Exercise 11: Numerical integration

The normal function (normalized to total area 1) with mean value zero and sigma 1 is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \tag{1}$$

Evaluate

$$V(t) = \int_{-t}^{t} f(x) dx, \tag{2}$$

where $t = 1, 2, 3, 4, 5$ with absolute precision $\epsilon = 10^{-7}$, using the trapezoidal and Simpson rule. Compare convergence speed by changing n ($n = 2, 4, 8, 16...$). Compare the results obtained with the values in the table to calculate the absolute precision. Note that the value of π should be given with an appropriate precision in your code.

t	$V(t)$
1	0.68268949
2	0.95449974
3	0.99730020
4	0.99993666
5	0.99999943

For an observable which follows the normal distribution (Eq. (1)), the probability to observe a value smaller than $-t$ or larger than t is actually $P = 1 - V(t)$.