statisticsbadscience

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1 Exercise 1: Statistics cannot save bad science

History of science suggests that the basic rate at which hypotheses are true is small, between 1 and 5%. If we assume that the probability of a positive finding, given that an hypothesis is true, is 0.95. What is the probability that an hypothesis is actually true given a positive finding?

We use bayesian inference, similar to what we did last week with the werewolves. We know $P(\text{true}) = x$, where $x \in [0.01, 0.05]$ and $P(\text{positive} \mid \text{true}) = 0.95$. Thus we can reason that $P(\text{false}) = 1 - x$ and $P(\text{negative} \mid \text{true}) = 0.05$

$$P(\text{true} \mid \text{positive}) = P(\text{positive} \mid \text{true})P(\text{true})/P(\text{positive}).$$

Next we see that

$$P(\text{positive}) = P(\text{positive} \mid \text{true})P(\text{true}) + P(\text{positive} \mid \text{false})P(\text{false}),$$

which gives us the relation

$$P(\text{true} \mid \text{positive}) = \frac{0.95x}{0.95x + y(1-x)}.$$

Beacuse the system is underdetermined, we set $P(\text{positive} \mid \text{false}) = y$.

[1]:
```
# We define the two known variables for the calculation
tru <- 0.05
pos_tru <- 0.95
```