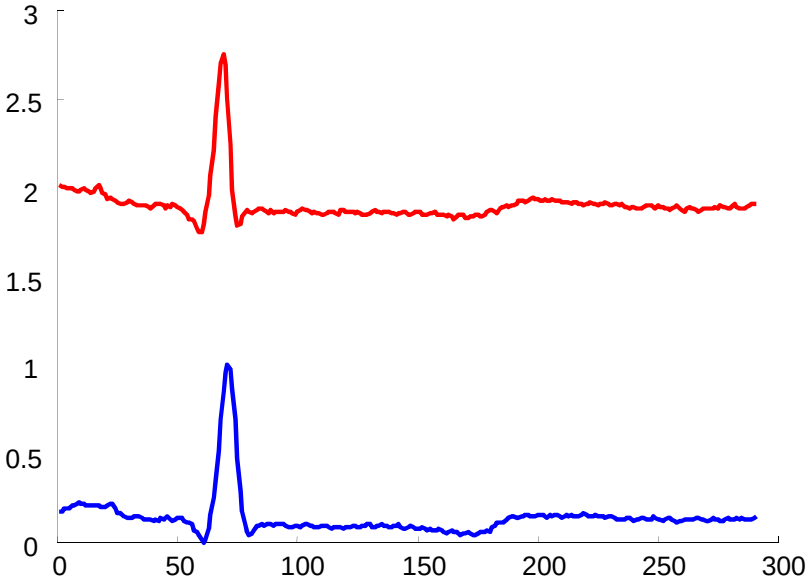


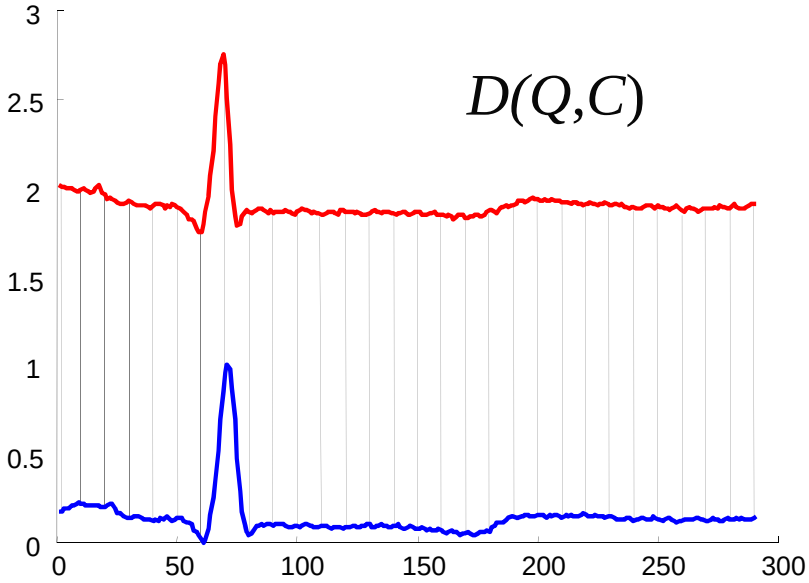
Early Abandon

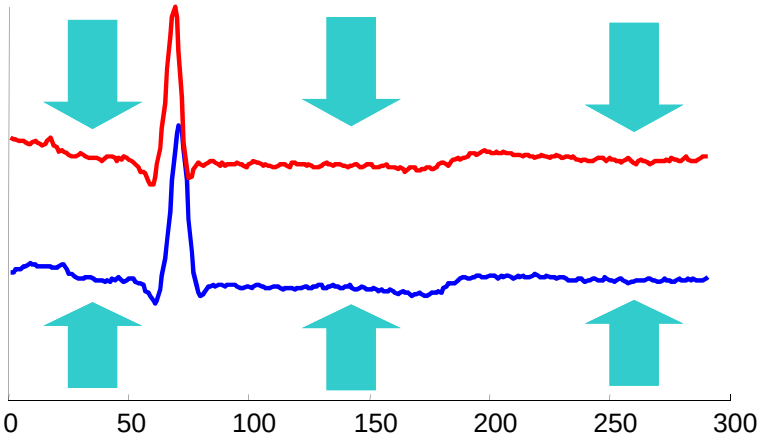
During the computation, if current sum of the squared differences between each pair of corresponding data points exceeds r^2, we can safely stop the calculation.



First translate offsets



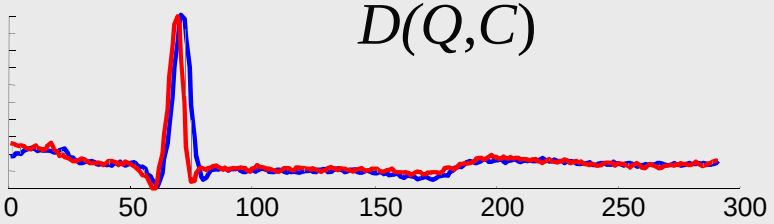




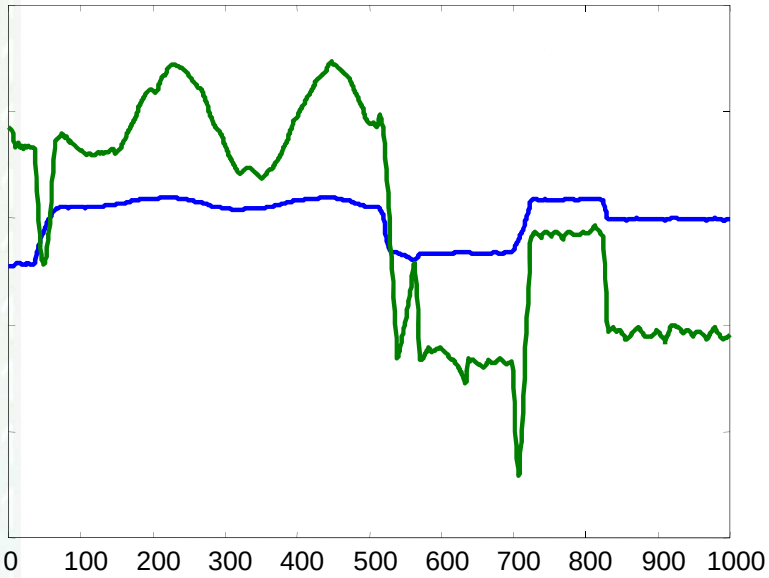
$Q = Q - \text{mean}(Q)$

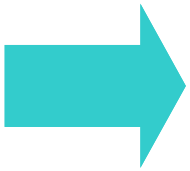
$C = C - \text{mean}(C)$

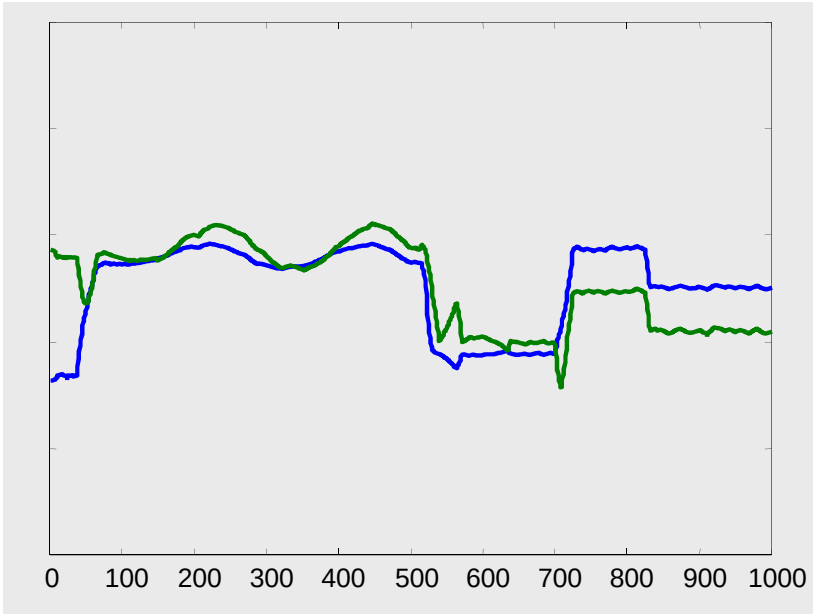
$D(Q,C)$



Then scale (normalize) signals





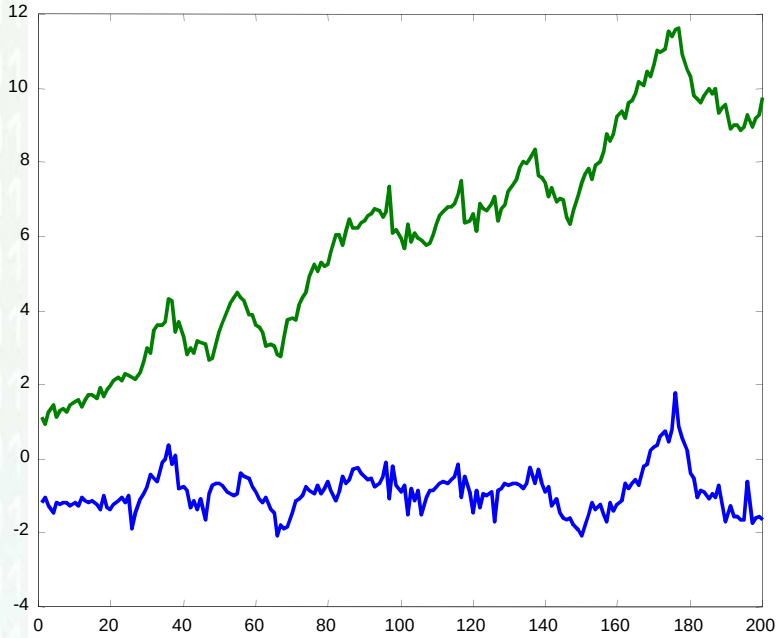


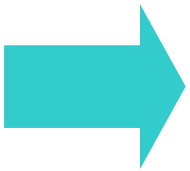
$Q = (Q - \text{mean}(Q)) / \text{std}(Q)$

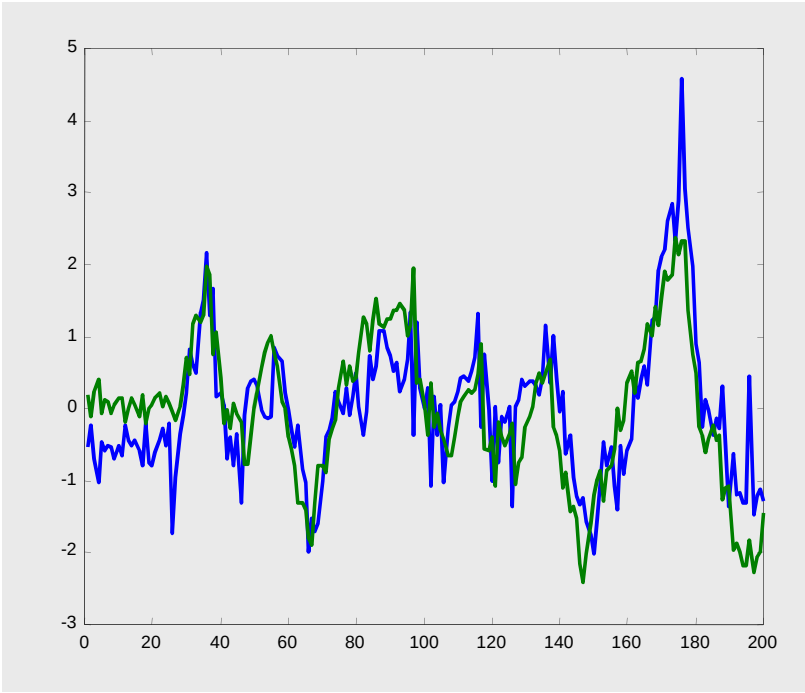
$C = (C - \text{mean}(C)) / \text{std}(C)$

$D(Q,C)$

Then remove trend (optional)





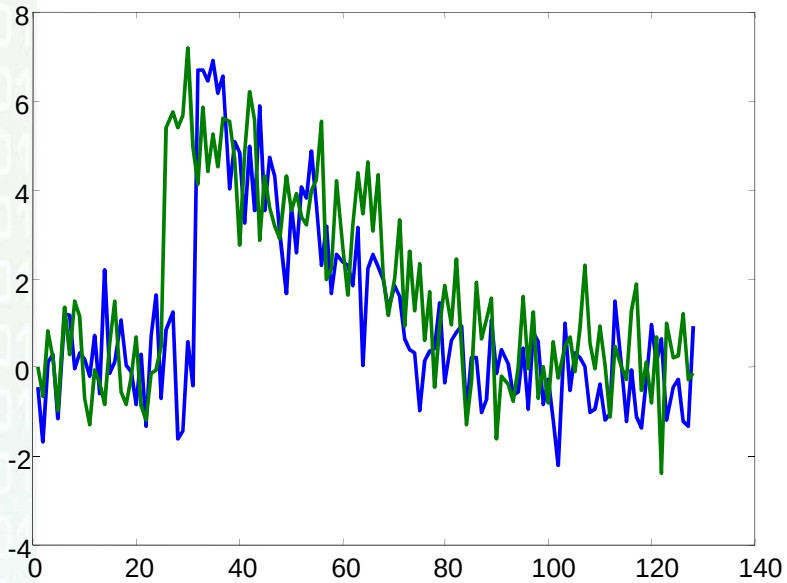


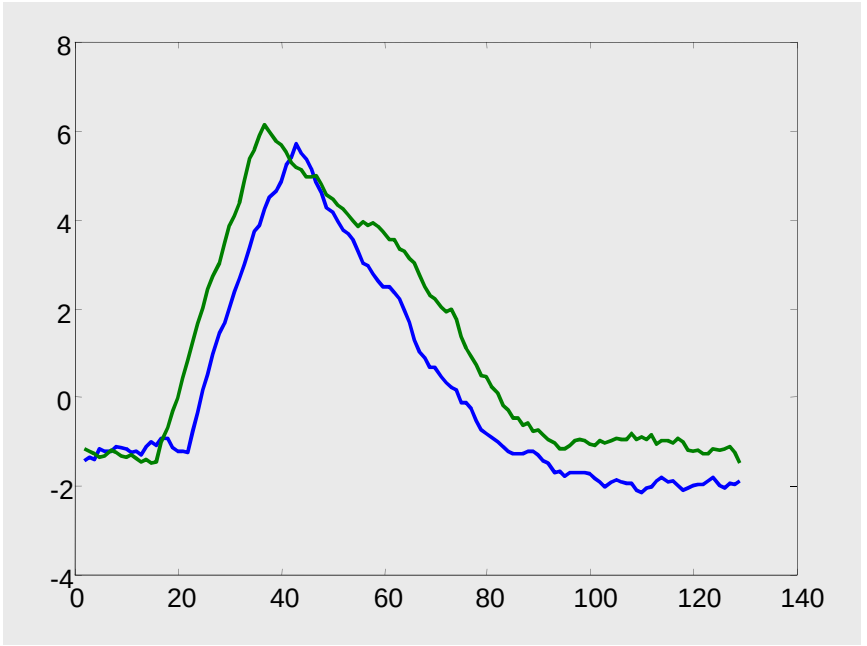
Removed **linear trend**

Removed offset translation

Removed amplitude scaling

Remove noise





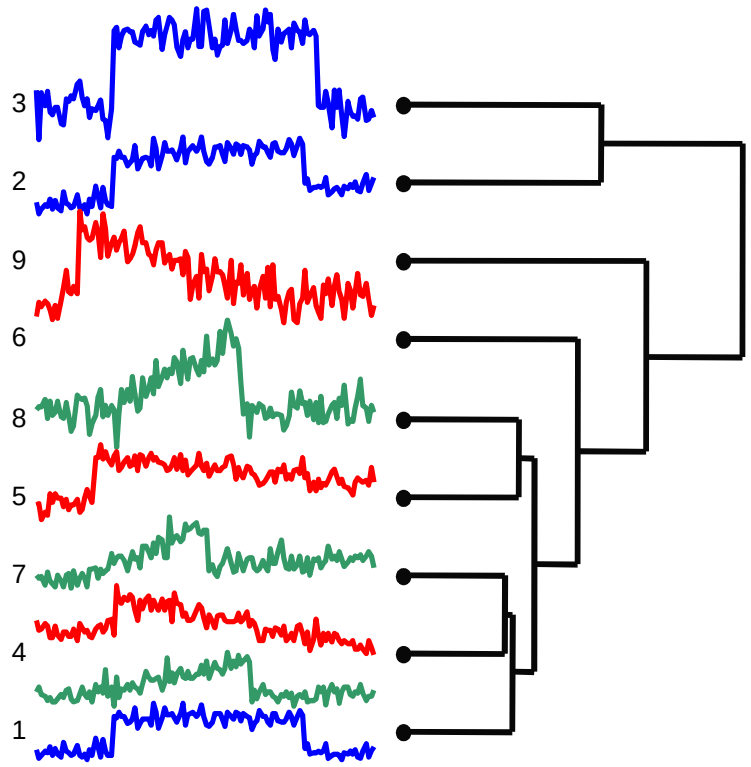
$Q = \text{smooth}(Q)$

$C = \text{smooth}(C)$

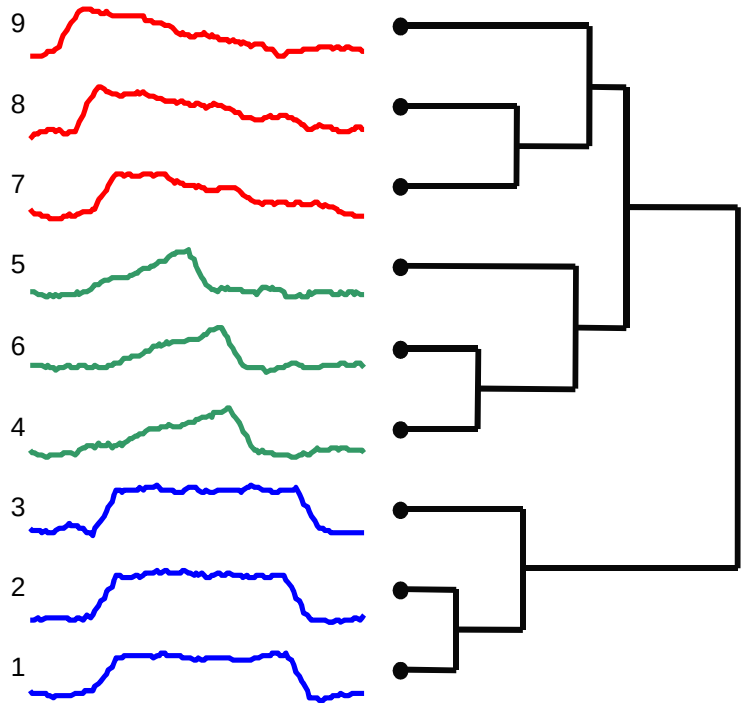
$D(Q,C)$

Agglomerative clustering using Euclidian distance

- Without preprocessing



- Noise removed
- Trend removed
- Offset translated
- Signals normalized



Wedge

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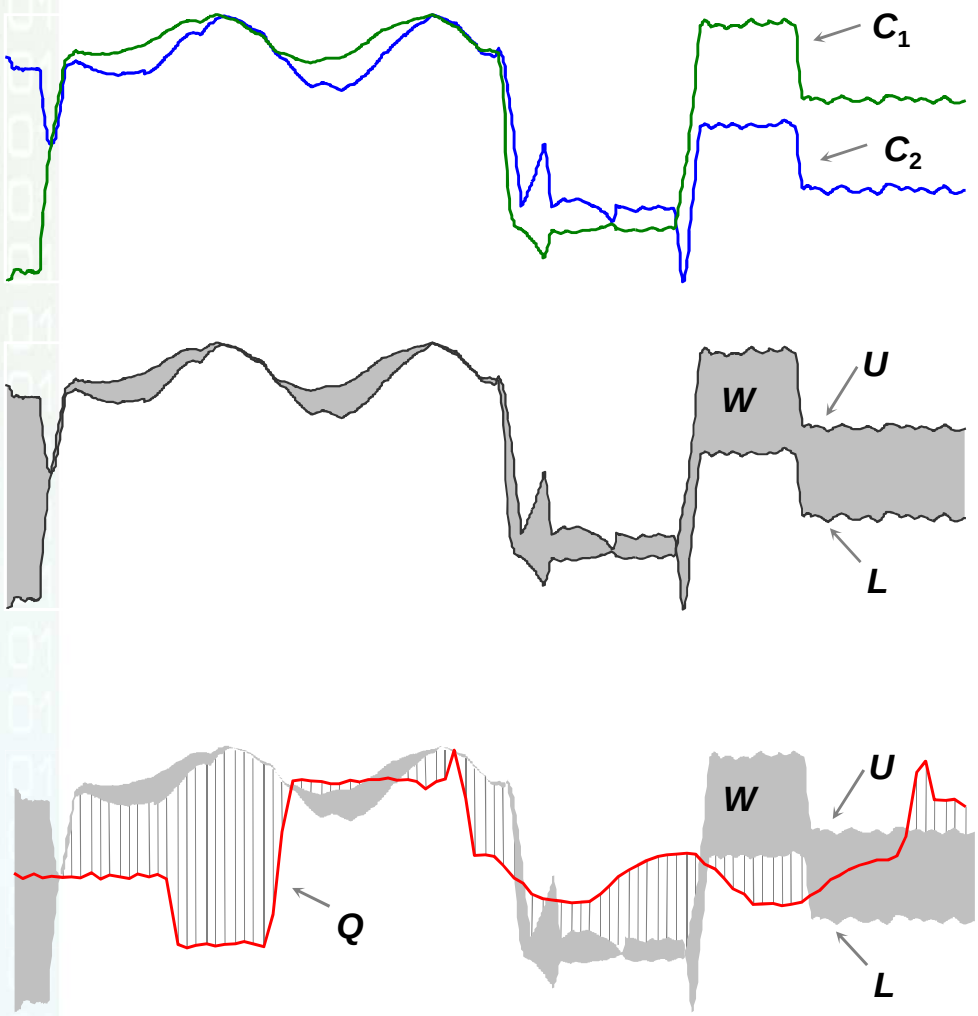


Having candidate sequences $C_1, .. , C_k$, we can form two new sequences U and L :

$$U_i = \max(C_{1i}, .. , C_{ki})$$
$$L_i = \min(C_{1i}, .. , C_{ki})$$

They form the smallest possible bounding envelope that encloses sequences $C_1, .. , C_k$.

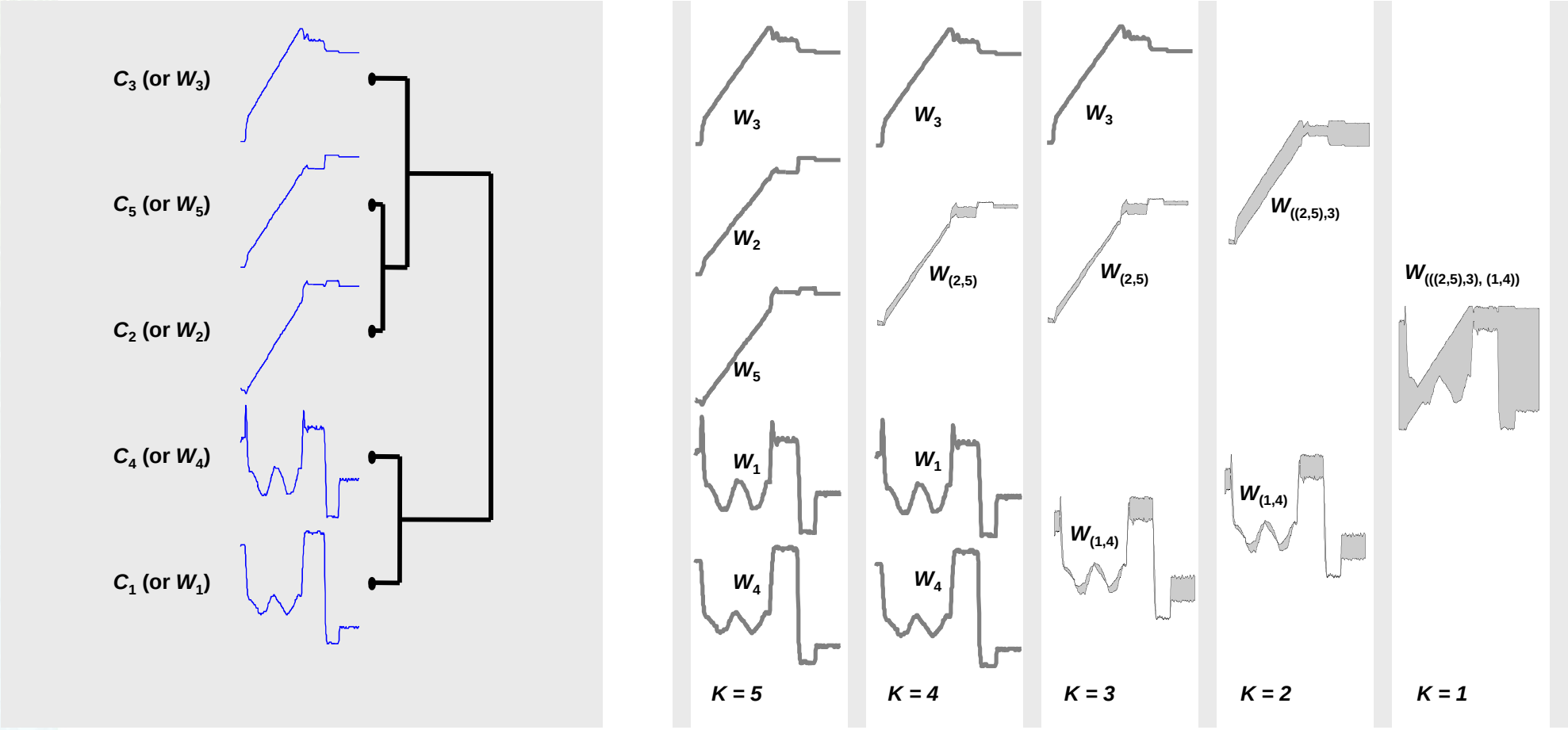
We call the combination of U and L a *wedge*, and denote a wedge as W.

$$W = \{U, L\}$$

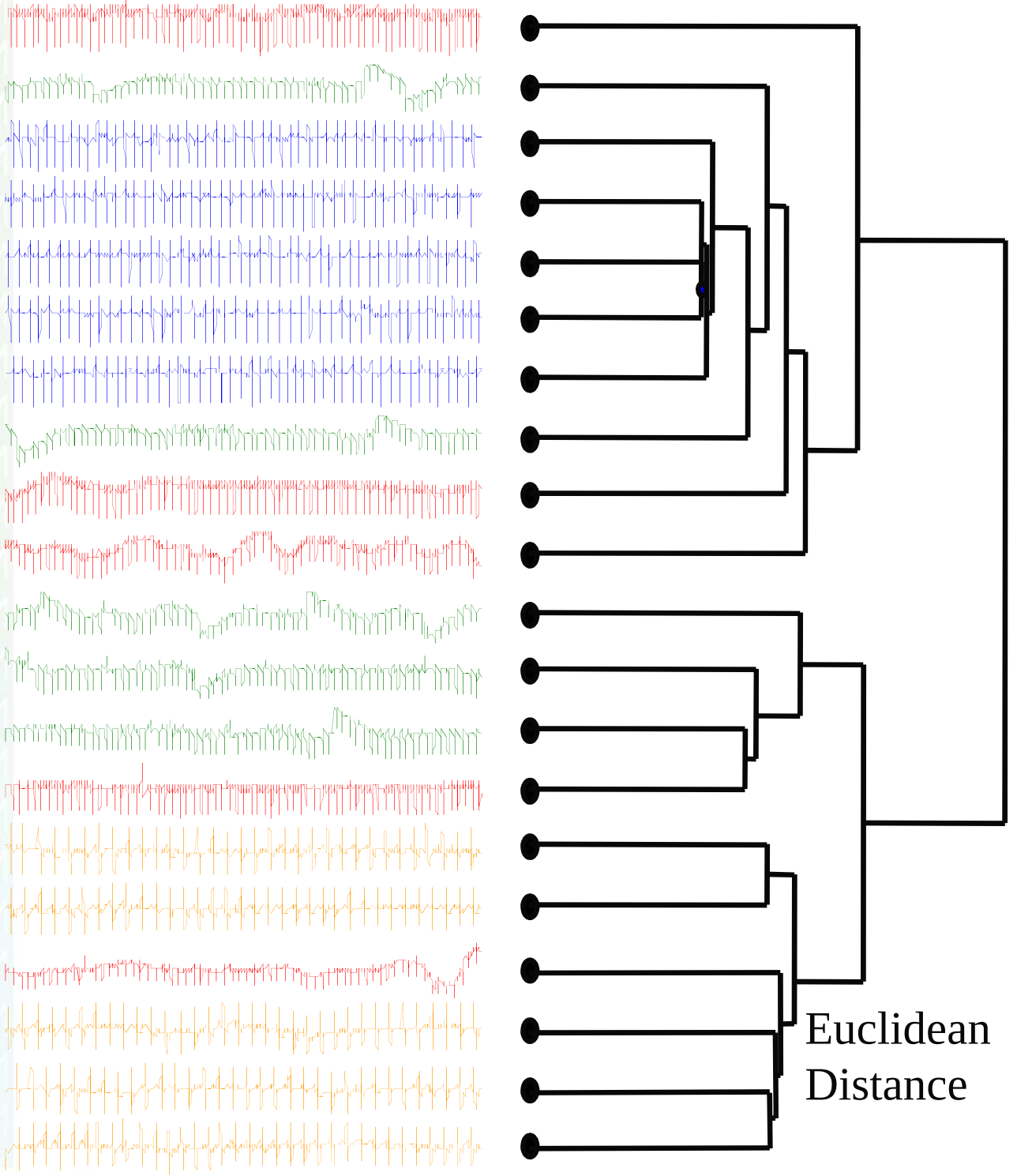
A lower bounding measure between an arbitrary query Q and the entire set of candidate sequences contained in a wedge W:

$$LB\_Keogh(Q,W) = \sqrt{\sum_{i=1}^{n} \begin{cases} (q_i - U_i)^2 & \text{if } q_i > U_i \\ (q_i - L_i)^2 & \text{if } q_i < L_i \\ 0 & \text{otherwise} \end{cases}}$$

Hierarchal Clustering with Wedges



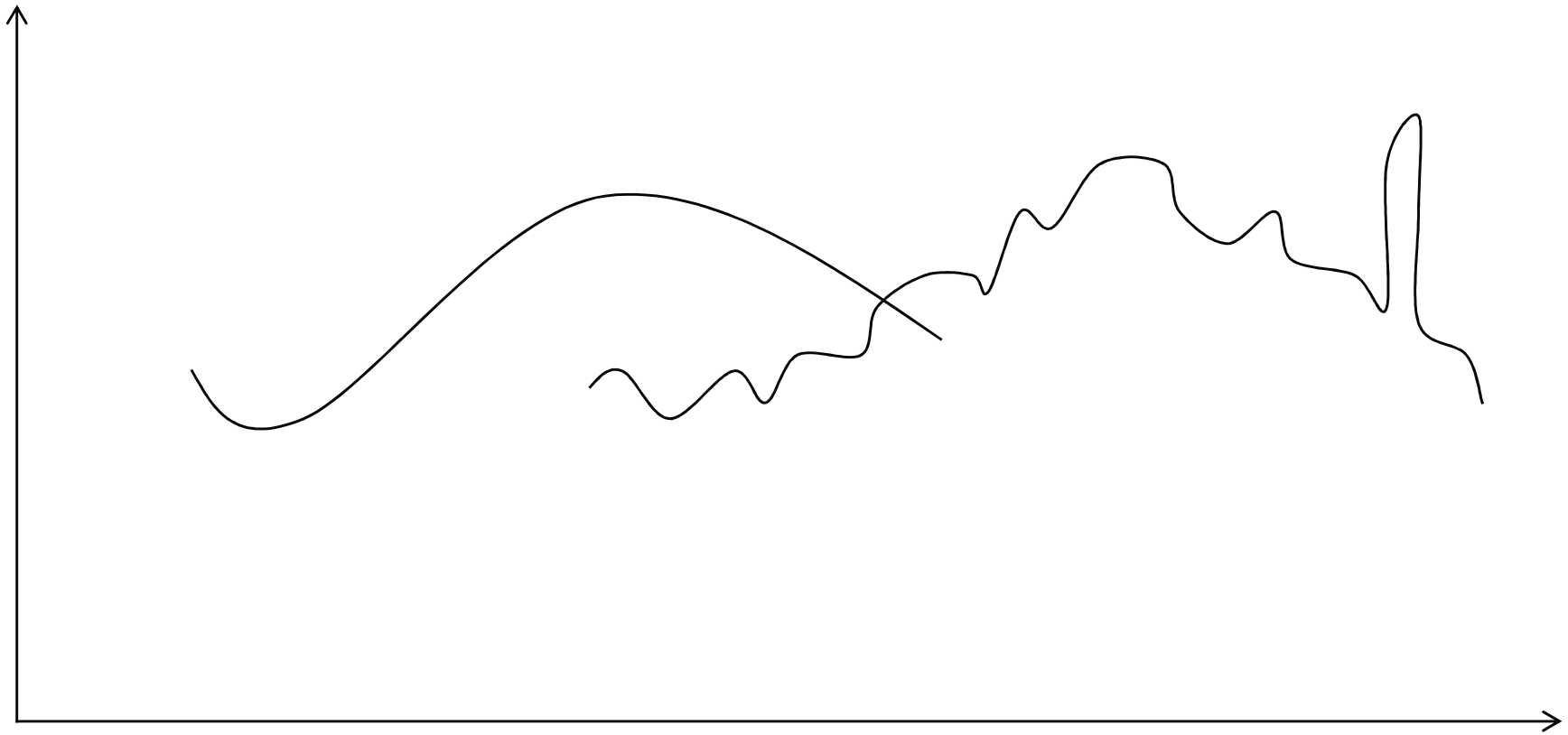
Wedges can be used for filtering or querying.



But!

- Euclidian distance not universally applicable!
- Consider:
  - Phase shift
  - Delays
  - Unsynchronized signals
  - ...

Example



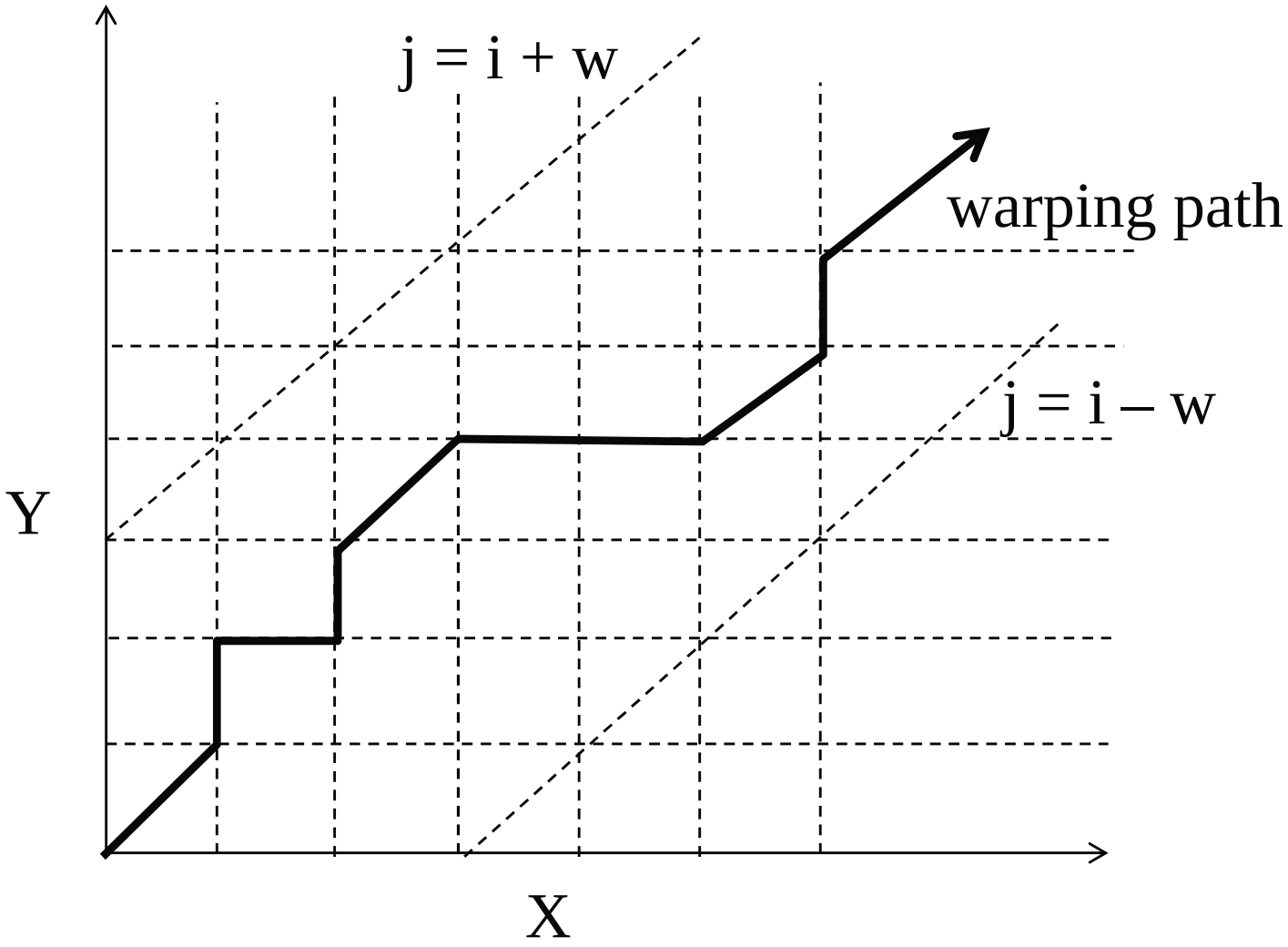
Dynamic Time Warping
[Berndt, Clifford, 1994]

- Allows acceleration-deceleration of signals along the time dimension

- Basic idea
  - Consider X = $x_1, x_2, \ldots, x_n$, and Y = $y_1, y_2, \ldots, y_n$
  - We are allowed to extend each sequence by repeating elements
  - Euclidean distance now calculated between the extended sequences X’ and Y’
  - Matrix M, where $m_{ij} = d(x_i, y_j)$

Dynamic Time Warping

[Berndt, Clifford, 1994]



Restrictions on Warping Paths

- Monotonicity
  - Path should not go down or to the left
- Continuity
  - No elements may be skipped in a sequence
- Warping Window

$$|i - j| <= w$$

Formulation

- Let D(i, j) refer to the dynamic time warping distance between the subsequences

$$x_1, x_2, \ldots, x_i$$
$$y_1, y_2, \ldots, y_j$$

$$D(i, j) = | x_i - y_j | + \min \{ D(i - 1, j), D(i - 1, j - 1), D(i, j - 1) \}$$

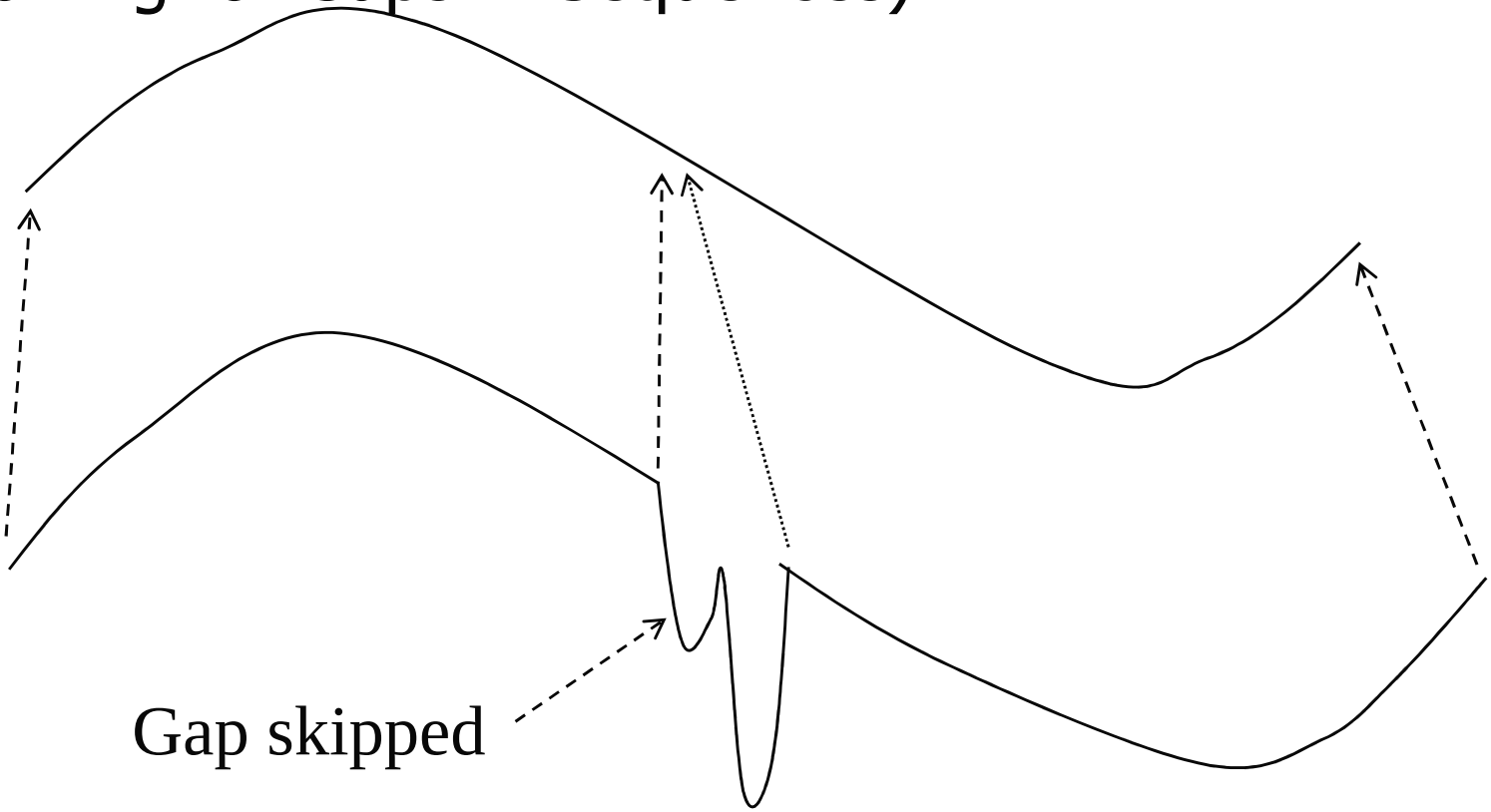
Solution by Dynamic Programming

- Basic implementation = $O(n^2)$ where n is the length of the sequences
  - will have to solve the problem for each (i, j) pair
- If warping window is specified, then O(nw)
  - Only solve for the (i, j) pairs where

  $| i - j | <= w$

Longest Common Subsequence Measures

(Allowing for Gaps in Sequences)



Basic LCS Idea

X = 3, **2**, **5**, **7**, 4, 8, **10**, 7

Y = **2**, **5**, 4, **7**, 3, **10**, 8, 6

LCS = **2**, **5**, **7**, **10**

$$Sim(X,Y) = |LCS| \text{ or } Sim(X,Y) = |LCS| / n$$

Edit Distance is another possibility

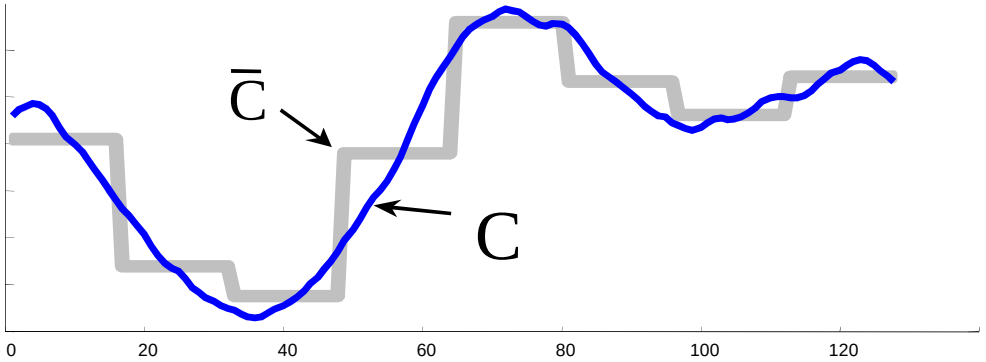
Landmarks
[Perng et. al., 2000]

- Similarity definition much closer to human perception (unlike Euclidean distance)
- A point on the curve is a n-th order landmark if the n-th derivative is 0
  - Thus, local max and mins are first order landmarks
- Landmark distances are tuples (e.g. in time and amplitude) that satisfy the triangle inequality
- Several transformations are defined, such as shifting, amplitude scaling, time warping, etc

PAA and APCA

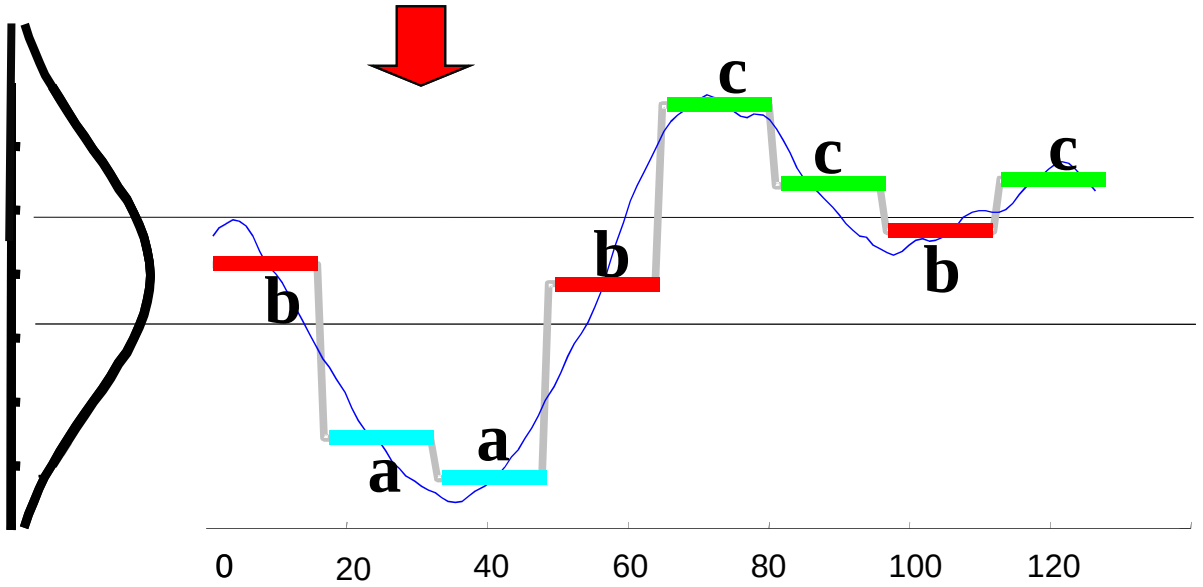
- **P**iecewise **A**ggregate **A**pproximation
  - Segment the time series into equal parts, store the average value for each part.
- **A**daptive **P**iecewise **C**onstant **A**pproximation
  - Parts are of adaptive length

Symbolic Aggregate approXimation



First convert the time series to PAA representation, then convert the PAA to symbols

It take linear time



baabccbc

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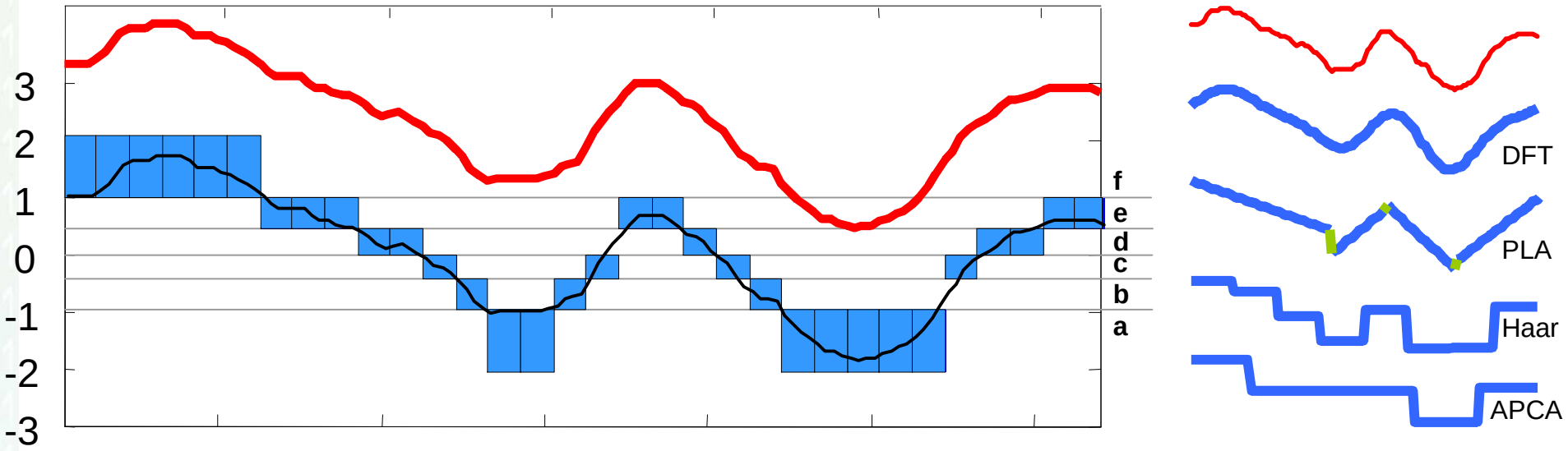
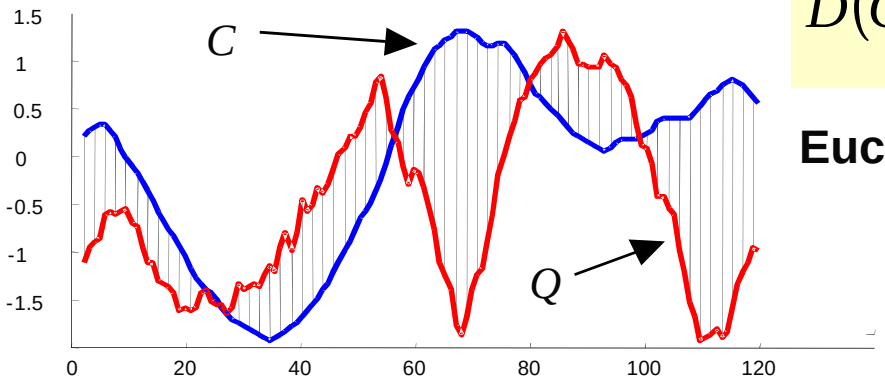
Visual Comparison



A raw time series of length 128 is transformed into the word "**fffffeeeddcbaabceedcbaaaaacddee**."

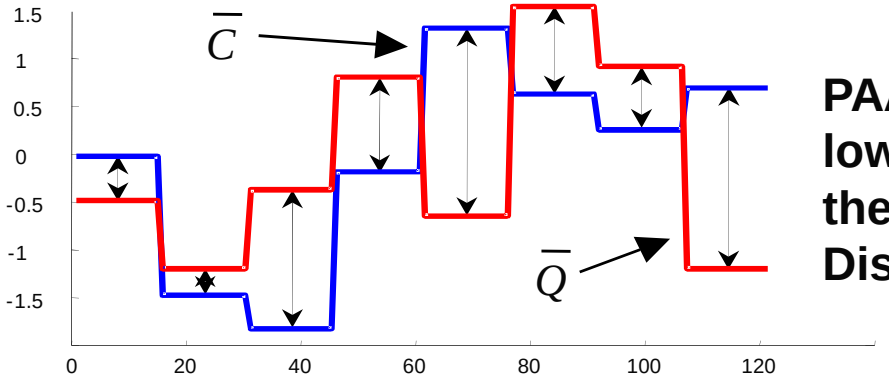
- We can use more symbols to represent the time series since each symbol requires fewer bits than real-numbers (float, double)



$$D(Q,C) \equiv \sqrt{\sum_{i=1}^{n} (q_i - c_i)^2}$$

Euclidean Distance

$$DR(\overline{Q},\overline{C}) \equiv \sqrt{\frac{n}{w}} \sqrt{\sum_{i=1}^{w} (\overline{q}_i - \overline{c}_i)^2}$$



PAA distance lower-bounds the Euclidean Distance

$\hat{C}$ = **baabccbc**

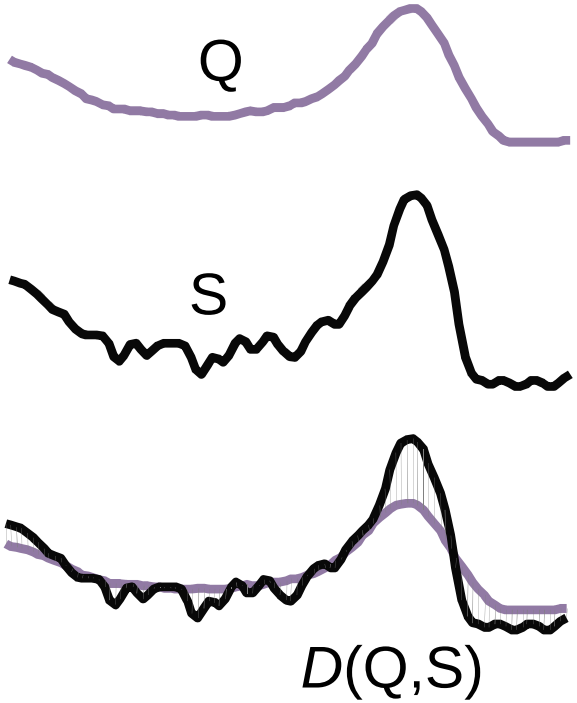
$\hat{Q}$ = **babcacca**

$$MINDIST(\hat{Q},\hat{C}) \equiv \sqrt{\frac{n}{w}} \sqrt{\sum_{i=1}^{w} (dist(\hat{q}_i, \hat{c}_i))^2}$$

dist() can be implemented using a table lookup.

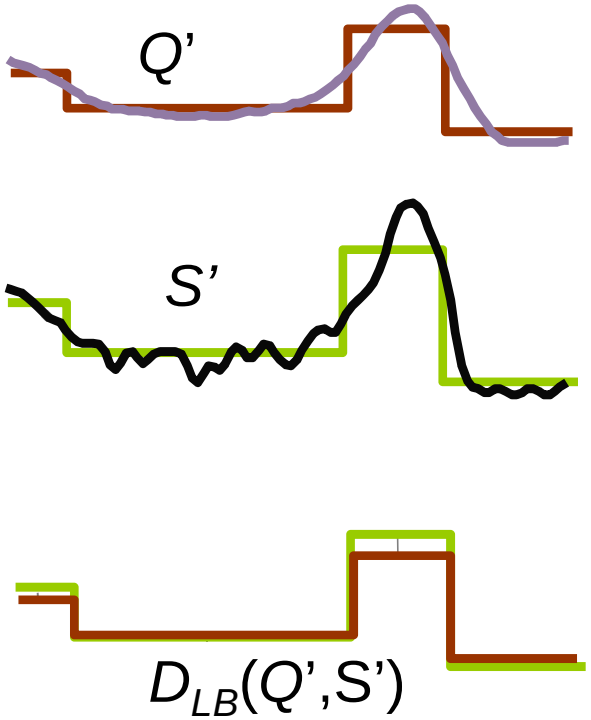
What is lower bounding?

Exact (Euclidean) distance D(Q,S)



$$D(Q,S) \equiv \sqrt{\sum_{i=1}^{n} (q_i - s_i)^2}$$

Lower bounding distance $D_{LB}(Q,S)$



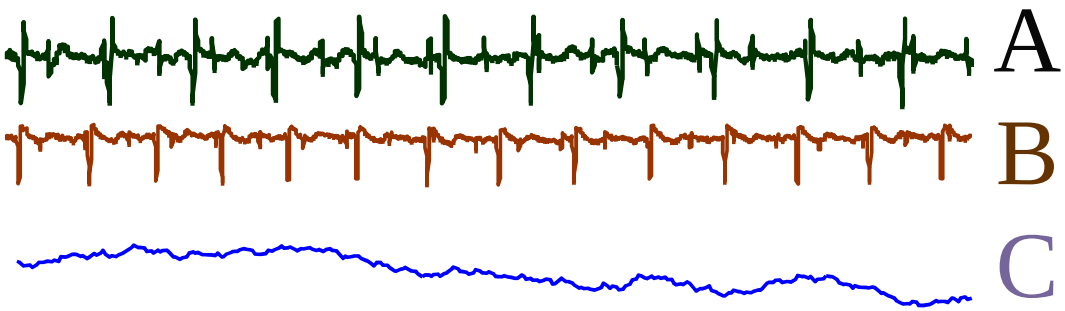
$$D_{LB}(Q',S') \equiv \sqrt{\sum_{i=1}^{M} (sr_i - sr_{i-1})(qv_i - sv_i)^2}$$

Lower bounding means that for all Q and S, we have…

$$D_{LB}(Q',S') \leq D(Q,S)$$

Structure and model based similarity

- Extract *global* features from the time series, create a feature vector, and use these feature vectors to measure similarity and/or classify



Feature \ Time Series	A	B	C
Max Value	11	12	19
Autocorrelation	0.2	0.3	0.5
Zero Crossings	98	82	13
…	…	…	…

ARIMA Models and Forecasting

- If we can describe the way the points in the series are related to each other (the autocorrelations), then we can describe the series using the relationships that we've found
- AutoRegressive Integrated Moving Average Models (ARIMA) are mathematical models of the autocorrelation in a time series
- One way to describe time series

Autocorrelation

- The major statistical tool for ARIMA models is the sample autocorrelation coefficient

$$r_k = \frac{\sum_{t=k+1}^{n}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}$$

Autocorrelations

- r_1 indicates how successive values of Y relate to each other,
- r_2 indicates how Y values two periods apart relate to each other,
- and so on.

Autoregressive Models

- The autoregressive process of order p is denoted AR(p), and defined by

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t$$

$$Y_t = \sum_{r=1}^{p} \phi_r \, Y_{t-r} + w_t$$

where $\phi_1, \ldots, \phi_p$ are parameters to be estimated and $\{w_t\}$ white noise, a sequence of independent (or uncorrelated) random variables with mean 0 and variance σ^2

An AR(p) model is a **regression model with lagged values** of the dependent variable in the independent variable positions, hence the name **autoregressive** model.

Moving Average Models

- The moving average process of order *q*, denoted MA(q), includes lagged error terms *t*–1 to *t*–*q*, written as

$$Y_t = \mu + w_t - \sum_{r=1}^{q} \theta_r w_{t-r}$$

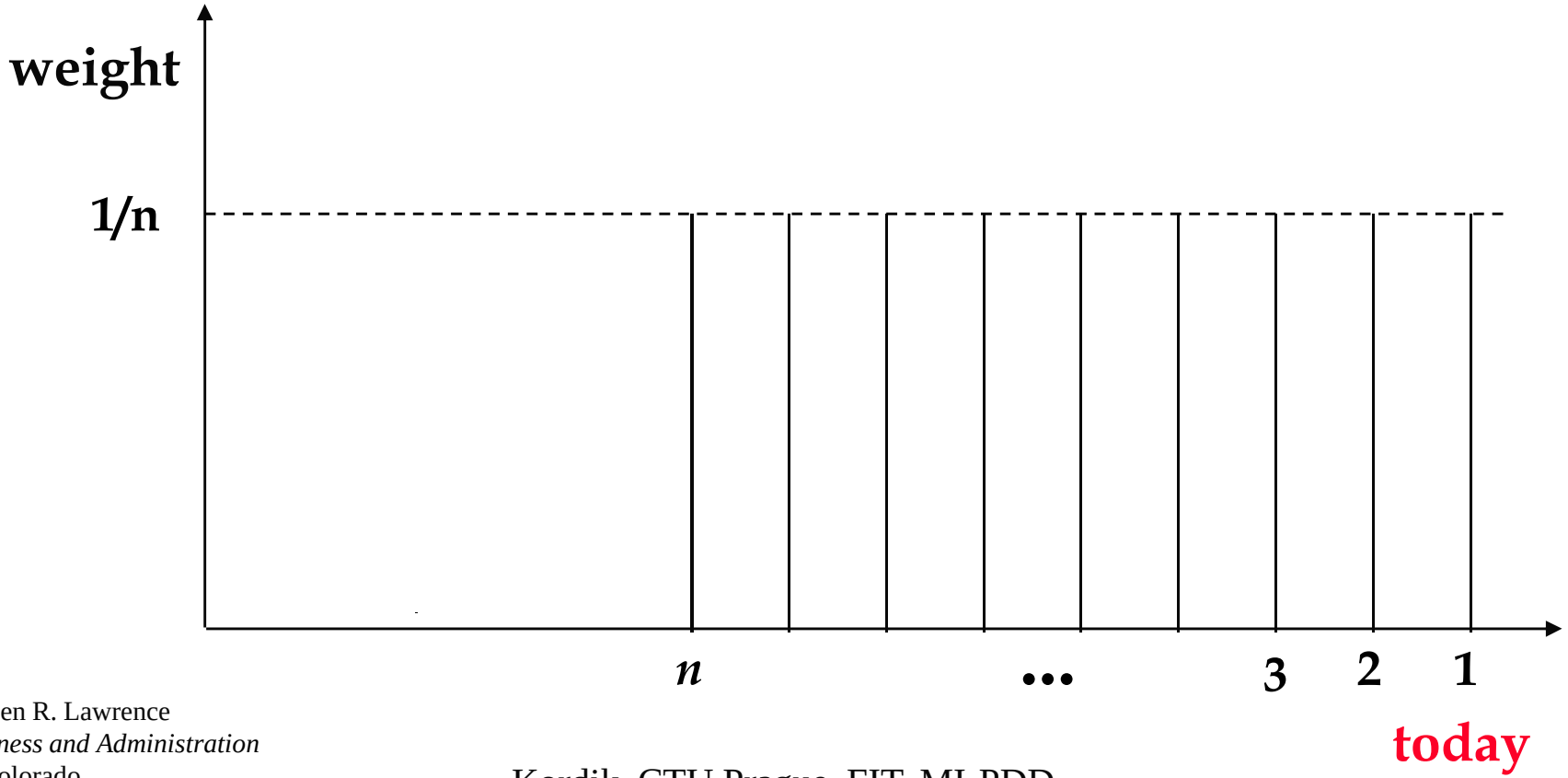
The term Moving Average is historical and should not be confused with the moving average smoothing procedures.

where θ_1 , θ_2 , … , θ_q are the MA parameters and w_t is white noise

An MA(q) model is a regression model with the dependent variable, **Y_t, depending on previous values of the errors** rather than on the variable itself.

Simple Moving Average

- Include *n* most recent observations
- Weight equally
- Ignore older observations



Professor Stephen R. Lawrence
College of Business and Administration
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Boulder, CO 80309-0419

Simple Moving Average

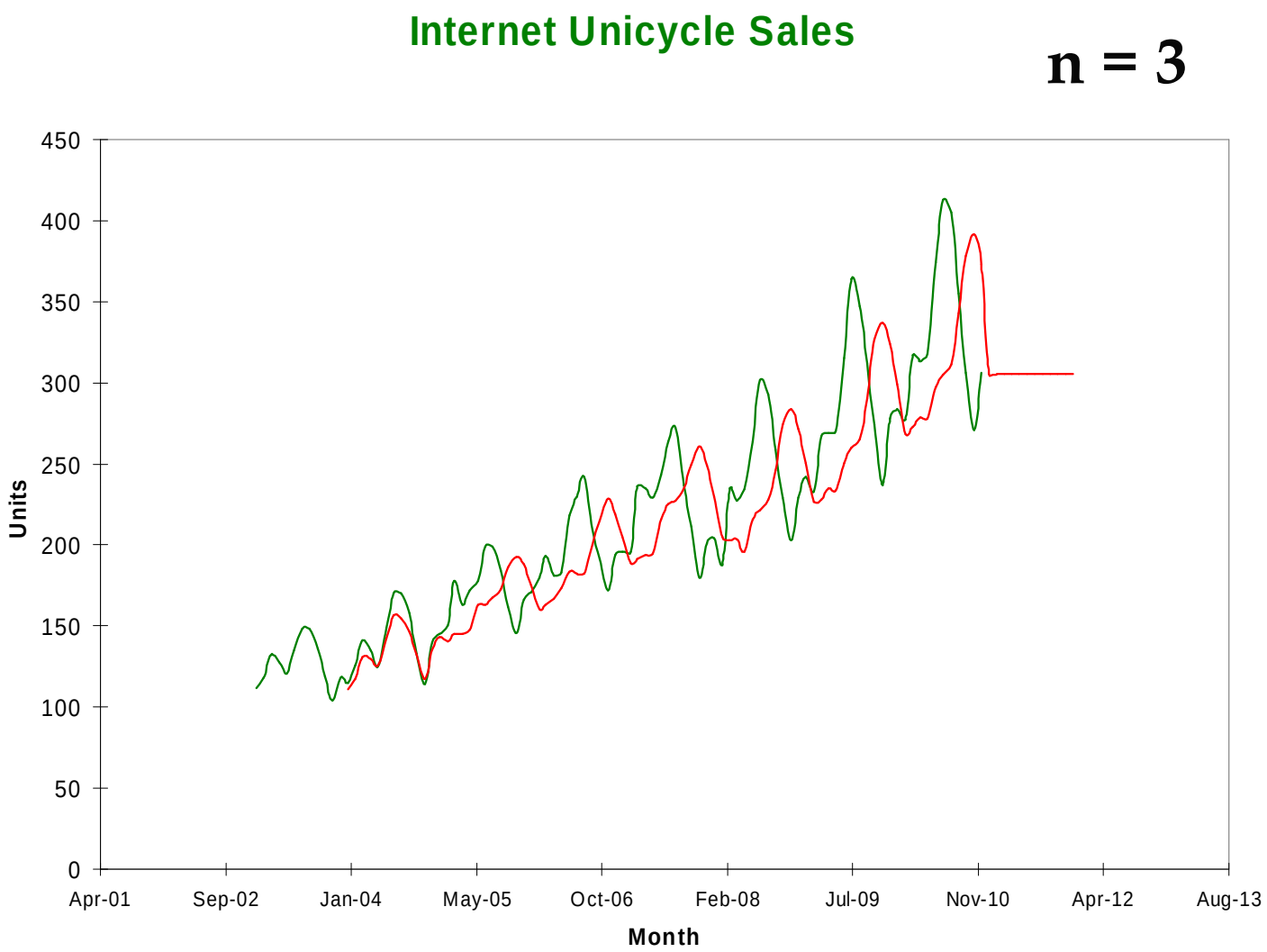
- Forecast F_t is average of n previous observations or actuals D_t:

$$F_{t+1} = \frac{1}{n}(D_t + D_{t-1} + \cdots + D_{t+1-n})$$

$$F_{t+1} = \frac{1}{n}\sum_{i=t+1-n}^{t} D_i$$

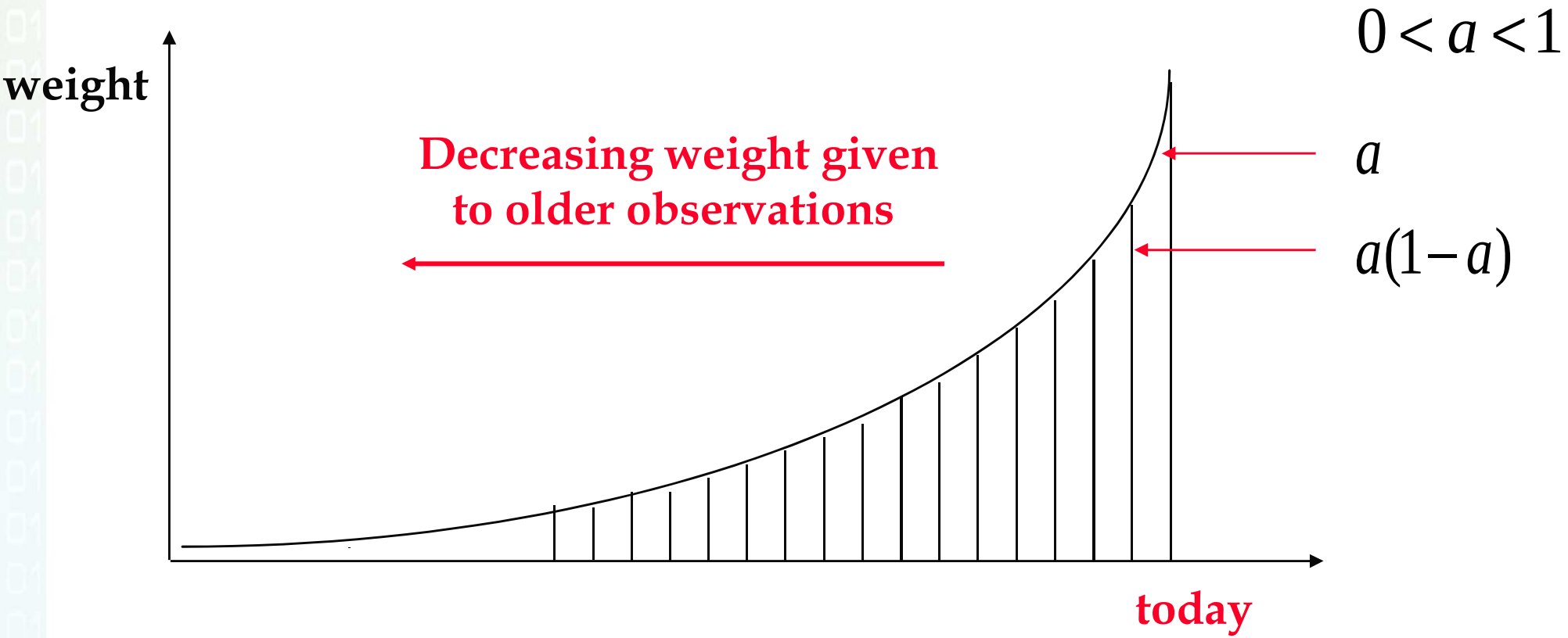
- Note that the n past observations are equally weighted.
- Issues with moving average forecasts:
  - All n past observations treated equally;
  - Observations older than n are not included at all;
  - Requires that n past observations be retained;
  - Problem when 1000's of items are being forecast.

Moving Average



Exponential Smoothing, idea

- Include all past observations
- Weight recent observations much more heavily than very old observations:



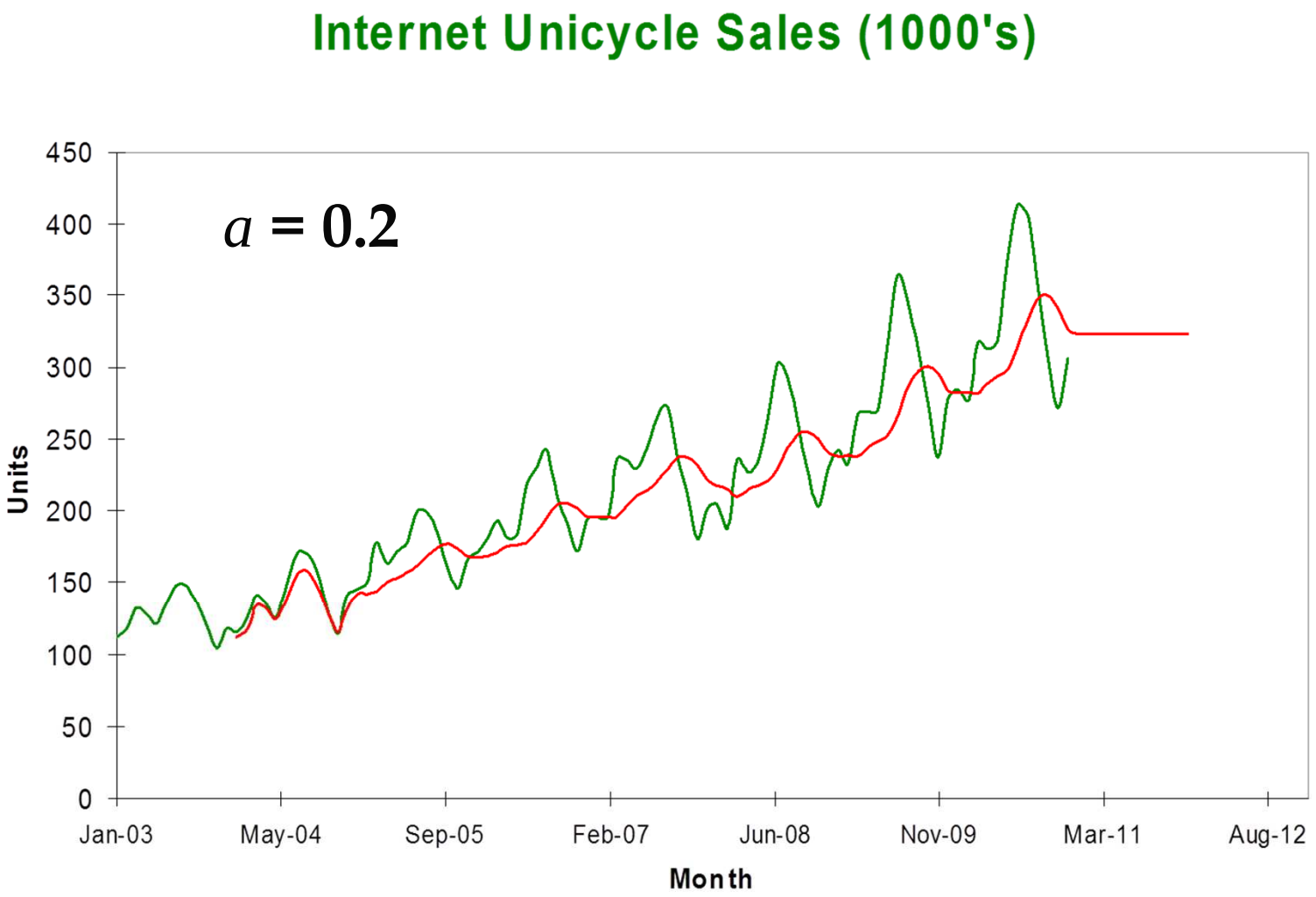
Exponential Smoothing, math

$$F_t = aD_t + a(1-a)D_{t-1} + a(1-a)^2 D_{t-2} + \cdots$$

$$F_t = aD_t + (1-a)F_{t-1}$$

- Thus, new forecast is weighted sum of old forecast and actual demand
- Notes:
  - Only 2 values (D_t and F_{t-1}) are required, compared with n for moving average
  - Parameter a determined empirically (whatever works best)
  - Rule of thumb: $a < 0.5$
  - Typically, $a = 0.2$ or $a = 0.3$ work well

Exponential Smoothing, example



Time honored linear models

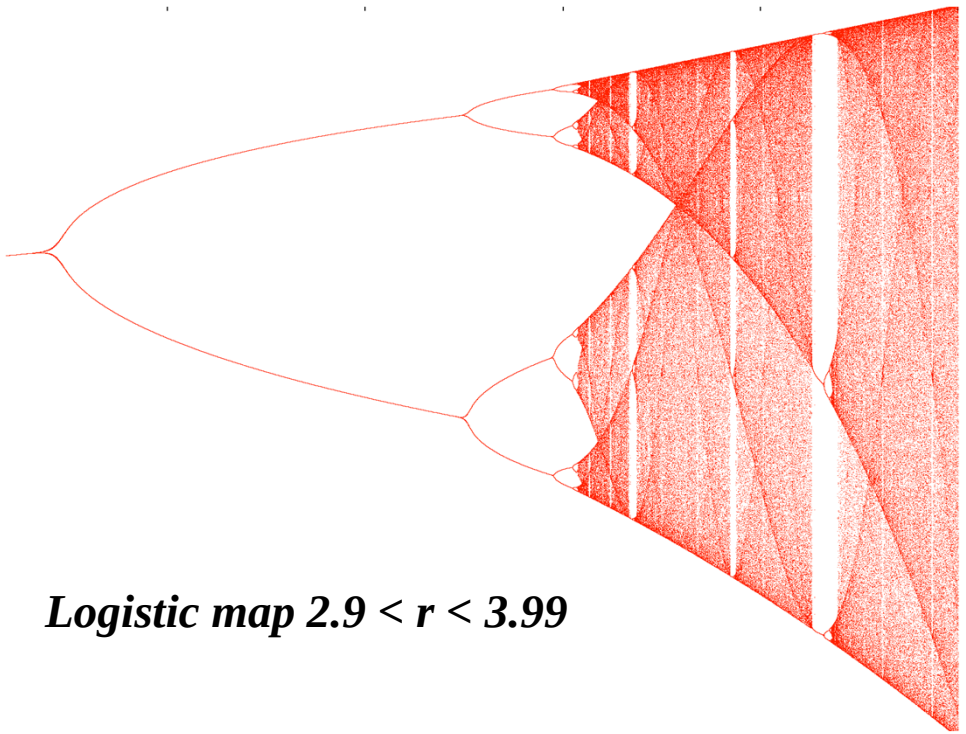
$$y[t+1] = \sum_{i=0}^{N_{AR}} a_i \cdot y[t-i] + \sum_{j=0}^{N_{MA}} b_j \cdot x[t-i]$$

- Auto Regressive Moving Average (**ARMA**)
- Many linear estimation techniques based on Least Squares, or Least Mean Squares
- Power spectra, and Autocorrelation characterize such linear systems
- Randomness comes ***only*** from forcing function $x(t)$

Simple nonlinear systems can exhibit chaotic behavior

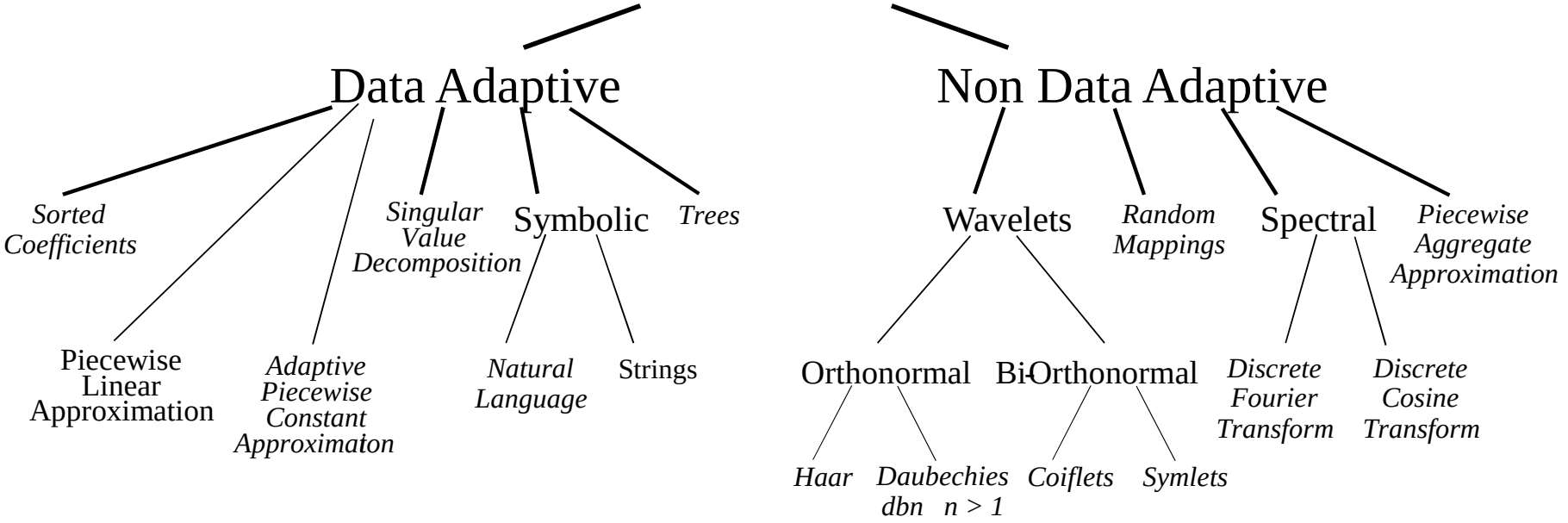
$$x[t+1] = r \cdot x[t](1 - x[t])$$

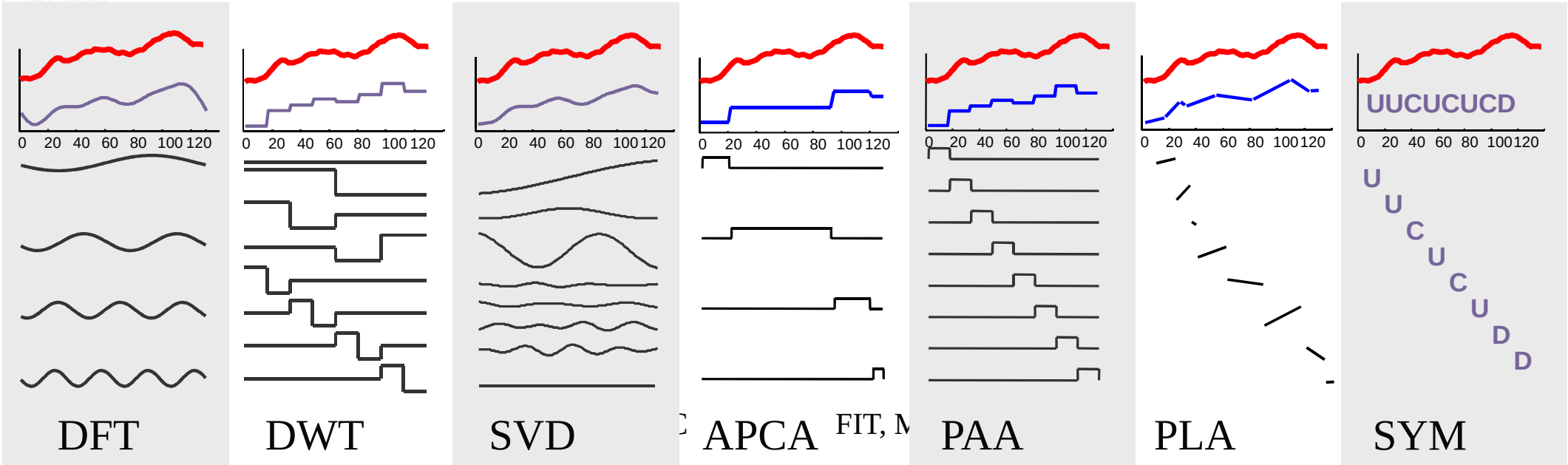
- Spectrum, autocorrelation, characterize ***linear*** systems, not these
- Deterministic chaos looks random to linear analysis methods
- Logistic map is an early example (*Elam 1957*).



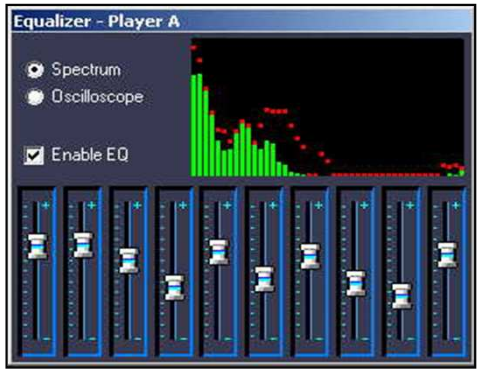
Logistic map 2.9 < r < 3.99

Time Series Representations





Spectrum and Shape Descriptors



Centroid
Rolloff
Flux
Bandwidth
Moments
....

=

M

F

Centroid

Feature Space

Feature vector

Fourier transform

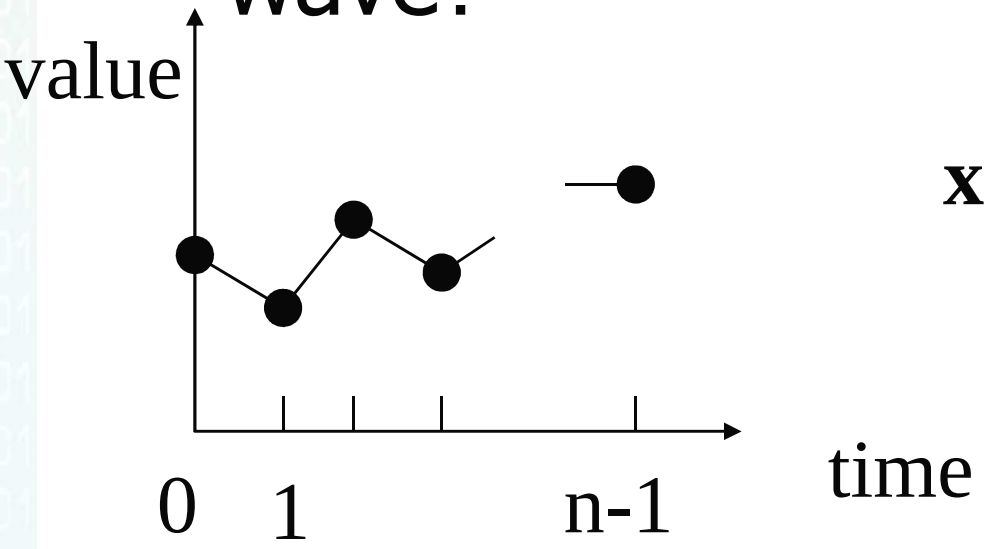
- DFT (Discrete Fourier Transform)
- Transform the data from the time domain to the frequency domain
- Useful for signals with periodicities
  - sales patterns follow seasons;
  - economy follows 50-year cycle (or 10?)
  - temperature follows daily and yearly cycles ...

How does it work?

Based on Slides by D. Gunopulos (UCR)

Decomposes signal to a sum of sine (and cosine) waves.

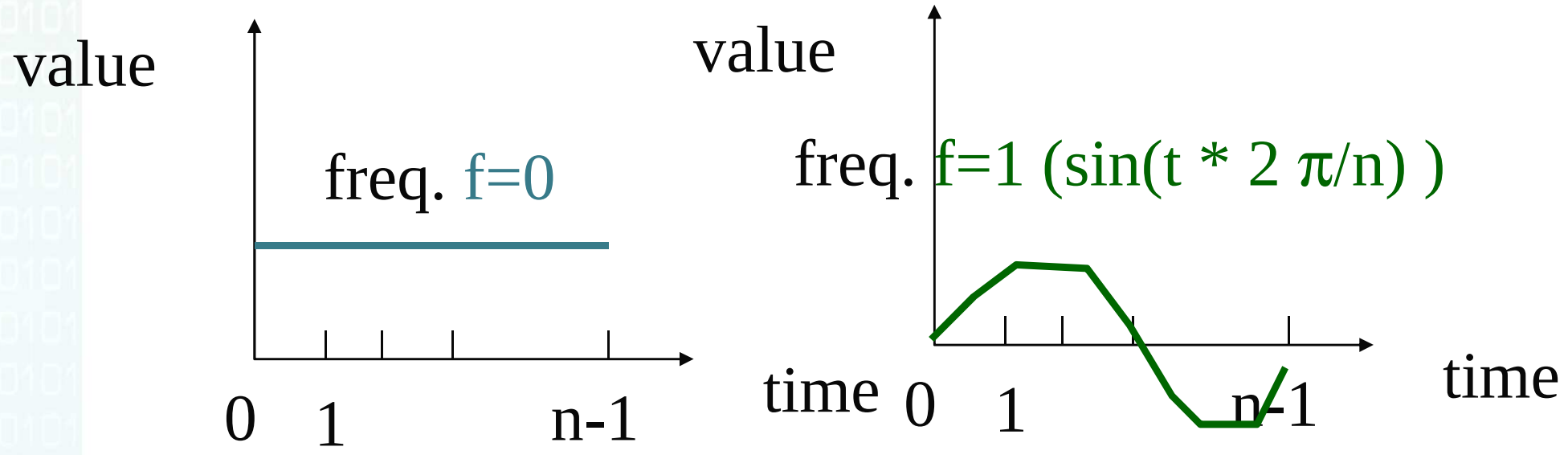
Q:How to assess 'similarity' of **x** with a wave?



$$\mathbf{x} = \{x_0, x_1, \ldots x_{n-1}\}$$

How does it work?

A: consider the waves with frequency 0, 1, ...; use the inner-product (~cosine similarity)



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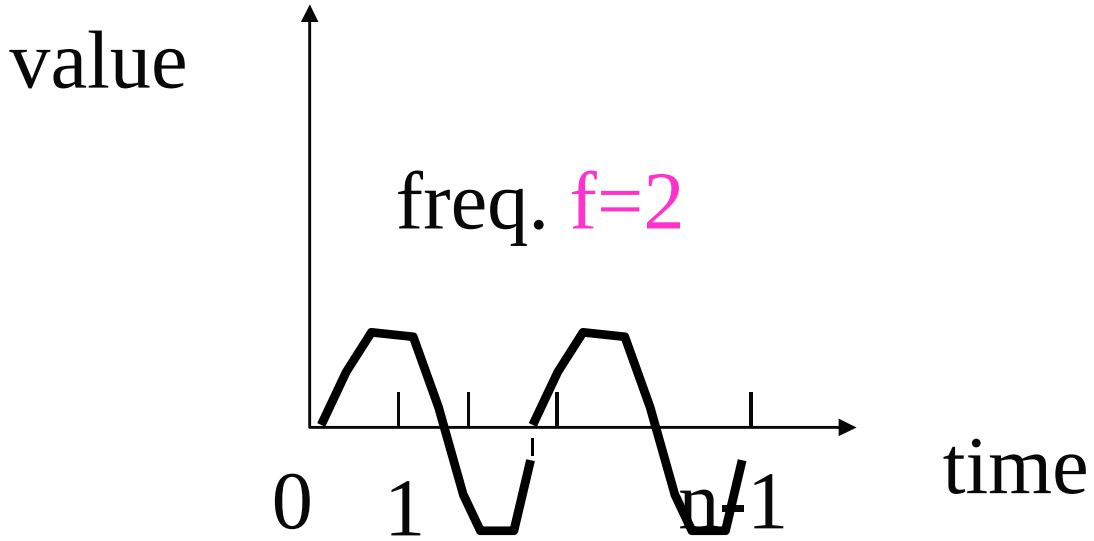
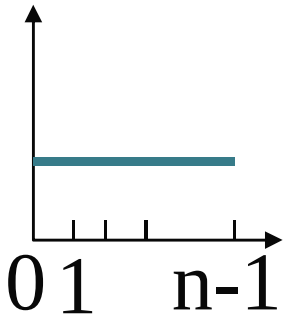
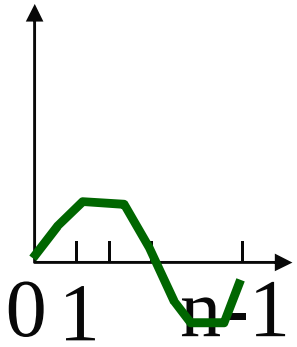


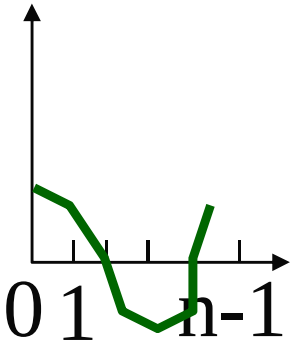
How does it work?

'basis' functions



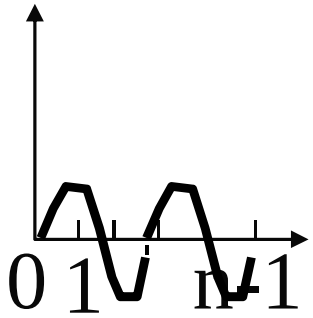
sine, freq =1

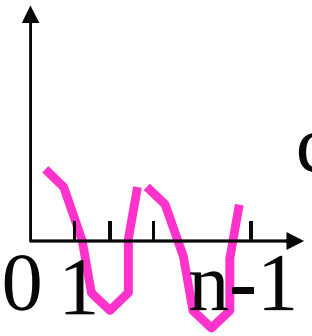




cosine, f=1

sine, freq = 2





cosine, f=2

How does it work?

- Basis functions are actually n-dim vectors, **orthogonal** to each other
- 'similarity' of **x** with each of them: inner product
- DFT: ~ all the similarities of **x** with the basis functions

How does it work?

Since $e^{jf} = cos(f) + j\,sin(f)$ ($j=sqrt(-1)$),

we finally have:

DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf / n)$$

$$(j = \sqrt{-1})$$

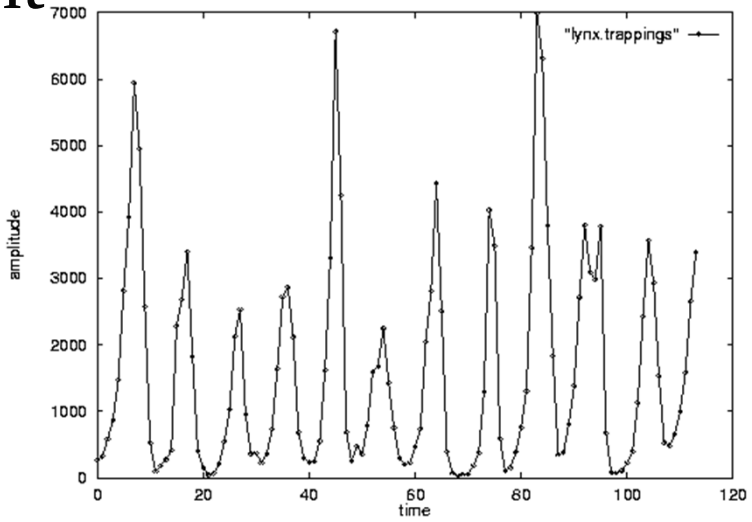
inverse DFT

$$x_t = 1/\sqrt{n} \sum_{t=0}^{n-1} X_f * \exp(+j2\pi tf / n)$$

DFT: Amplitude spectrum

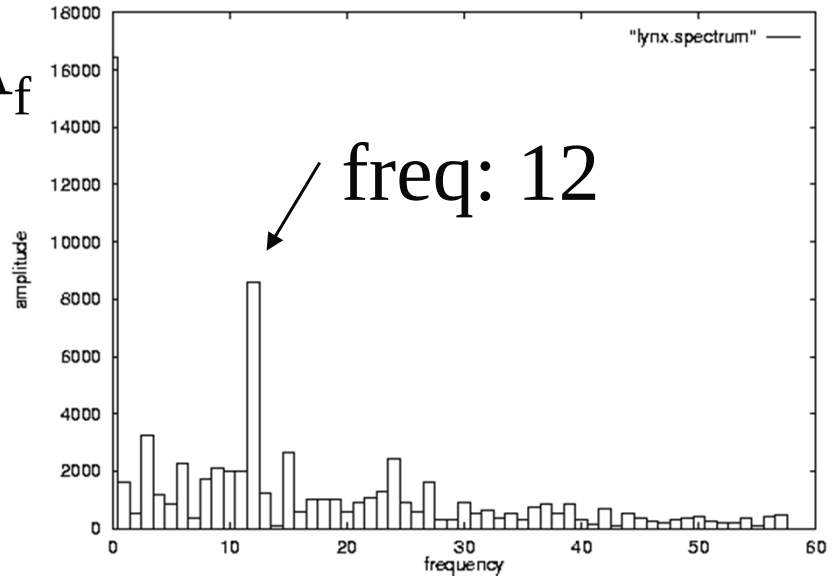
- Amplitude $A_f^2 = \mathrm{Re}^2(X_f) + \mathrm{Im}^2(X_f)$
- Intuition: strength of frequency '*f*'

count



time

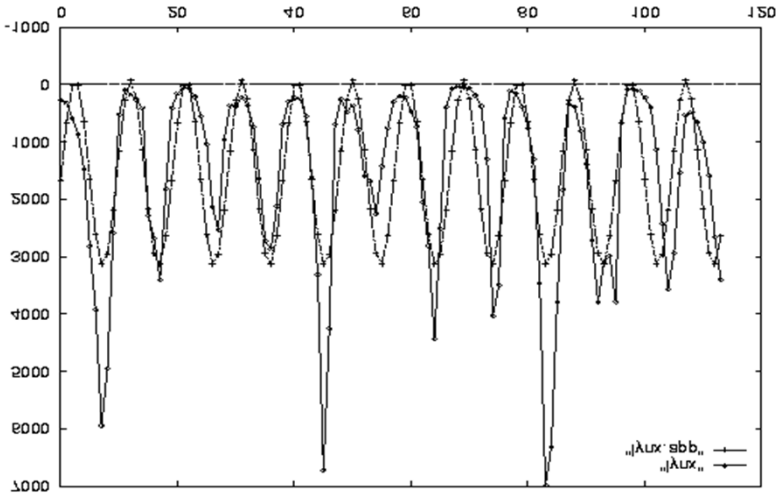
A_f

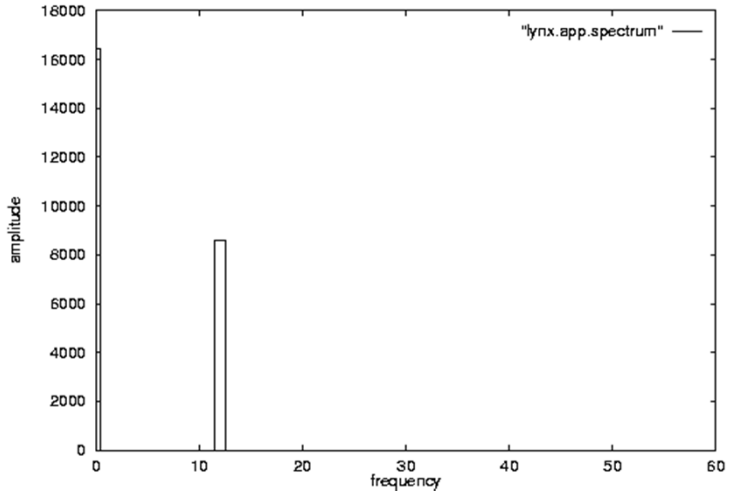


freq. *f*

DFT: Amplitude spectrum

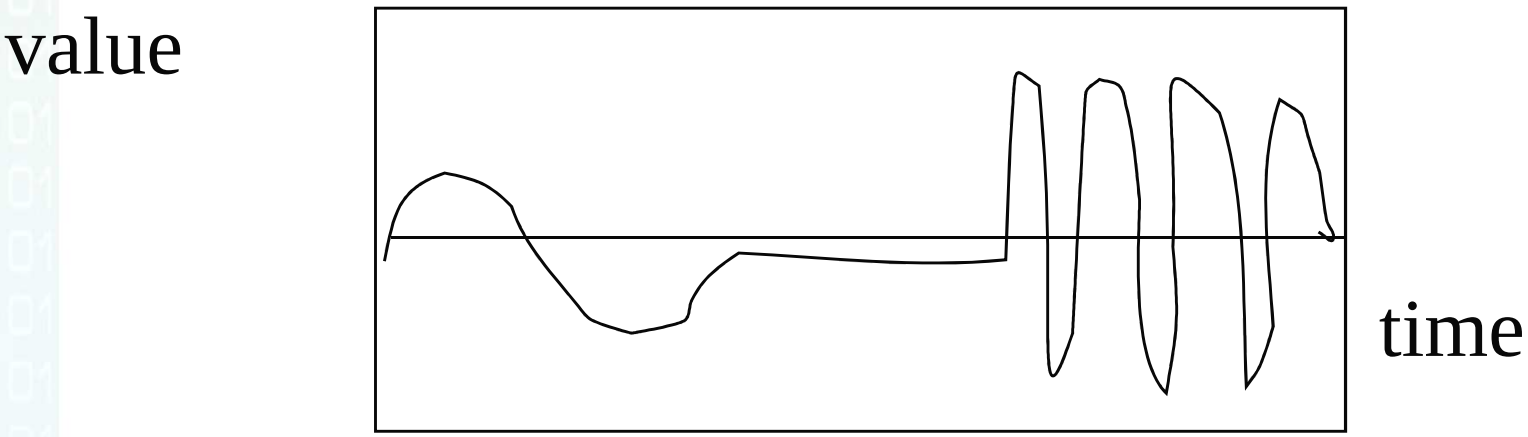
- excellent approximation, with only 2 frequencies!





Varying frequencies

- DFT is great - but, how about compressing opera? (baritone, silence, soprano?)



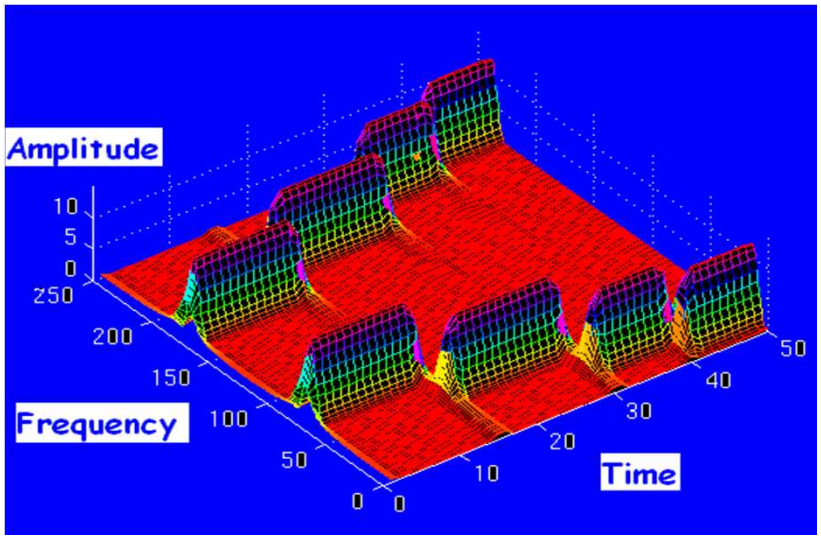
Short time Fourier transform (STFT)

- Solution#1: Short time Fourier transform
  - Apply DFT to sliding window
- But: how short should be the window?

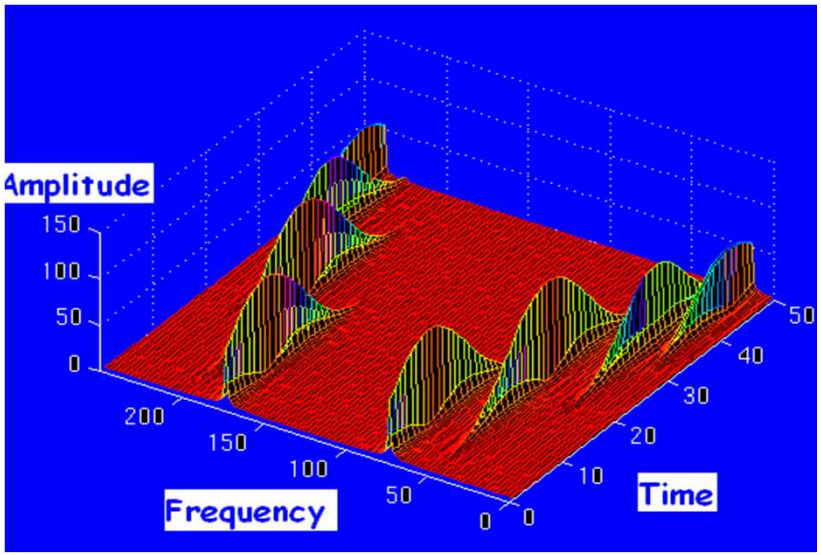
STFT drawbacks

- Unchanged Window
- Dilemma of Resolution
  - Narrow window -> poor frequency resolution
  - Wide window -> poor time resolution
- Heisenberg Uncertainty Principle
  - Cannot know what frequency exists at what time intervals

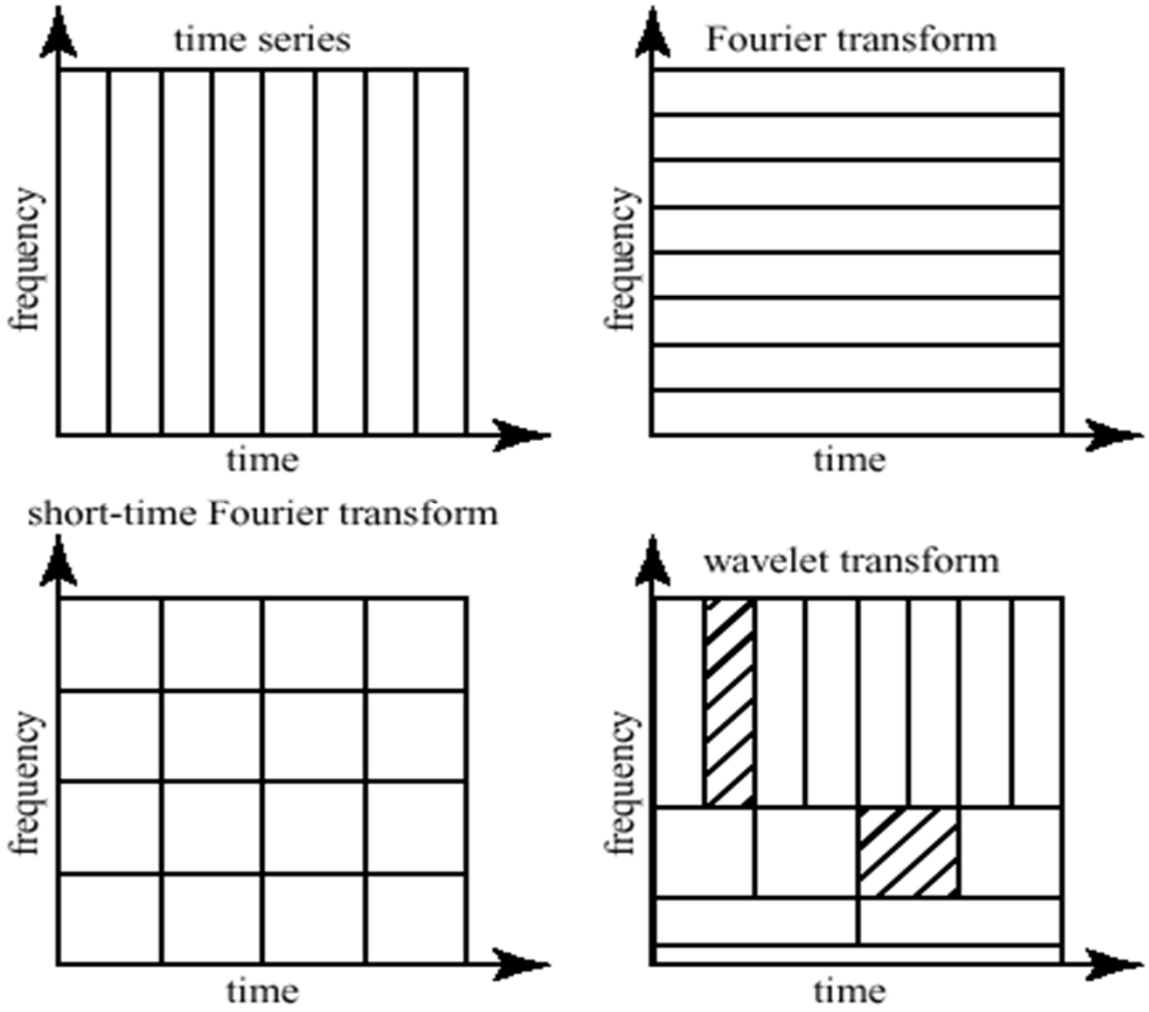
Via Narrow Window



Via Wide Window



Transformations

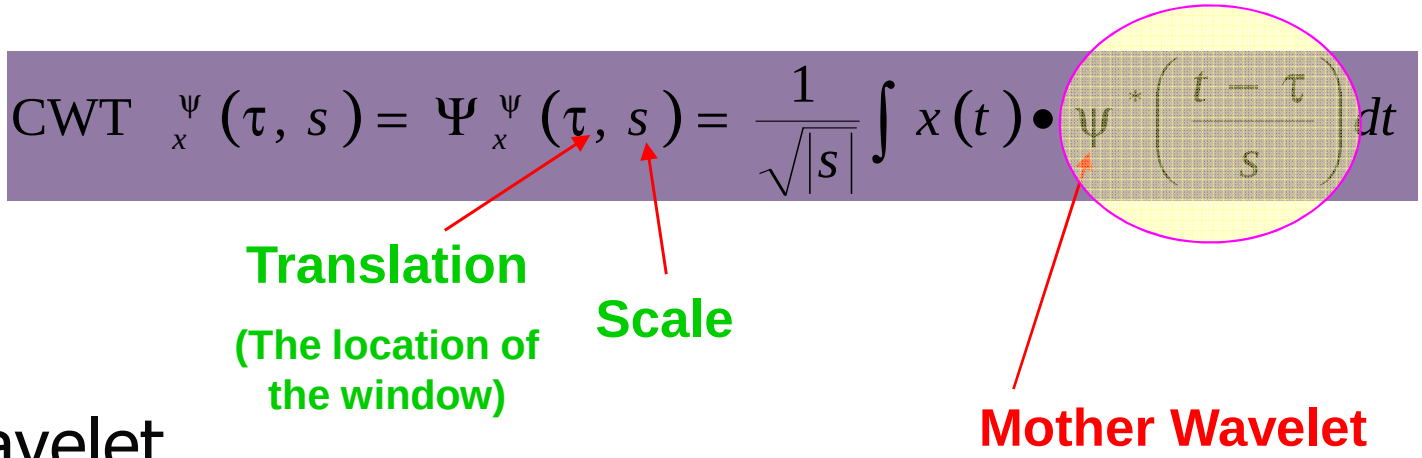


From http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10

Wavelet transform

- Split Up the Signal into a Bunch of Signals
- Representing the Same Signal, but all Corresponding to Different Frequency Bands
- Only Providing What Frequency Bands Exists at What Time Intervals

DEFINITION OF CONTINUOUS WAVELET TRANSFORM



- Wavelet
  - Small wave
  - Means the window function is of finite length
- Mother Wavelet
  - A prototype for generating the other window functions
  - All the used windows are its dilated or compressed and shifted versions

SCALE

- Scale
  - S>1: dilate the signal
  - S<1: compress the signal
- Low Frequency -> High Scale -> Non-detailed Global View of Signal -> Span Entire Signal
- High Frequency -> Low Scale -> Detailed View Last in Short Time
- Only Limited Interval of Scales is Necessary

CWT computation

$$\mathrm{CWT}_x^{\Psi}(\tau, s) = \Psi_x^{\Psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \bullet \psi^* \left(\frac{t-\tau}{s} \right) dt$$

Step 1: The wavelet is placed at the beginning of the signal, and set s=1 (the most compressed wavelet);

Step 2: The wavelet function at scale "1" is multiplied by the signal, and integrated over all times;

Step 3: Shift the wavelet to $t = \tau$, and get the transform value at $t = \tau$ and $s = 1$;

Step 4: Repeat the procedure until the wavelet reaches the end of the signal;

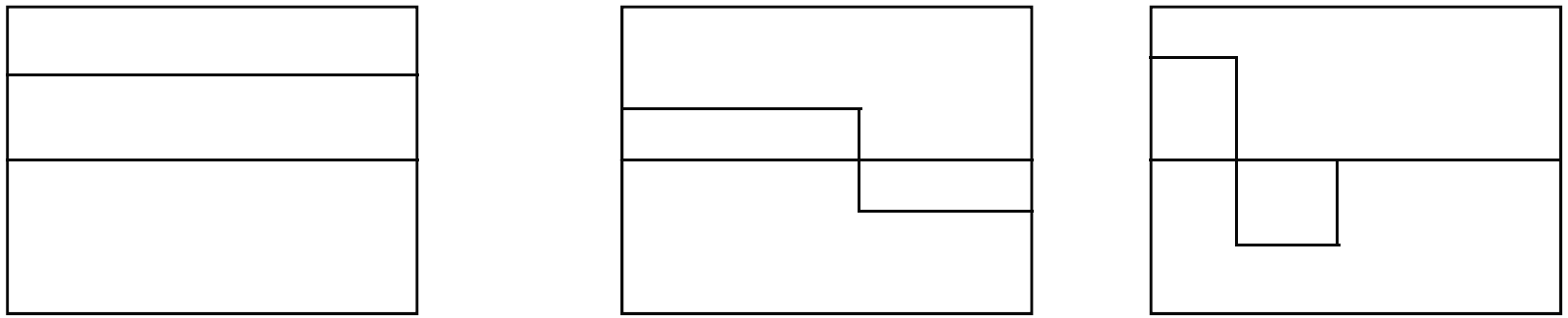
Step 5: Scale s is increased by a sufficiently small value, the above procedure is repeated for all *s;*

Step 6: Each computation for a given *s* fills the single row of the time-scale plane;

Step 7: CWT is obtained if all s are calculated.

Haar Wavelets

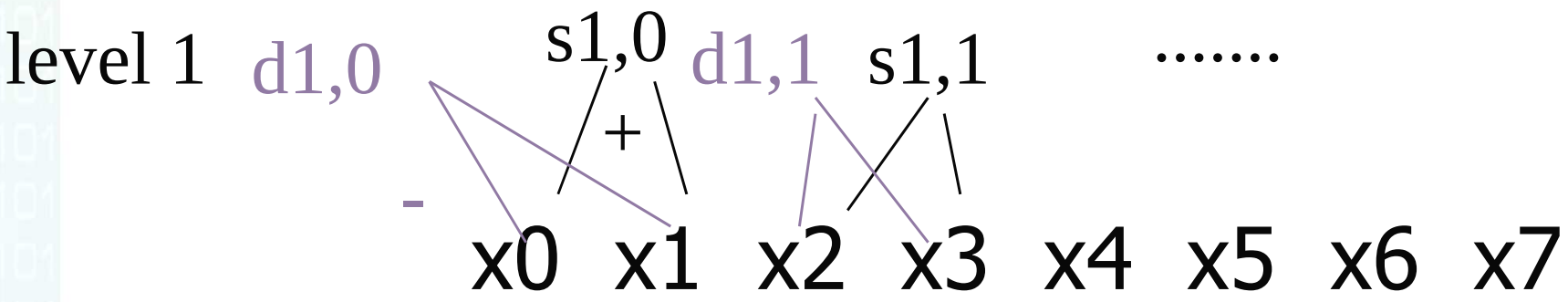
- subtract sum of left half from right half
- repeat recursively for quarters, eightths ...



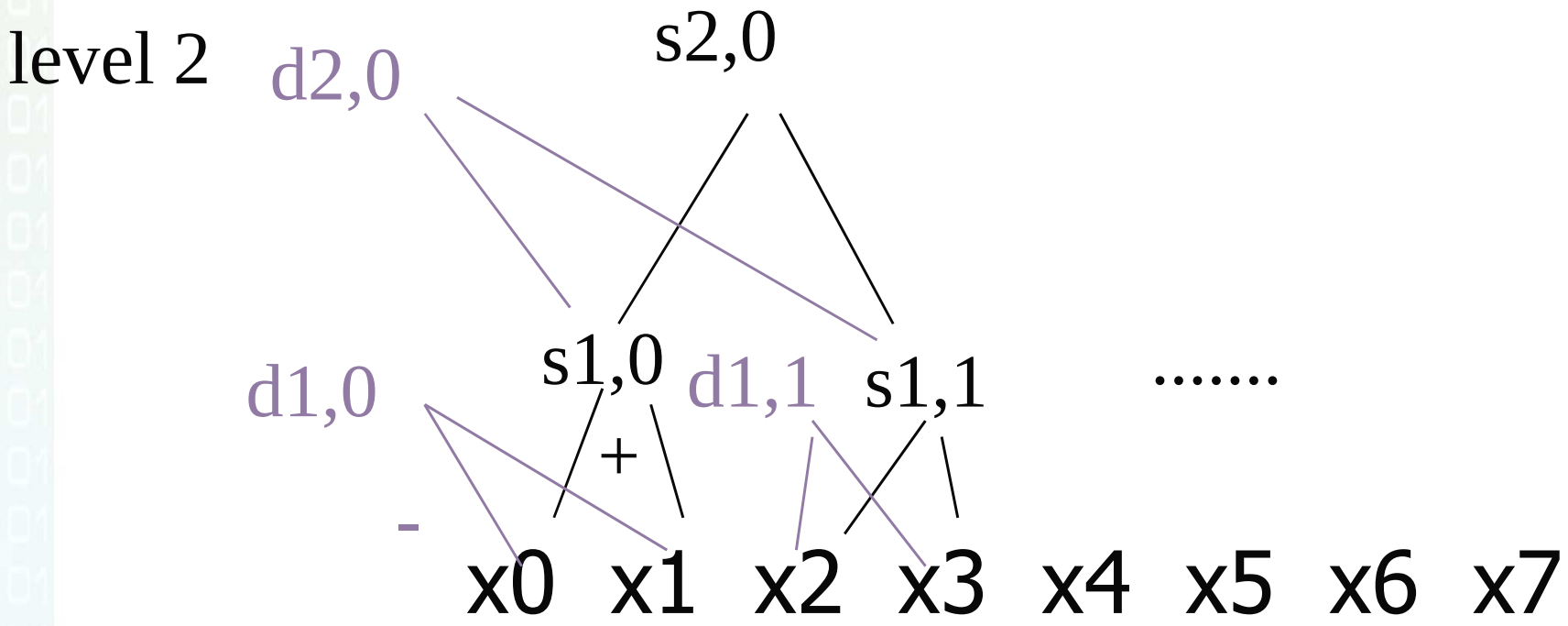
Wavelets - construction

x0 x1 x2 x3 x4 x5 x6 x7

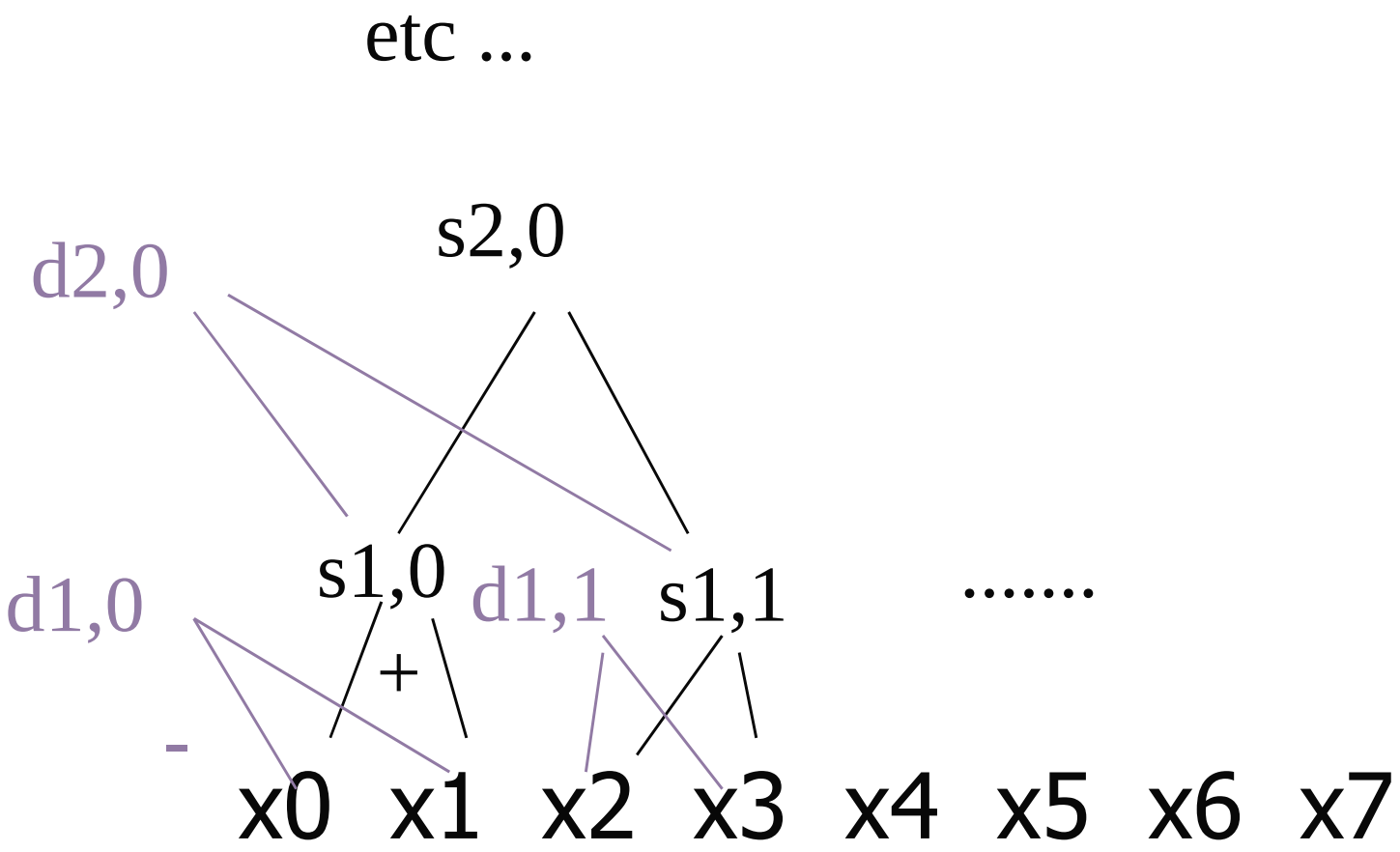
Wavelets - construction



Wavelets - construction



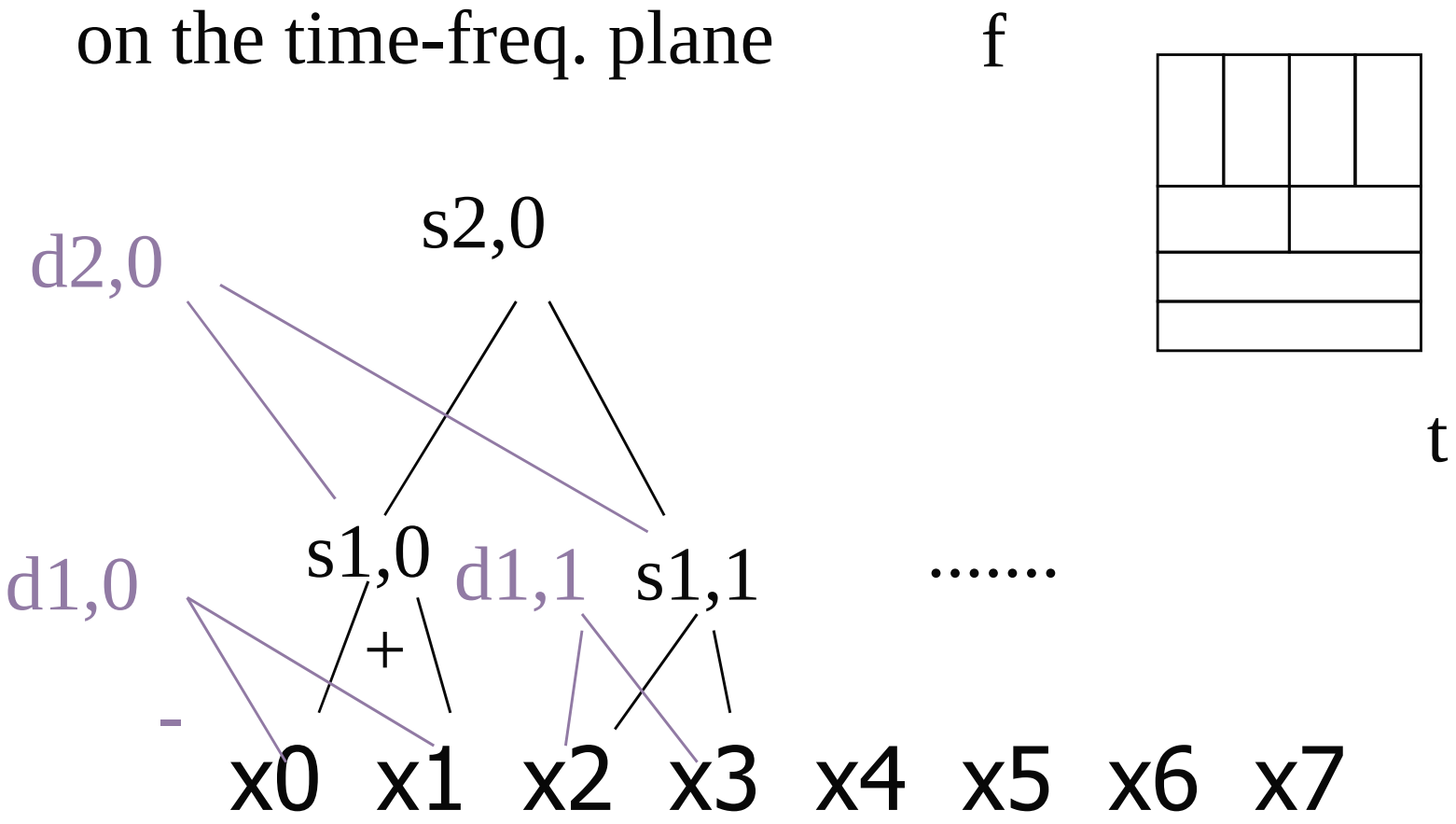
Wavelets - construction



Wavelets - construction

Q: map each coefficient

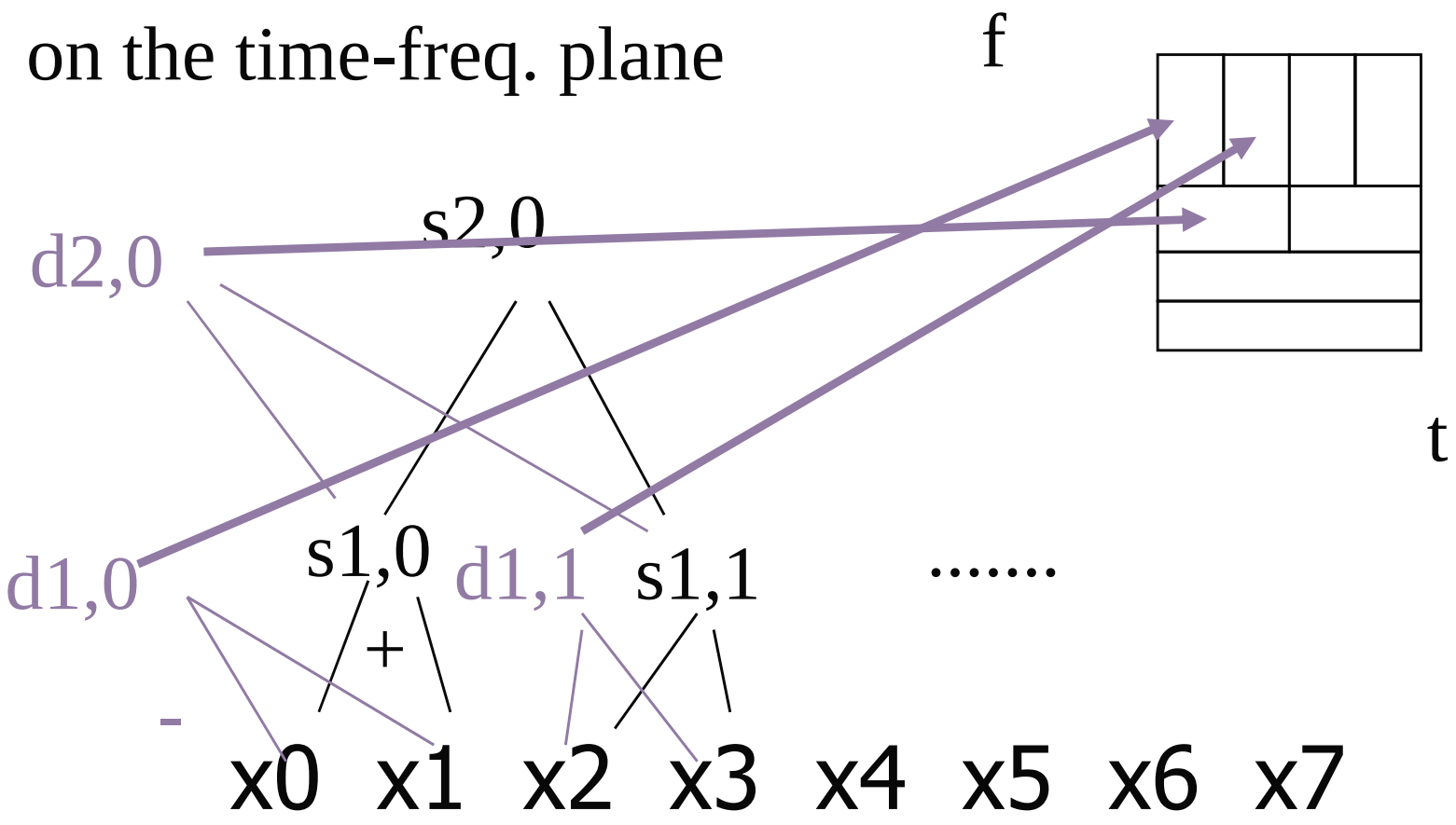
on the time-freq. plane



Wavelets - construction

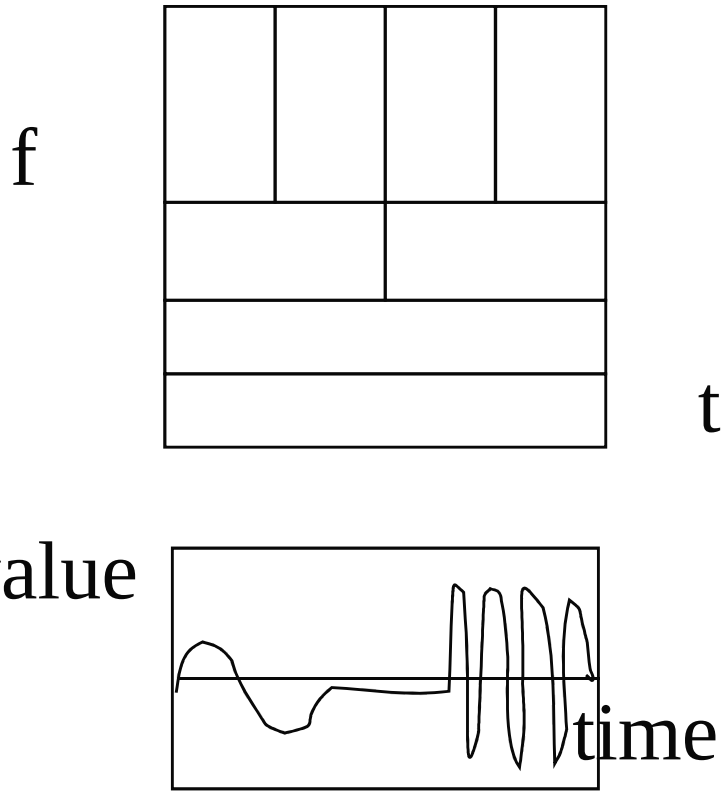
Q: map each coefficient

on the time-freq. plane



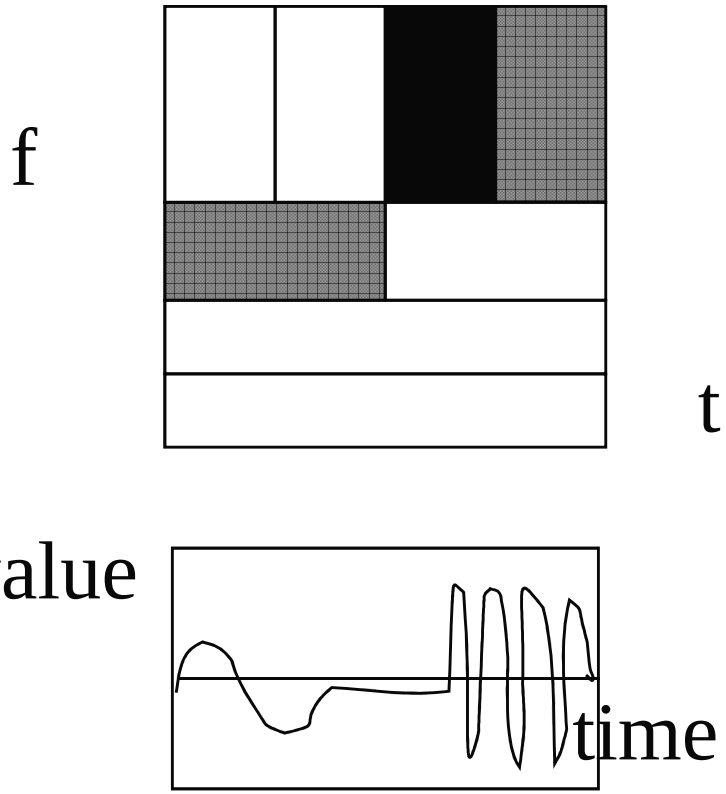
Wavelets - Drill:

- Q: baritone/silence/soprano - DWT?



Wavelets - Drill:

- Q: baritone/soprano - DWT?



Wavelets - construction

Observation1:

- ‘+’ can be some weighted addition
- ‘-’ is the corresponding weighted difference (‘Quadrature mirror filters’)

Observation2: unlike DFT/DCT,

- there are *many* wavelet bases: Haar, Daubechies-4, Daubechies-6, ...

Advantages of Wavelets

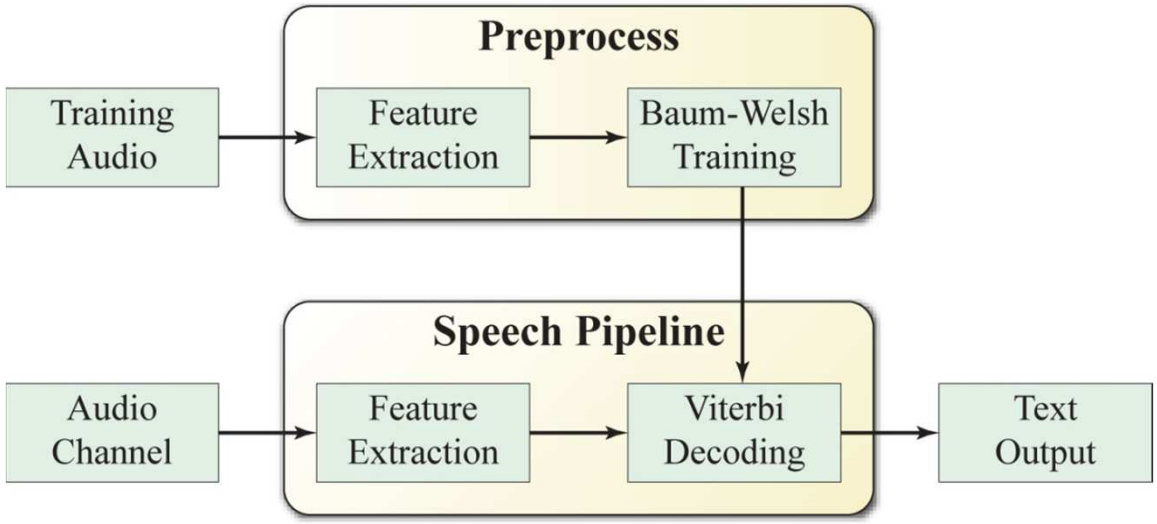
- Better compression (better RMSE with same number of coefficients)
- closely related to the processing of the mammalian eye and ear
- Good for progressive transmission
- handle spikes well
- usually, fast to compute (O(*n*)!)

Feature space

- Keep the d most “important” wavelets coefficients
- Normalize and keep the largest

Speech extraction and recognition

- System Components
  - **Feature Extraction**
    - Convert audio stream into feature vectors
  - Baum-Welsch Training
    - Calibrate HMMs using known data.
  - **Viterbi Decoding**
    - Recognition using HMMs.



How it is implemented at IBM Cell processors?

- Feature Extraction
  - Pipeline (12 stages):
    - Window Extraction
    - Zero Mean
    - Energy Computation
    - Preemphasis Filter
    - Hamming Window
    - Spectrum Computation (FFT)
    - Mel Frequency Computation
    - Cepstrum Computation
    - Discrete Cosine Transform
    - Lifter (Cepstral Filter)
    - Cepstrum Energy Normalization
    - First and Second Temporal Derivatives
  - → **Convert 25 ms frame of speech (with 15 ms overlap) into 39 coefficients (MFCCs).**
- Viterbi Decoding
  - Evaluate likelihood that a HMM produced a particular sequence of frames.
  - Dynamic Programming
  - HMM parameters
    - Left-Right model (only forward and self transitions; transition matrix is upper bi-diagonal)
    - Gaussian Mixture Model (PDFs are approximated by a set of Gaussians)
  - → **Very expensive computation; <u>bottleneck in speech pipeline</u>**

References

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